Pergamon

Cement and Concrete Research 33 (2003) 265–270

CEMENT AND CONCRETE RESEARCH

A model for the flow of cement pastes

H.W. Chandler[a], D.E. Macphee[b,*]

[a]Department of Engineering, Kings College, University of Aberdeen, Aberdeen AB24 3UE, UK
[b]Department of Chemistry, Kings College, University of Aberdeen, Meston Walk, Aberdeen AB24 3UE, UK

Received 4 January 2002; accepted 15 October 2002

Abstract

This paper presents a simple mathematical model of the flow behaviour of fresh cement pastes and mortars based on a soil mechanics approach. A soil mechanics model can mimic many of the features of the pastes rheological behaviour indicating that physical, as well as chemical, factors might control many of these.

Specifically, we have developed a simple mathematical model of the evolution of the packing of the clinker or sand grains and its effect on the flow of the paste or mortar. This not only incorporates the role of the pore water but also copes with the large deformations typical of the mixing and placement of mortars. This model is used here to simulate small cyclic deformations with a large drift accumulating in one direction, and is able to capture both the liquefaction and locking phenomena seen in practice when mortars are vibrated.

Keywords: Modelling; Rheology; Vibration; Workability; Mortar

1. Introduction

One of the most exciting challenges left to the mathematical modeller of the mechanical properties of materials is to understand and model the sometimes bizarre behaviour of fresh cement pastes and mortars. Anyone who has worked with fresh mortars or fresh concrete will have noticed some of these rather odd properties. Perhaps the most surprising are their ability to liquefy while vibrated and, while in this state, to 'lock-up' if deformed too quickly. Hence, one finds that it is much easier to move the vibrating poker slowly. This paper suggests one way of explaining these properties and presents a simple mathematical model of the flow behaviour of fresh cement pastes and mortars. It addresses the deformation of a paste made from inert particles so issues such as surface chemistry and continuing hydration reactions are ignored. The model can therefore only give a partial understanding of these materials. But it is notable, that even so, much of the behaviour of cement pastes can be mimicked in this way.

Previous studies have consistently found that, other than for very small strains [1], the phenomenological models best suited to the flow of cement pastes are either the Bingham fluid [2] or its variant the Herschel–Bulkley fluid [3]. Both these modes contain a yield strength. However, there appears to be no consensus concerning the reasons why the paste even has a yield strength, never mind why this reduces during vibration [4,5]. While issues of surface chemistry may dominate the rheology of pastes with moderate solids concentrations, cement pastes normally have a sufficiently high solids concentration that direct interparticle contacts become important. Such pastes appear to be better modelled using developments from soil mechanics [6].

The soil mechanics approach to the rheology of concentrated pastes is not new. The principle of effective stress [7] splits the externally applied pressure supported by the paste (p) into that carried by the particle skeleton (p') and that supported by the fluid (P). That is

$$p = p' + P \tag{1}$$

Only the pressure carried by the skeleton affects the mechanical properties and is known as the effective pressure. Originally, this idea was applied, very success-

* Corresponding author. Tel.: +44-1224-272941; fax: +44-1224-272921.

E-mail address: d.e.macphee@abdn.ac.uk (D.E. Macphee).

0008-8846/02/$ – see front matter © 2002 Published by Elsevier Science Ltd.
PII: S0008-8846(02)01024-4

