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A generalized approach for the determination of yield stress by slump and slump flow

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Abstract

The slump test (ASTM C-143) is the most common method for assessing the flow properties of fresh concrete. Although slump provides a qualitative measure of workability, the relationship between slump and more quantitative rheological parameters is not fully understood. In this study, a dimensionless model relating slump to yield stress is further developed and generalized as a function of cone geometry. Yield stress measurements of cement paste are performed using a vane technique and compared with slump measurements using cylindrical and conical geometries. The cylindrical slump model is in excellent agreement with the experimental yield stress data obtained using the vane method. The data for the conical slump measurements fit the cylindrical model at low yield stress values, but the results deviate as the yield stress of the paste increases. Most of the other slump models available in the literature, including finite element models, predict the same yield stress for a given slump when converted to dimensionless form. The results suggest that a fundamental relationship exists between yield stress and slump that is independent of the material under investigation and largely independent of cone geometry.

Keywords: Rheology; Workability; Cement paste; Fresh concrete

1. Introduction

Among the most important properties of fresh concrete is "workability." Although the definition of workability has been debated between scientist and engineers for several decades, it generally refers to the consistency, flowability, pumpability, compactability, and harshness of a mix. Several tests have been developed to assess workability, including the slump, flow table, compacting factor, Vebe consitometer, and Kelly ball penetration test [1]. Although these methods are useful as quality control tools, they are largely qualitative measures based on arbitrarily defined scales.

As concrete construction applications become more demanding, there is increasing pressure on engineers to ensure high workability while at the same time maintain the structural properties necessary to meet design specifications. For advancements to be made in understanding and controlling the workability of fresh concrete, testing procedures and industrial standards must move to a more fundamental quantitative basis. Accordingly, workability should be defined in terms of established measurable rheological parameters such as yield stress and viscosity.

Several authors have acknowledged the need for a more quantitative measure of the fluidity of fresh concrete [1–4]. However, measuring the fundamental rheological properties of concrete is experimentally challenging due to the large particle size of the aggregates. Furthermore, the equipment used for field testing must be relatively inexpensive, easy to use, and sufficiently small to be of practical use at construction sites. For these reasons, there is renewed interest in understanding how the slump test correlates to the rheological properties of fresh concrete.

The slump test is the most widely used method to evaluate the fluid properties of fresh concrete. It was approved as an ASTM standard in 1922 and is also used throughout Europe and Asia as a standard technique to assess concrete workability [1]. The simplicity of the measurement and low cost are among the reasons why the slump test remains the most common method for quality control evaluations of fresh concrete in the field.

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The equipment for the slump test consists of a hollow frustum of a cone and a measuring device (e.g., ruler). The height of the cone specified in the ASTM standard concrete slump test (C-143) is 30.48 cm (12 in.). The diameter of the top and bottom of the cone is 10.48 cm (4 1/8 in.) and 20.64 cm (8 1/8 in.), respectively [5]. The cone is filled with fresh concrete and then lifted vertically. The height difference between the concrete (once it has come to rest) and the cone is called the "slump height" or "slump" (Fig. 1).

With the recent advances in self-compacting concrete, the slump height is often so large that it is difficult to distinguish different batches. The "slump flow" or diameter of the base of the slumped material is often used instead of the height measurement (Fig. 1).

Although the slump test has been used in the United States for over eight decades, analysis of the test results has never moved past a qualitative understanding. Fresh concrete is often described as "workable," "harsh," or "fluid" based on the results from the slump test [3]. It is extremely difficult, if not impossible, to fully understand the flow properties of fresh concrete using such qualitative descriptions.

Using the "two-point" method developed by Tattersall [6], Scullion concluded that slump has a negative power law dependence on yield stress and is largely independent of viscosity. Morinaga [7] also found an inverse relationship between slump and yield stress using a concentric cylinder concrete rheometer. Murata [8] confirmed the results of Morinaga [7] using normal and lightweight concrete and further suggested that slump is not influenced by viscosity.

Murata [8] also related slump to yield stress based on a simple force balance model. Christensen [9] corrected the integration errors in the original Murata [8] model and converted the units to dimensionless quantities. Christensen's [9] model is independent of the particular material under investigation and the size of the slump cone. However, Christensen [9] did not experimentally confirm the accuracy of the model.

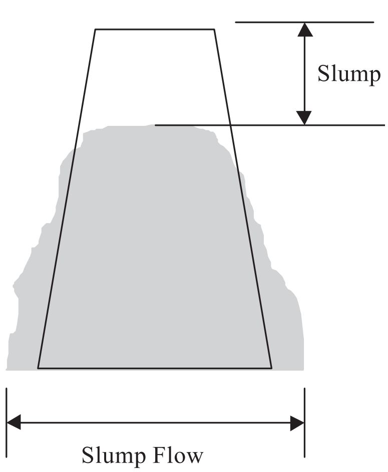

Fig. 1. ASTM standard slump test. The cone is filled with concrete and then lifted. The height difference between the top of the cone and the deformed material is called "slump." The diameter of base of the slumped material is often measured and designated "slump flow."

Recently, Pashias et al. [10] adapted the dimensionless slump model for cylindrical geometries. Using various types of clays, Pashias et al. [10] found excellent agreement between the model's predictions and experimental measurements of slump and yield stress. The study showed that the final slump of the material is largely independent of the lifting rate and surface on which the slump test is performed.

The dimensionless models developed by Christensen [9] for the ASTM standard slump cone and Pashias et. al. [10] for cylindrical slump provides several advantages over the traditional concrete modeling techniques. As will be discussed later, the use of dimensionless variables makes the slump–yield stress relationship material independent. Furthermore, the data suggest that the size and geometry of the slump cone (or cylinder) are not critical to the fundamental slump–yield stress relationship.

2. Yield stress measurements

One of the great challenges in modeling the relationship between yield stress and slump is accurately measuring yield stress. The most common method used to determine the yield stress of suspensions such as concrete is by extrapolating the stress–strain rate flow curve to zero strain rate through the use of an appropriate model [4]. The most widely used model is Bingham's linear approximation for flow of a viscoelastic material [11]. Although the simplicity of the Bingham model is attractive, the stress–strain rate relationship of concentrated suspensions, such as cement paste and concrete, is highly psuedoplastic and only approximates linearity as the particles become deflocculated or at very high shear rates [12]. At best, the Bingham model provides an estimate of yield stress at low solids concentrations and loses accuracy as the solids concentration increases [13].

Directly measuring the stress required to initiate flow is the best method for determining yield stress. This type of test is performed by applying a stress ramp or by a similar type of creep recovery experiment using a rheometer. Another common approach for measuring yield stress is to apply a constant rotational rate and monitor the development of stress as a function of time. Both methods have been shown to produce similar values for yield stress [14].

A controlled stress (or strain rate) rheometer with smooth-walled coaxial cylinders is commonly used for yield stress measurements. However, these systems often show pronounced "wall slip" due to the displacement of the disperse phase away from the solid boundaries [15]. Slip is the result of both static effects and dynamic forces (e.g., hydrodynamic, shear force gradient) that develop during testing [16].

A vane, as shown in Fig. 2, is often used to avoid slip. In this test, yielding occurs along the localized cylindrical

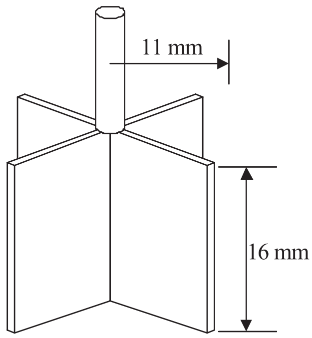

Fig. 2. Vane used for yield stress measurements (cup inner radius = 22 mm).

surface circumscribed by the vane. This prevents the possibility of slip since the material yields within itself. Finite element modeling and experimental data have confirmed the cylindrical conformity of the yielded surface [17]. Haimoni and Hannant [18] showed that the yield stress of cement paste determined using a vane was approximately twice the value obtained when smooth-walled coaxial cylinders were used. These results have been confirmed in our laboratory and the influence of slip has been shown to be more pronounced as the concentration of solids increases [19].

3. Objectives

The objectives of this study are threefold. First, the dimensionless slump model originally developed by Murata [8] is generalized, making it applicable for any cone geometry. Secondly, the accuracy of the model is experimentally confirmed. Conical and cylindrical geometries are used to measure the slump behavior of cement paste, and yield stress is directly measured using a vane to eliminate the possibility of wall slip. Finally, the results are compared with other models recently developed for concrete relating slump to yield stress.

4. Generalized slump model

Using the assumptions of the original Murata [8] model, the pressure exerted on any horizontal layer at a distance z in a slump cone is given by:

$$p_z = \frac{W_z}{\pi r_z^2} \quad (1)$$

where W_z is the weight of the material from the top of the slump cone to z, and r_z is the radius of the cone at z (Fig. 3). The weight of the sample is the product of the sample density, volume, and the acceleration due to gravity.

$$W_z = \rho g V_{z'} \quad (2)$$

The volume of a cone is expressed as:

$$V = \frac{1}{3}\pi r^2 h \quad (3)$$

where r is the radius of the cone and h is the height.