fully, to systems without surface tension but Norton [8] realised that the surface tension was important in many circumstances. The curvature of fluid menisci at the surface induces an internal fluid tension that, even when the externally applied pressure is zero, has the effect of pulling clay particles together. The friction between these particles then gives the paste its yield strength. The higher the fluid tension, the higher the strength. This idea is useful enough to provide soil physicists [9] with an adequate understanding of soil's tensile strength and can be used to understand the behaviour of not just soils but also green ceramics and cement pastes or mortars. The effective pressure is always important even when surface tension is not. For cement or mortar in a mould, there can be excess water on the top surface so the surface tension will not induce a significant fluid tension. Instead, the weight of the cement provides sufficient interparticle contact forces, resulting in a positive p', to provide some shear strength. This, however, may vary during deformation. For example, cyclic shear during vibration starts to pack the particles more efficiently. This happens at the bottom layers first, which expel water. This water moves upward fluidising the upper layers. For pastes with most of their surface open to the atmosphere, say in a slump test after the mould has been removed, surface tension will have an important role. The curvature of the fluid menisci between the particles at the surface of the paste will control the fluid tension. This curvature is clearly dependent on the particle size, but the precise fluid content and the efficiency of particle packing are also important. It is well known that efficiency of packing reduces during monotonic deformation: a phenomenon known as dilatancy [10]. This can have a major effect on the pore tension. As the internal porosity increases, fluid is pulled in from the free surface to fill the space, increasing the interface curvature and so the fluid tension and strength of the paste. Conversely, cyclic deformation increases the efficiency of packing and has the reverse effect, lowering the fluid tension and so the strength. These ideas can predict some other familiar behaviour. For example, it is well known that a vibrating poker can only be moved slowly through fresh concrete as otherwise the resistance to lateral movement rises rapidly. This can be understood as follows. While the vibration produces cyclic shear that induces an *increase* in packing efficiency, the lateral movement produces a *reduction*, drying of the surface and consequently, a rapid increase in strength. It is the competition between these two processes, we suggest, that controls the rate at which the poker can usefully be moved.

In order to make these ideas more definite, we have developed a simple mathematical model of the evolution of packing and its effect on the strength of the paste. While many detailed constitutive models of soil behaviour have been developed to do this, they are largely restricted to small deformations. The problem of the flow of cement paste, whether vibrated or not, is certainly a problem of finite deformation and so most soil models are inappropriate. The work reported in this paper uses a recently developed one-dimensional model that, while giving similar results to existing soil models for small cyclic and monotonic deformation, also copes with large deformations. This model is used here to simulate small cyclic deformation with a large drift accumulating in one direction. This seems to be at least a start towards developing a model for the flow of vibrated cement pastes.

2. Development of the dry model

2.1. Evolution of volume

As identified above, tracking the evolution of packing efficiency is a key element in understanding the behaviour of pastes. To this end, consider an assembly of dry rigid particles occupying a volume v. When subjected to a simple shear (γ), the volume will change in various ways. For example,

- As the assembly of dry particles is initially sheared the volume usually starts to decrease and, depending on the initial state of packing, will either continue to reduce its volume at a decreasing rate, or start to increase its volume [11].
- If shearing is continued in one direction indefinitely, then the volume will reach the critical state where the volume remains constant. The relative density of this state is dependent on the effective pressure but not on the initial state of packing [11].
- On subsequent strain reversal the volume reduces rapidly [14].
- If the assembly is given small cyclic shears oscillating about any central value, then the volume at the central position will slowly decrease towards a limiting dense state [12].
- Ignoring the slow decrease in volume, the small volume changes either side of the central value of shear are approximately reversible [13].

In order to produce a model that simulates this diverse behaviour, first postulate a reference state whose motion lags behind the real deformation. Let its position, in terms of shear strain, be denoted by γ_m, which is updated by the differential equation

$$\dot{\gamma}_m = c_1(\gamma - \gamma_m)|\dot{\gamma}|, \tag{2}$$

where c_1 is a constant, and where $\dot{\gamma}$ represents the rate of change of γ. (This is the same for any variable throughout this paper.) Eq. (2) allows γ_m to lag behind γ while it attempts to catch up whenever $\dot{\gamma}$ is nonzero.

A further equation updates the current volume (v)

$$\dot{v}/v = c_2(v_D - v)|\dot{\gamma}| + c_3(\gamma - \gamma_m)(v_L - v)\dot{\gamma} \quad (3)$$

Consider the role of each term in turn:

- The first term of the right-hand side is made up from the product of:

(i) a material constant c_2;
(ii) the difference between the current volume and the volume of the densest state (v_D); and
(iii) the modulus of the increment in shear strain.

Overall, the term produces a tendency to reduce volume, until the densest state is reached, whatever the direction of strain.