Referring to Fig. 3a, the volume of the top section (h_t) and the entire cone ($h_t + H$) is given by:

$$V_t = \frac{1}{3}\pi r_t^2 h_t \quad (4)$$

$$V_c = \frac{1}{3}\pi r_c^2 (h_t + H) \quad (5)$$

The volume at any position z in a cone of height ($h_t + z$) is then expressed as:

$$V_z = \frac{1}{3}\pi r_z^2 (h_t + z) \quad (6)$$

The total volume of sample in a slump cone at any position z is given by subtracting Eq. (4) from Eq. (6):

$$V_{z'} = \frac{1}{3}\pi r_z^2 (h_t + z) - \frac{1}{3}\pi r_t^2 (h_t) \quad (7)$$

Rearranging yields:

$$V_{z'} = \frac{1}{3}\pi (r_z^2 z + r_z^2 h_t - r_t^2 h_t) \quad (8)$$

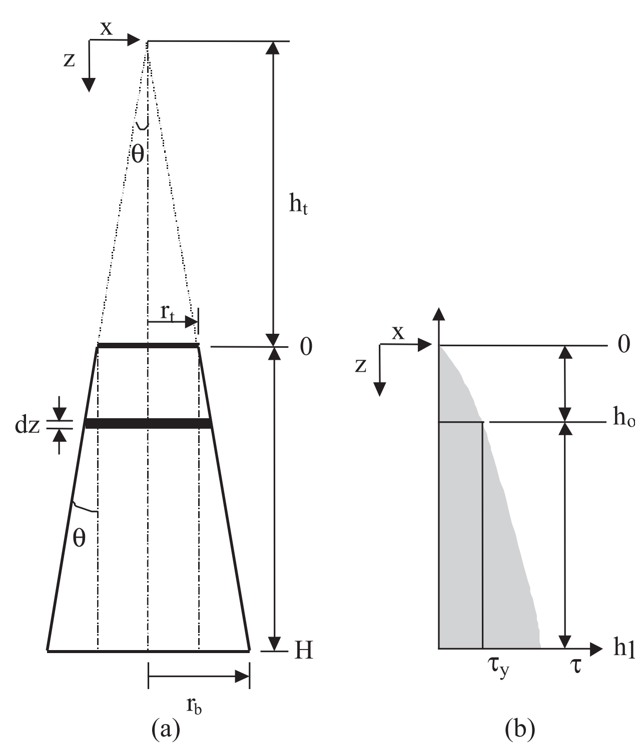

Fig. 3. (a) Slump cone and (b) stress distribution before lifting. The stress is greatest at the bottom of the cone due to the weight of the sample. The stress in the sample is equal to the material's yield stress at a distance h_o from the top of the cone.

Combining (Eqs. (1), (2), and (8) gives:

$$p_z = \left(\frac{\rho g}{3}\right)\frac{\pi(r_z^2 z + r_z^2 h_t - r_t^2 h_t)}{\pi r_z^2} \tag{9}$$

which can be reduced to:

$$p_z = \left(\frac{\rho g}{3}\right)\left[(z+h_t) - \left(\frac{r_t^2}{r_z^2}\right)h_t\right] \tag{10}$$

The radius at the top of the slump cone (r_t), bottom of the slump cone (r_H), and at any position z (r_z) are given by:

$$r_t = h_t \tan\theta \tag{11}$$

$$r_H = (h_t + H)\tan\theta \tag{12}$$

$$r_z = (h_t + z)\tan\theta \tag{13}$$

respectively. Substituting Eqs. (11) and (13) into Eq. (10) yields:

$$p_z = \left(\frac{\rho g}{3}\right)\left[(z+h_t) - \left(\frac{(h_t)\tan\theta}{(z+h_t)\tan\theta}\right)^2 h_t\right] \tag{14}$$

which can be reduced to:

$$p_z = \left(\frac{\rho g}{3}\right)\left[(z+h_t) - \frac{(h_t)^3}{(z+h_t)^2}\right] \tag{15}$$

The maximum shear stress acting on a plane is one half the applied pressure, assuming the Tresca criteria. This is expressed as:

$$\tau_z = \left(\frac{\rho g}{6}\right)\left[(z+h_t) - \frac{(h_t)^3}{(z+h_t)^2}\right] \tag{16}$$

Normalizing the shear stress by the height of the slump cone and material density produces the dimensionless shear stress given by:

$$\tau_z' = \frac{\tau_z}{\rho g H} = \left(\frac{1}{6H}\right)\left[(z+h_t) - \frac{(h_t)^3}{(z+h_t)^2}\right] \tag{17}$$

Thus, the dimensionless shear stress experienced by a material at a position z is given only in terms of the cone geometry, independent of the specific material under examination.

Assuming the material is incompressible, the volume of layer dz will be the same both before and after the slump cone is lifted (Fig. 4). Thus, the volume can be expressed as:

$$dz(\pi r_z^2) = dz_1(\pi r_{z_1}^2) \tag{18}$$

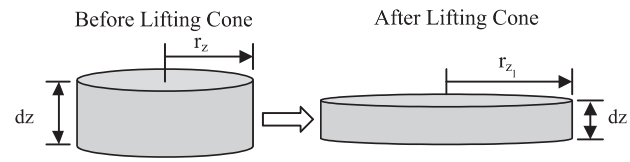

Fig. 4. Change in layer of thickness dz before and after lifting the cone.

where dz and dz_1 are the height of a layer initially at distance z before and after the slump cone is lifted, respectively. Eq. (18) can be written as:

$$dz_1 = dz\left(\frac{r_z}{r_{z_1}}\right)^2 \tag{19}$$

Referring to Figs. 4 and 5, the height h_1 of the deformed section of the slumped material can be expressed as the sum of all elements dz_1. Mathematically, this gives the integral:

$$h_1 = \int_{h_o}^{H} dz_1 \tag{20}$$

Substituting Eq. (19) into Eq. (20) yields:

$$h_1 = \int_{h_o}^{H}\left(\frac{r_z}{r_{z_1}}\right)^2 dz \tag{21}$$

It is assumed that there is no flow between layers of the material and the material is incompressible. Thus, the cross-sectional area of a layer in region h_1 will increase until the applied shear stress is equal to the yield stress. This relationship can be expressed as:

$$\tau_z(\pi r_z^2) = \tau_{z_1}(\pi r_{z_1}^2) = \tau_y(\pi r_{z_1}^2) \tag{22}$$

$$\tau_z r_z^2 = \tau_y r_{z_1}^2 \tag{23}$$

$$\frac{r_z^2}{r_{z_1}^2} = \frac{\tau_y}{\tau_z} \tag{24}$$

Substituting Eq. (24) into Eq. (21) gives:

$$h_1 = \int_{h_o}^{H}\left(\frac{\tau_y}{\tau_z}\right)dz = \int_{h_o}^{H}\left(\frac{\tau_y'}{\tau_z'}\right)dz \tag{25}$$

Combining Eqs. (25) and (17) provides the solution in terms of dimensionless yield stress:

$$h_1 = \int_{h_o}^{H}\left\{(6H)\frac{\tau_y'}{\left[(z+h_t) - \frac{(h_t)^3}{(z+h_t)^2}\right]}\right\}dz \tag{26}$$

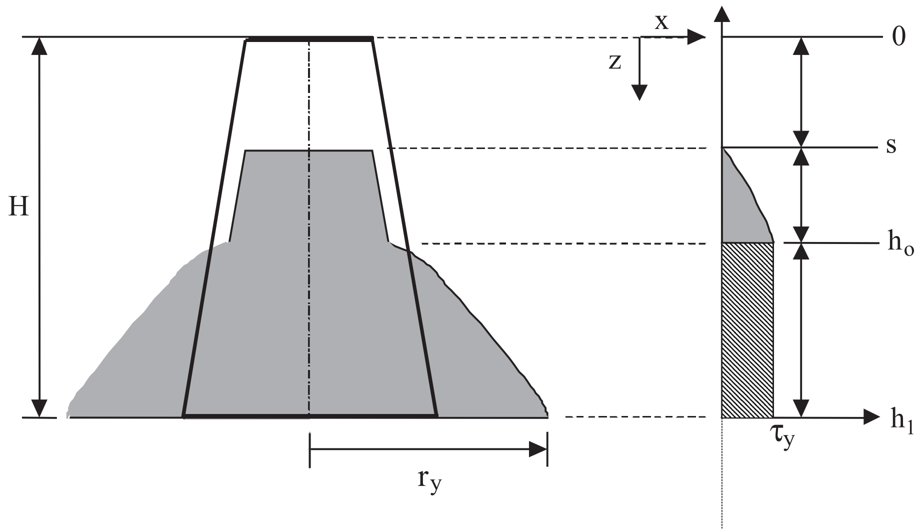

Fig. 5. "Slump" of a material after the cone is lifted and associated stress distribution.

Integrating Eq. (26) yields:

$$h_1 = (2H\tau_y')\left[\ln\left(\frac{H}{h_o}\right) + \ln\left(\frac{H^2 + 3h_t(H + h_t)}{h_o^2 + 3h_t(h_o + h_t)}\right)\right] \quad (27)$$

Referring to Fig. 5, slump is the distance between the top of the cone and the height of the material at rest after the cone is lifted. This can be expressed as:

$$s = H - h_o - h_1 \quad (28)$$

Normalizing slump to the height of the cone (H) yields an equation for dimensionless slump.

$$s' = \left(\frac{s}{H}\right) = 1 - \left(\frac{h_o}{H}\right) - \left(\frac{h_1}{H}\right) \quad (29)$$

The shear stress at h_o will be equal to the material's yield stress (Fig. 5).