- In the second term of the right-hand side c_3 is a constant. Therefore, if the value of γ_m, remains fixed, the contribution of $c_3(\gamma - \gamma_m)\dot{\gamma}$, on its own, gives exactly reversible parabolic volume changes with shear. However, the second term is modified by: (i) the closeness of the current volume to the upper limit of volume (v_L); and (ii) the fact that γ_m continually changes. The first of these produces a nonparabolic, but still reversible, volume change. The second makes the volume change irreversible.

The two differential equations (Eqs. (2) and (3)) therefore evolve the memory of past deformation held by γ_m and the current volume. With a suitable choice of constants, the results of simple shear experiments on a two-dimensional assembly of discs [14] can be reproduced as shown in Figs. 1 and 2.

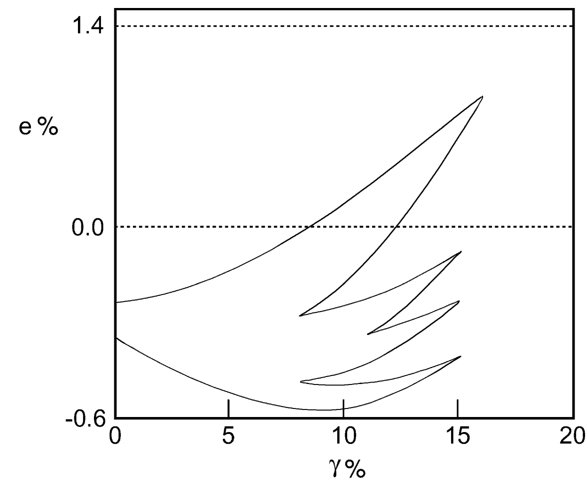

Fig. 2. Simulation of experimental results given in Fig. 1.

2.2. Resistance to flow

The shear stress τ producing the shear deformation, and the pressure restraining any volume increase, must do work on the assembly [15]. This gives rise to:

- The material storing energy by increasing the elastic energy and increasing the volume against an external pressure. The work done in increasing these can be recovered during strain reversal at constant pressure.
- The material dissipating energy by friction as the particles slide past one another. It should be noted that this is pressure dependent and the work done cannot be recovered.

If the effects of elasticity are ignored, then a rigid plastic approach leads to

$$\tau = p\mu + \frac{p\dot{v}}{v\dot{\gamma}} \quad (4)$$

where p is the applied pressure and μ is the coefficient of internal friction. However, a much closer approximation can be obtained by incorporating some elastic response as in the

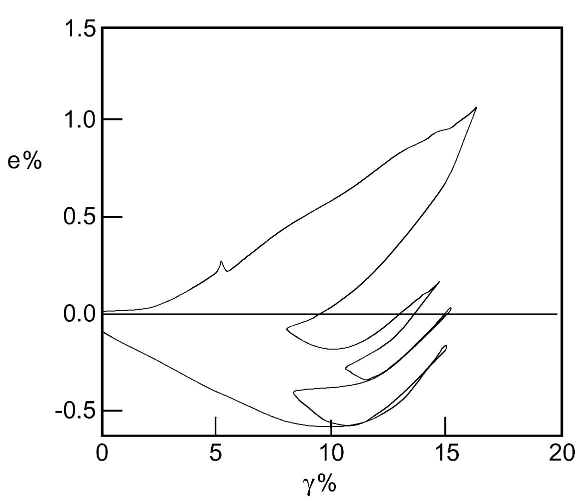

Fig. 1. Experimentally determined volume changes with simple shear deformation of a random assembly of cylinders. (Redrawn from Ref. [14].)

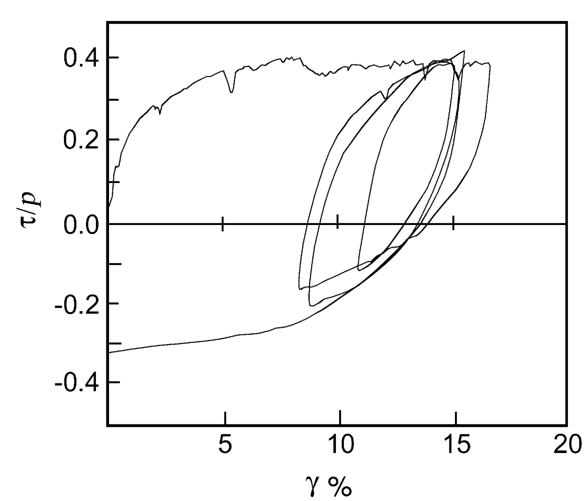

Fig. 3. Experimentally determined shear stress with simple shear deformation of a random assembly of cylinders. (Redrawn from Ref. [14].)