Using Eq. (17), the dimensionless yield stress can be expressed in terms of h_o as:

$$\tau_y' = \left(\frac{1}{6H}\right)\left[(h_o + h_t) - \frac{(h_t)^3}{(h_o + h_t)^2}\right] \quad (30)$$

Rearranging Eq. (30) provides the solution for h_o in terms of dimensionless yield stress and the geometry of the cone.

$$h_o = 2H\tau_y + \frac{(2H\tau_y)^2}{a} + (a) - h_t \quad (31)$$

where:

$$a = \left[\left(\frac{1}{2}\right)(h_t^3 + 16(H\tau_y)^3 + \sqrt{h_t^3[h_t^3 + 32(H\tau_y)^3]})\right]^{1/3} \quad (32)$$

It is desirable to express h_t in terms of easily measured slump cone dimensions. Combining Eqs. (11) and (12) gives:

$$\frac{r_H}{r_t} = \frac{(h_t + H)\tan\theta}{(h_t)\tan\theta} \quad (33)$$

$$r_H h_t - r_t h_t = r_t H \quad (34)$$

$$h_t = H\left(\frac{r_t}{r_H - r_t}\right) \quad (35)$$

Slump can then be related to yield stress by substituting Eqs. (27), (31), (32), and (35) into Eq. (29). The units are dimensionless for both slump and yield stress, normalized by the height of the slump cone and material density. This generalized, three-dimensional model for slump provides an advantage over the earlier work of Murata [8], Christensen [9], and Pashias et al. [10] in that the slump of any material can be related to yield stress as a function of cone geometry.

5. Experimental procedure

Type I Portland cement and deionized water were used for all experiments at water to cement ratios (w/c) ranging from 0.30 to 0.45. Various amounts of fly ash, silica fume, and superplasticizers were incorporated into the paste to give a wide range of yield stress values. Each batch was mixed in a mechanical mixer conforming to the specifications of ASTM C 305 [20]. The water was first placed in the mixing bowl and the cement was added over a 1-min period while the mixer was at setting No. 1. The batch was then mixed at setting No. 2 for 2.5 min, after which the sides of the bowl were scraped with a rubber paddle. Finally, the sample was mixed for an additional 2.5 min for a total of 5 min of mixing at setting No. 2. This mixing procedure was strictly followed to avoid any experimental complications arising from incomplete mixing [21].

A Haake Rheostress 150 rheometer was used for all of the testing. The dimensions of the vane used for the yield stress experiments are given in Fig. 2. The inner radius of the cup used with the vane was 22 mm, conforming to the suggested procedure of Dzuy and Boger [15]. A solvent trap was used to prevent drying of the samples during testing.

5.1. Yield stress measurements

Cement paste was placed in the rheometer immediately after mixing. Yield stress was measured via a stress growth experiment [10]. A constant rotational speed of 0.01 rad/s was inputted into the rheometer test program. Shear stresses developed in the sample as the rheometer tried to move the spindle to meet the designated rotational speed. Initially, the sample deforms elastically due to stretching of the bonds in the network structure (Fig. 6). At some point, the network begins to break under the shear stress as some of the bonds reach their elastic limit. The development of viscoelasticity is represented by the departure from linearity of the shear stress time profile as shown in Fig. 6. At some point, the structure breaks down completely and a maximum stress is obtained (i.e., the yield stress). Finally, the stress decays to some equilibrium value as the vane begins to rotate. For this paper, the reported yield stress refers to the peak of the shear stress time profile as represented in Fig. 6.

5.2. Slump measurements

Both cylinder and cone were used for the slump tests. The cylinder was made of PVC plastic and the height to diameter ratio was 0.96 (Fig. 7). The slump cone was made of brass with the dimensions given in Fig. 7.

Slump measurements were conducted immediately after mixing. The cone (or cylinder) was filled gradually using a small stirring rod to consolidate the material in the mold. After the top of the cone was leveled off, it was slowly lifted by hand. The speed of lifting was slow enough as to not

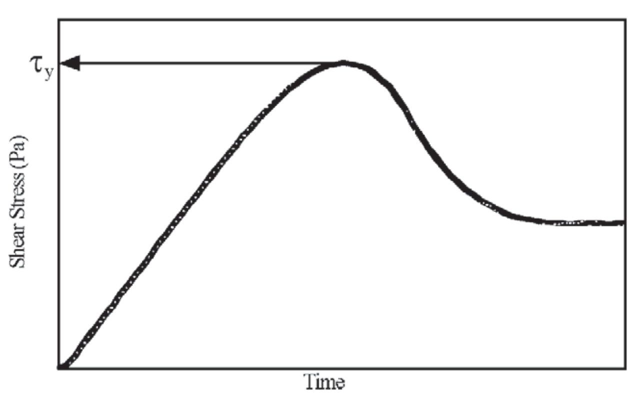

Fig. 6. Stress response for constant rotational speed yield stress measurements. The peak stress corresponds to the yield stress (τ_y) of the material.