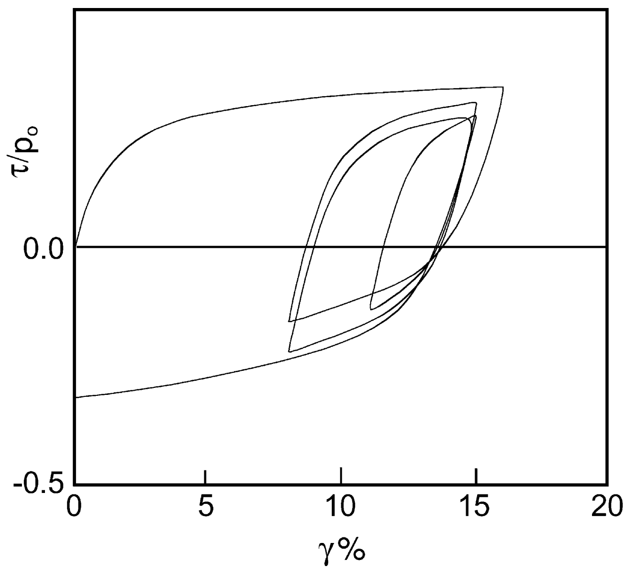

Fig. 4. Simulation of experimental results given in Fig. 3.

model developed earlier [16]. The evolution of τ is then governed by the differential equation

$$\dot{\tau} = \dot{\gamma}G - \frac{G\left(\tau - \frac{p\dot{v}}{v\dot{\gamma}}\right)|\dot{\gamma}|}{p\mu} \quad (5)$$

where G can be interpreted as the initial elastic shear modulus. This is an essentially rigid plastic model modified by the addition of elastic terms that have the effect of smoothing some of the abrupt changes in slope normally associated with a rigid plastic idealisation. Using this equation in conjunction with Eqs. (2) and (3), stress–strain curves can mimic the experimental results of Calvetti et al. [14] (see Figs. 3 and 4).

3. Application to wet systems and pastes

As mentioned in the Introduction, Terzaghi's [7] principle of effective stress governs the behaviour of granular materials. If the applied pressure is p and P is the pressure in the fluid, which may come from surface tension or from pressure build-up as a result of viscous resistance to the fluid flow through the paste, then the effective pressure (p') is given by

$$p' = p - P \quad (6)$$

which replaces the applied pressure p in all the constitutive equations. With this modification, Eq. (5) becomes

$$\dot{\tau} = \dot{\gamma}G - \frac{G\left(\tau - \frac{p'\dot{v}}{v\dot{\gamma}}\right)|\dot{\gamma}|}{p'\mu} \quad (7)$$

For the case of fluid stress induced by surface tension, a useful way forward is to relate the fluid tension to the volume nominally occupied by the particle assembly. A useful approximation, as far as these computations are concerned, is

$$P = P_o \exp(\lambda(v - v_o)/v_o) \quad (8)$$

where the subscript o indicates the values before deformation begins and λ can be thought of as a material constant. The fact that P does not reach zero, or take negative values, is a convenient computational device avoiding the consequences of division by zero.