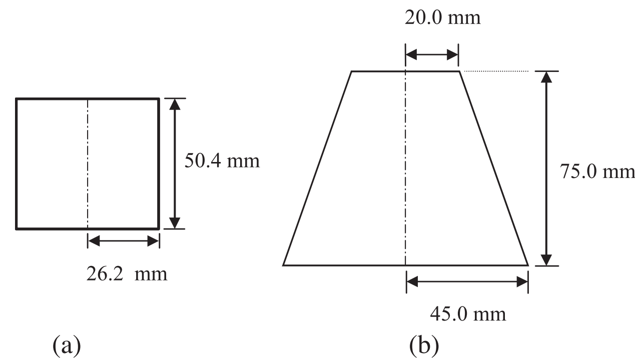

Fig. 7. (a) PVC cylinder and (b) brass cone used for slump testing.

disturb the natural slumping of the material. The slump height was taken as the maximum height in the center of the material. Measurements were made to the nearest 0.5 mm.

5.3. Slump flow measurements

The area of the slumped material over the surface was also determined. In most cases, the slumped area was circular, indicating a uniform deformation throughout the material. The area (or radius) at the base is commonly called the slump spread or slump flow. This measurement technique is used extensively for evaluation of self-compacting concrete due to the low yield stress of such materials.

Approximately 24 h after the slump test was performed, a trace of the base of the hardened slumped paste was made. The trace was then converted to a digital image and an image analysis program was used to determine the area of the final spread. The original area of the bottom of the slump cone (or cylinder) was divided by the final spread area to give dimensionless slump flow area. The relationship has the form:

$$f' = \frac{A_c}{f_a} \qquad (36)$$

where f' is the dimensionless slump flow area, A_c is the area of the base of the cone (or cylinder), and f_a is the slump flow area of the hardened paste.

6. Results and discussion

The data from the slump measurements made using the cylinder are plotted with the results of Pashias et al. [10] in Fig. 8. The fit of the data to the model is excellent. Furthermore, the data are in agreement with the results for clays and ceramic suspensions from Pashias et al. [10]. Given the wide range of materials used and difference in cylinder size in comparison to the work of Pashias et al. [10], the data suggest that the dimensionless scaling of yield stress and slump is correct.

The results from the cone slump experiments are given in Fig. 9. The data do not fit the model for the cone.

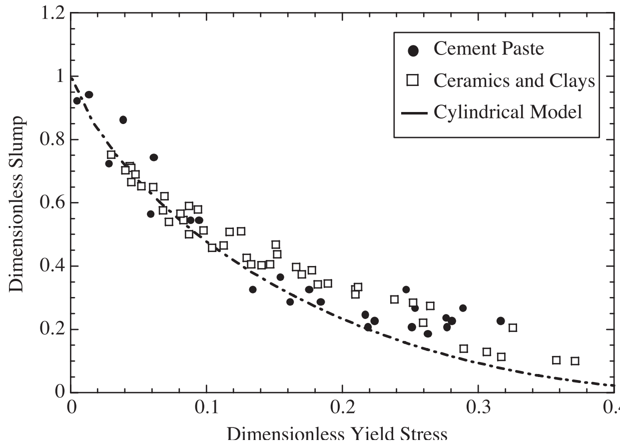

Fig. 8. Dimensionless yield stress versus dimensionless slump for cylinder. Data are from Pashias et al. [10] for clays and ceramic suspensions. Solid line represents dimensionless model prediction for cylinder.

Surprisingly, at low yield stress values, the cone data fit the cylinder model much better. At higher yield stresses (dimensionless yield stresses above 0.15), the data deviate significantly from the cylinder model.

Some understanding of why the cone data do not follow the predicted behavior is achieved by looking at the slumped material. At low yield stresses, the diameter of the top portion of the slumped paste is the same as the bottom diameter of the cone as shown in Fig. 10. The dimensionless model does not predict this result (see Fig. 5). Instead, the model assumes that the top portion of the slumped material is undeformed, retaining the shape of the cone. Experimentally, it appears as if the entire sample yielded, to some extent, as the cone was lifted. This indicates that each layer of material of thickness *dz* (Fig. 4) significantly deforms due to viscous forces acting in conjunction with the downward gravitational force. Thus, the material is effectively extruded from the cone through a die of constant diameter. This is analogous to the cylindrical, not conical, slump test. Accordingly, the low yield stress data fit the cylindrical slump model.