Given a suitable choice of constants, the model of a paste comprising (Eqs. (2), (3) and (6)–(8) simulates quite well the cyclic shear stress–strain response of undrained tests on sands [17]. An attempt can therefore be made to simulate progressive flow of a cyclically sheared paste as time progresses. The cyclic shear is assumed to be sinusoidal with amplitude $C/2$ and frequency f while the drift in shear per complete cycle is denoted by D, giving an evolution with shear strain

$$\gamma = \left(\frac{C}{2}\right)\cos(2\pi ft) + Dft \quad (9)$$

and rate of shear strain

$$\dot{\gamma} = -\pi fC\sin(2\pi ft) + Df \quad (10)$$

For the purposes of illustration, three cases are presented in some detail. These are labelled I, II and III. In each case, C takes the value 0.05 and D takes the values: $\pi C/3$, $2\pi C/3$ and πC, respectively. It should be noted that with $D > \pi C$ that there is no reverse shear and, because the simulation is rate independent, the stress–strain curve in this case will

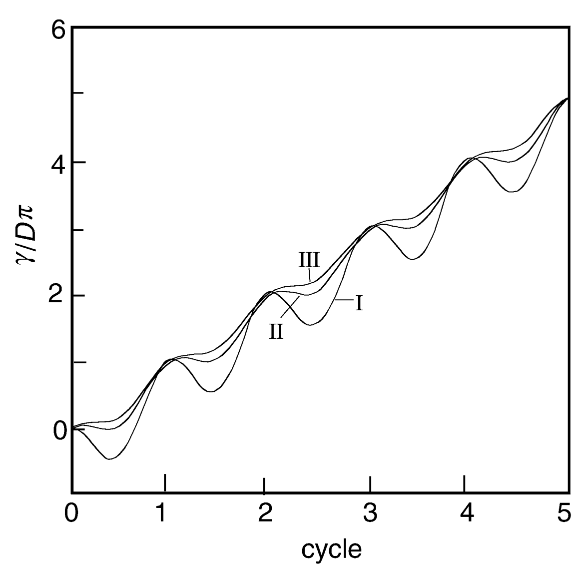

Fig. 5. Change in dimensionless shear strain with number of cycles.

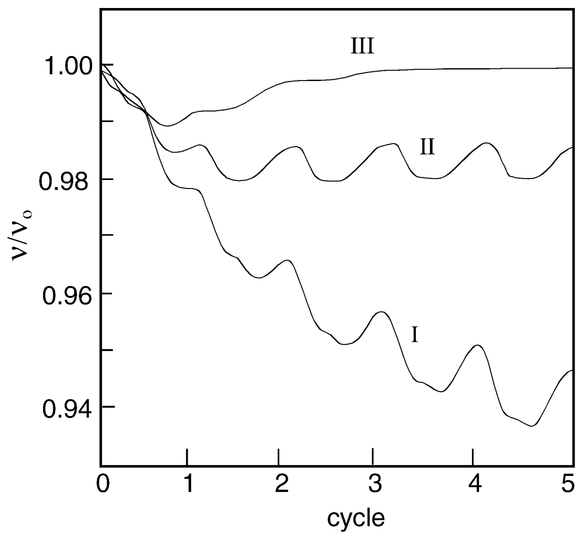

Fig. 6. Volume evolution with cycles.

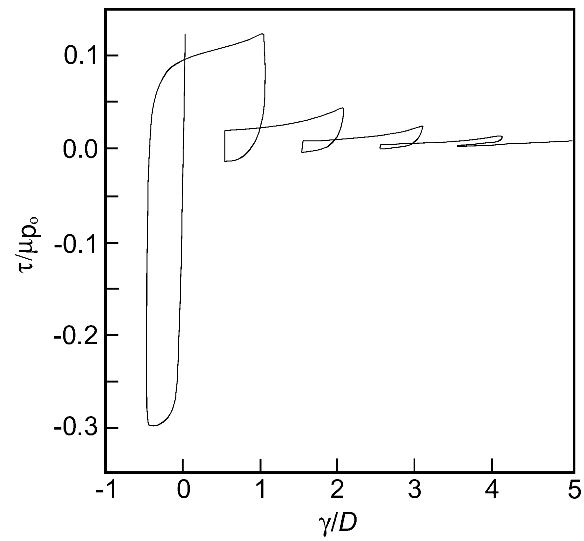

Fig. 8. Cyclic stress–strain plot for Case I.

be the same as a uniform drift with no sinusoidal contribution.