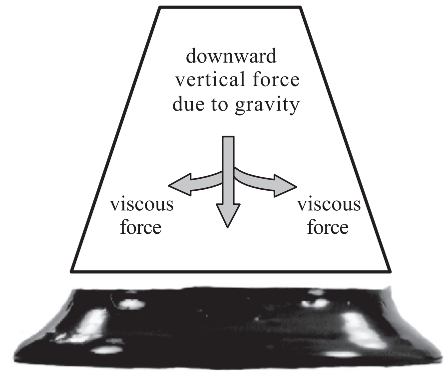

Fig. 10. Photograph of low yield stress, slumped cement pastes taken after the cone was lifted. Outline of slump cone is superimposed on top of slumped material. Note that the diameter of the top of the slumped paste is the same as the diameter of the bottom of the cone.

At higher dimensionless yield stresses (above 0.15), it was difficult to properly fill the cone and large air voids were present. Further tamping using the stirring rod did not reduce the number of voids (it should be noted that filling the cylinder was much easier than filling the cone, even at high yield stress values). The shape of the slumped material did not have the appearance of the cylindrical slumps and the upper portion of the material did not yield. Thus, lifting the cone did little to deform the already stiff material, and at high yield stress, the accuracy of the slump test breaks down. Oil was added to the sides of the cone in an effort to reduce any complicating effects due to wall friction during lifting. The difference between the values recorded for slump with and without the lubricating oil was not statistically significant.

Dimensionless slump flow area is plotted versus dimensionless slump in Fig. 11. Although there is some scatter in the data at low slump values (i.e., low yield stress), a near linear relationship exists between the two measurements.

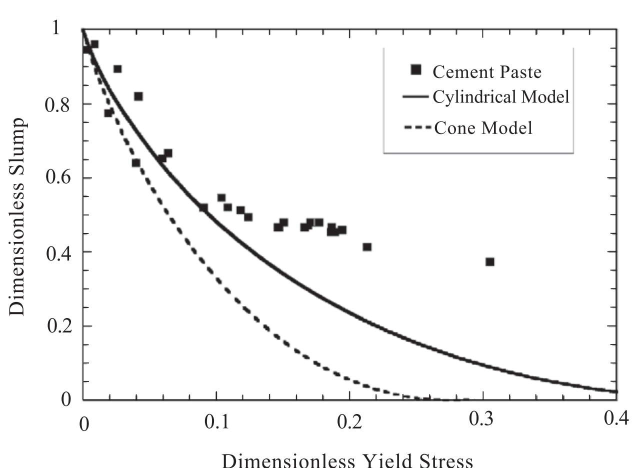

Fig. 9. Dimensionless yield stress versus dimensionless slump for cone. Solid and dashed lines represent dimensionless model prediction for cylinder and cone, respectively.

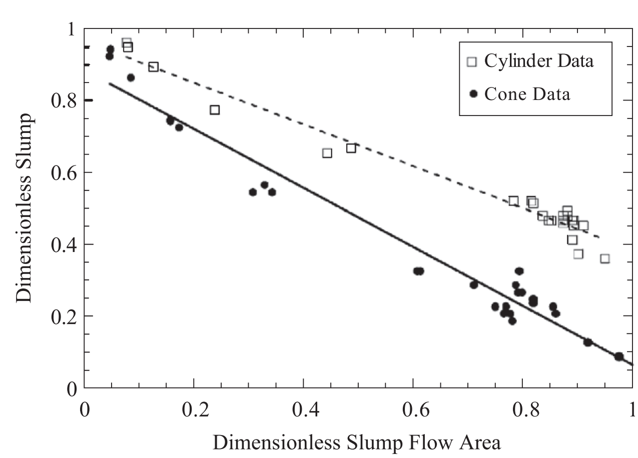

Fig. 11. Dimensionless slump versus dimensionless slump flow area for cone and cylinder geometry.

The aspect ratio (major axis radius/minor axis radius) for the base of slumped material was determined using an image analysis program. The average aspect ratio for the conical and cylindrical slumped material was 1.02 and 1.03, with standard deviations of 0.02 and 0.04, respectively. The calculations were made from sample sets of at least 25 experimental data. The results suggest that using an average diameter to characterize the final spread (slump flow) is an acceptable experimental approach since the aspect ratio is near unity for both geometries. The results further suggest that the deformation of the paste was uniform around the circumference of the slump cone.