Euler's backward method was found to be satisfactory for obtaining numerical solutions to the differential equations which give accumulated shear strain, volume and shear stress varying with time. Typical plots of the change in overall shear strain with cycles of deformation are given in Fig. 5. The evolution of volume is shown in Fig. 6 and the development of shear stress needed to produce the deformation is shown in Fig. 7. This demonstrates that the shear stress for steady flow can be very much lower then when C is nonzero, i.e. cyclic deformation is superimposed on monotonic deformation. A shear stress–strain plot for Case I is given in Fig. 8.

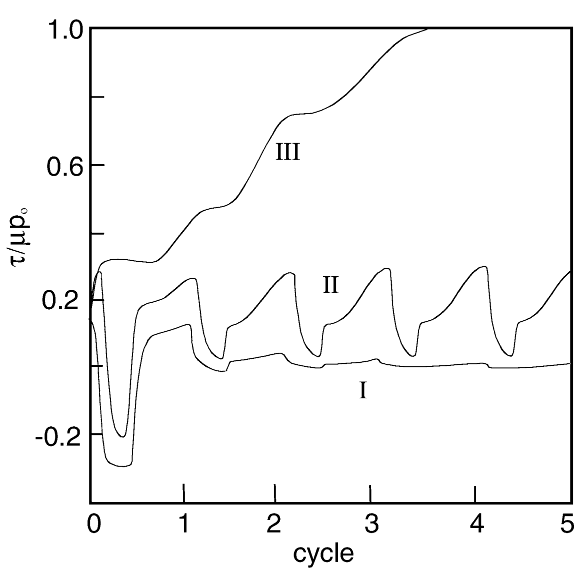

Fig. 7. Evolution of shear stress with cycles.

It is important to note that the simulation is rate independent. That is, the results do not depend on f, only on C and D, because no viscosity terms are included. For maximum effect, the drift and cyclic amplitude need to be in the appropriate ratio. If the drift per cycle becomes too large in comparison to the cyclic amplitude, then the steady state shear stress becomes large. Additionally, the cyclic amplitude can either be too small to have much effect or so large that little increase in packing efficiency is produced.

4. Discussion

The key development in being able to model the flow of pastes is the introduction of a moveable 'reference state' that allows 'fluid-like' and 'solid-like' properties to exist simultaneously. This idea was originally invented for pitch or polymer melts [18], where the memory of past deformation decayed with time. Such materials appear to be solid if deformed quickly, but fluid if deformed slowly. Armstrong and Frederick [19] developed the idea of a memory in the context of the kinematic hardening model for metal plasticity. In this case, rate effects are not important neither is the memory erased by the passing of time, but by further plastic deformation. Nixon [20] appears to be the first to apply this idea to granular materials.

As mentioned in the Introduction, the most striking behaviour of cement pastes and mortars is their ability to liquefy while being vibrated, and once in this state, to 'lock-up' if deformed too quickly. The model presented above reproduces this, and is based on the quasi-static behaviour of hard, chemically inert particles and gives results that reproduce a realistic response to cyclic deformation. This therefore provides a possible framework for an extensive experimental programme that would hopefully make clearer the interplay between chemical and physical factors in the rheology of cement pastes and give some more direct indication of whether or not the approach of this paper is a useful one.

The model suggests a number of further effects that could be reproduced experimentally and the results compared to the predictions of the model. The segregation processes taking place in a vibrated mould require considerable further work. The idea of a progressive layer of actively densifying material in a mould should be able to be tested experimentally. One obvious prediction would be that liquefaction would stop if the active layer reached the top free surface. Injecting water or changing the permeability could test the role of flowing water, and the role of surface tension could be investigated by the addition of detergents.

How important are the many influences not considered in this model?

- The inertia of particles is ignored, as is the possibility of particles temporarily losing contact with each other as they vibrate individually.
- The repulsive/attractive nature of the interparticle chemistry is ignored as is any change in these induced by vibration. Any change in the state of flocculation will influence whether or not the particles will agglomerate and so influence the permeability to flow of water.
- The beginnings of the hydration reaction will clearly replace some of the pore water with solid phase.
- The viscosity of the water is ignored so the plastic viscosity seen in unvibrated pastes cannot be reproduced.
- Brownian motion is also ignored, though for pastes containing very small particles this might produce a ‘natural’ cyclic deformation helping to keep the packing efficiency high even during monotonic deformation.

5. Conclusions

If suitably modified, models based on soil mechanics can reproduce at least some of the features of the finite deformation of cement pastes and mortars. In particular, the liquefying effects of cyclic deformation are linked to changes in packing efficiency, the expulsion of pore water and the principle of effective stress. It therefore seems clear that models developed from soil mechanics may have a useful role in understanding concentrated pastes. At the very least, they could help in identifying purely mechanical aspects of the rheology of pastes and so make the chemical contribution of additives more easily determined.

References

[1] L. Nachbaur, J.C. Mutin, A. Nonat, L. Choplin, Dynamic mode rheology of cement and tricalcium silicate pastes from mixing to setting, Cem. Concr. Res. 31 (2001) 183–192.
[2] G.H. Tattersall, P.F.G. Banfill, Rheology of Fresh Concrete, Pitman Advanced Publishing Programme, London, 1983.
[3] M. Cyr, C. Legrand, M. Mouret, Study of the shear thickening effect of superplasticizers on the rheological behaviour of cement pastes containing or not mineral additives, Cem. Concr. Res. 30 (2000) 1477–1483.
[4] M.F. Petrou, K.A. Harnes, F. Galdala-Maria, V.G. Kolli, A unique experimental method for monitoring aggregate settlement in concrete, Cem. Concr. Res. 30 (2000) 809–816.
[5] G.H. Tattersall, P.H. Baker, The effect of vibration on the rheological properties of fresh concrete, Mag. Concr. Res. 40 (13) (1988) 79–89.
[6] H.W. Chandler, S.D. George, J. Liddle, S.A. Nixon, Backlash, a feature of the cyclic deformation of ceramic pastes, J. Eur. Ceram. Soc. 20 (2000) 1699–1705.
[7] K. Terzaghi, Theoretical Soil Mechanics, Wiley, London, 1943.
[8] F.H. Norton, Fundamental study of clay: VIII. A new theory for the plasticity of clay–water masses, J. Am. Ceram. Soc. 31 (1948) 236–241.
[9] T.J. Marshall, J.W. Holmes, Soil Physics, Cambridge University Press, Cambridge, UK, 1979.
[10] O. Reynolds, On the dilatancy of media composed of rigid particles in contact, Philos. Mag. (Series 5) 20 (1885) 469–481.
[11] J.H. Atkinson, P.L. Bransby, The Mechanics of Soils: An Introduction to Critical State Soil Mechanics, McGraw-Hill, Maidenhead, 1978.
[12] T.L. Youd, Compaction of sands by repeated straining, J. Soil Mech. Found. Div. ASCE 98 (1972) 709–725.
[13] B. Balendran, S. Nemat-Nasser, Double sliding model for cyclic deformation of granular materials, including dilatancy effects, J. Mech. Phys. Solids 41 (1993) 573–612.
[14] F. Calvetti, G. Combe, J. Lanier, Experimental micromechanical analysis of a 2D granular material: Relation between structure evolution and loading path, Mech. Cohes.-Fric. Mater. 2 (1997) 121–163.
[15] S.A. Nixon, H.W. Chandler, On the elasticity and plasticity of dilatant granular materials, J. Mech. Phys. Solids 47 (1999) 1397–1408.
[16] H.W. Chandler, A Model for the Monotonic and Cyclic Deformation of Stiff Pastes, in preparation.
[17] F. Oka, A. Yashima, A. Tateishi, Y. Taguchi, S. Yamashita, A cyclic elasto-plastic constitutive model for sand considering a plastic strain dependence on the shear modulus, Geotechnique 49 (1999) 661–680.
[18] B. Bernstein, E.A. Kearsley, L.J. Zapas, A study of stress relaxation with finite strain, Trans. Soc. Rheol. 7 (1963) 391–410.
[19] P.J. Armstrong, C.O. Frederick, A mathematical representation of the multiaxial Bauschinger effect, Central Electricity Generating Board, Report No. RD/B/N/73, 1966.
[20] S.A. Nixon, Yield functions suitable for granular materials, PhD Thesis, University of Aberdeen, 1999.