6.1. Comparison of concrete slump models

Several groups have developed models relating the slump behavior of concrete to yield stress based on experimental data and finite element modeling. de Larrard et al. [22] developed a model relating concrete slump to yield stress based on data taken from tests using a BTRHEOM concrete rheometer. Their model is:

$$\tau_y = \frac{\rho}{A}(H - s) \tag{37}$$

where H is the height of the cone, s is the slump, A is a fitting constant, and ρ is the density. The model is easily converted to dimensionless units

$$\tau_y' = (C)(1 - s') \tag{38}$$

where the constant C=$(Ag)^{-1}$ and g is the gravitational constant.

Helmuth et al. [23] developed a slump model based on geometric constraints for the standard ASTM C-143 concrete slump cone. Yield stress was calculated based on Murata's [8] force balance approach.

$$\tau_y = \frac{\rho g V_c}{2\pi r_s^2} \tag{39}$$

The radius of the base of the slumped material was assumed to be a function of the final slump height. The relationship is expressed as:

$$r_s = \left(\frac{336}{(12 - s)} - 3\right)^{1/2} - 1 \tag{40}$$

for the standard ASTM cone where r_s is the radius at the base of the slumped material and s is the slump height. Both Eqs. (39) and (40) can be converted to dimensionless units having the form:

$$\tau_y' = \left[\frac{(1 - s')}{6}\right]\left(R^2 + R + 1\right) \tag{41}$$

$$r_s = \left(\frac{28}{(1 - s')} - 3\right)^{1/2} - 1 \tag{42}$$

where R is the ratio of the radius of the top of the slump cone to the final diameter of the base of the slumped material (r_t/r_s).

Tanigawa and Mori [24] used a finite element method to model the slump behavior of fresh concrete for the ASTM standard cone. They imputed yield stress, viscosity, and density into an FEM algorithm, with slump as the output. Given the density, yield stress, cone height, and modeled slump, it is very easy to convert the results of Tanigawa and Mori [24] to dimensionless units. The results indicate that viscosity has a minor effect on slump height. Christensen [9] conducted a more detailed finite element analysis using finite strain algorithms. Although the coding and boundary conditions were somewhat different, both finite element studies produced similar results for predicting concrete slump as a function of yield stress.

A plot of the various concrete slump models, normalized to dimensionless variables, is presented in Fig. 12, with the cylindrical slump model described earlier (note: the finite element model of Christensen [9] has been omitted from the graph for clarity). The model of de Larrard et al. [22] is the only one that does not fit the trend. The divergence of the de Larrard et al. [22] model from the other models may be due to differences in how yield stress was determined. de Larrard et al. [22] extrapolated flow curves to zero shear rate as a measure of yield stress in contrast to the direct measurements used in this study.

It is very encouraging to note that the two independent finite element models for concrete; experimental data for cement paste, clays, and ceramic suspensions; and two theoretical models fall on the same curve. This suggests that slump is material and cone geometry independent when analyzed in dimensionless units. Thus, the models developed and tested using cement paste should apply to concrete. Furthermore, these results provide a simple and straightforward method for determining yield stress from slump, moving the qualitative slump test to a more quantitative scale.

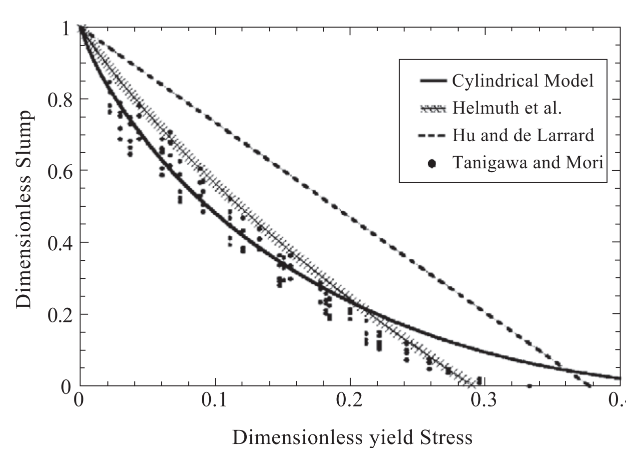

Fig. 12. Various slump models converted to dimensionless form and compared to cylindrical model (de Larrard et al. [22], Hemulth et al. [23], and Tanigawa and Mori [24]).

7. Conclusions

A three-dimensional model relating slump to yield stress has been derived as a function of cone geometry. The results indicate that the model fits the experimental data for cylindrical slump over a wide range of yield stress values for a variety of materials, including cement paste, clays, and ceramic suspensions [10]. Results from the cone tests do not fit the derived model but instead follow the cylindrical slump model at low yield stress values. This is consistent with the final shape of the slumped material. At high yield stress, the data deviate from the conical model as filling the slump cone becomes difficult and large air voids are present. The data also show a simple relationship between slump and the area of the final spread (slump flow). Multiple models predict identical yield stress for a given slump when converted to dimensionless form. These results suggest a fundamental relationship between yield stress and slump that is material independent and largely independent of cone geometry.

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