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Justification of Fédération International de Béton, fib, 2000 model for elastic modulus of normal and high-performance concrete, HPC

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Abstract

This article outlines an experimental and theoretical comparison between the fib 2000 model for elastic modulus of normal concrete, NC, and HPC and 144 laboratory tests. Two dimensions of specimen were studied (diameters 56 and 100 mm) and one climate (20 °C, sealed or with ambient relative humidity, RH, 60%). The age dependence did not seem to be well correlated between the derived and the measured elastic modulus. The fib 2000 model overestimated the elastic modulus of mature concrete and in contrast slightly underestimated the elastic modulus of young NC. As a whole, the derived modulus when using the fib model was about 110% of the elastic modulus measured at loading. A relationship to strength was found for this diversity with an increasing overestimation of the E-modulus at lower strength.

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1. fib 2000 model for NC and HPC

The model originated from European work in the field over 10 years [1–6]. The elastic modulus of concrete was derived according to the following formula:

$$E_c(t) = 21.5(f_{cm}/f_{cm0})^{1/3}\exp[1-(28/t/t_1)^{0.5}s/2] \quad (1)$$

where $f_{cm0} = 10$ MPa; f_{cm} denotes 28-day cylinder strength (MPa); t denotes age (days); $t_1 = 1$ day; and s denotes constant given in Table 1.

2. Methods

In all, 16 concretes were cast of two Portland cements, 1 slowly hardening (CEMI42.5BV/SR/LA Degerhamn, SL) and 1 normal hardening (CEMI42.5R Slite Std, N), Table 2. The w/b varied between 0.24 and 0.80 with 28-day strength varying between 24 and 141 MPa (100-mm cube). The aggregate content varied between 0.70 and 0.75 and consisted of quartzite sandstone, Table 3. Natural sand filler was used. The mix proportions also contained silica fume, Table 4 [8–16]. After demolding at 1-day-age measurement, points of steel screws were fixed into items cast in the concrete cylinders on three sides of them. Half of the specimens were sealed with adhesive aluminum foil. Half of the number of specimens were subjected to relative humidity (RH) = 60% in a climate chamber. Cylinders 56 or 100 mm in diameter were used. For cylinders 56 mm in diameter, three LVDT gauging devices were mounted on the side of the cylinder in order to determine the deformation. Mechanical devices were used when the deformation was measured on the 100-mm cylinders in turn calibrated to length by an INVAR rod. The loading for the 56-mm cylinders was applied were rapidly, after about 0.01 s, in order to obtain the “true” modulus of elasticity [17]. For the 100-mm cylinders, the loading was applied in a more traditional way with a rate of about 1 MPa/s. Over a period of 100 s, substantial creep took place, especially at young ages [8]. The observed elastic modulus was smaller at slow loading. However, after 100 s of loading

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Table 1
Constant *s* in Eq. (1)

Type of cement and concrete strength	Notification	*s*
Slowly hardening, concrete strength ≤ 60 MPa	SL	0.38
Normal or rapid hardening cement, concrete strength ≤ 60 MPa	N, R	0.25
Rapid hardening high-strength cement, concrete strength ≤ 60 MPa	RS	0.20
All types of cements, concrete strength > 60 MPa	SL, N, R, RS	0.20

Table 2
The chemical composition of the cements [8–16]

Chemical composition (%)	CEMI42.5BV/SR/LA Degerhamn (SL)	CEMI42.5R Slite Std (N)
CaO	65	62
SiO_2	21.6	20
Al_2O_3	3.5	4.4
Fe_2O_3	4.4	2.3
K_2O	0.58	1.4
Na_2O	0.05	0.2
MgO	0.78	3.5
SO_3	2.07	3.7
Ignition losses	0.47	2.4
CO_2	0.14	1.9
Clinker minerals		
C_2S	21	14
C_3S	57	57
C_3A	1.7	8
C_4AF	13	7
Physical properties		
Water demand (%)	25	28
Initial setting time (min)	145	154
Density (kg/m^3)	3214	3122
Specific surface (m^2/kg)	305	364

at which loading time the elastic modulus was determined, no difference between the measured elastic modules was observed dependent on the rate of loading, slow or rapid [8].

3. Results and Analyses

3.1. Results

Evaluations according to the fib 2000 model were plotted versus the measured laboratory results, Figs. 1 and 2. Cylinders 56 mm with quasi-instantaneous loading were used from 1 day age up to 500 days age, Fig. 1. Loading of air and sealed cured specimens showed no significant difference between the measured modules of elasticity, Fig. 1. Fig. 2 shows an overestimation with the fib model increased with age. Both Figs. 1 and 2 shows that an overestimation took place when the fib model was used for 500-day-old concrete compared with the measured results even through the significance of the 500-day results was low. Fig. 3 shows the ratio of the elastic modulus derived with the fib model to measured elastic modulus versus strength. Calculations according to the fib

Table 3
Properties of the aggregate (quartzite sandstone) [7]

Compressive strength (MPa)	Split tensile strength (MPa)	Elastic modulus (GPa)	Ignition losses (%)
333	15	60	0.3

Table 4
Mix proportions and strength of concretes, cylinder diameter, d (kg/m^3, MPa, cm)

Mix	Cement, c	Cement type	Silica fume, *s*	w/b	Sand filler	Air (%)	28-Day strength	*d*
1	460	SL	21	0.36		4.8	69	6
2	440	SL	44	0.34		1.1	85	6
3	445	SL	45	0.34		4	69	6
4	455	SL	23	0.3		0.9	89	6
5	495	SL	50	0.28		1.1	99	6
6	530	SL	51	0.27		1.2	106	6
7	490	SL	49	0.27		1	112	6
8	545	SL	55	0.24		1.3	114	6
27	500	SL	50	0.24	50	1.3	141	10
32	389	N		0.32	106	12	55	10
38N	360	N		0.38	68	12	42	10
38S	400	N		0.38	145	1.4	86	10
50N	285	N		0.50	33	13	30	10
50S	340	N		0.50	165	3.5	61	10
80N	250	N		0.80		1.2	24	10
80S	260	N		0.80	185	1.9	27	10

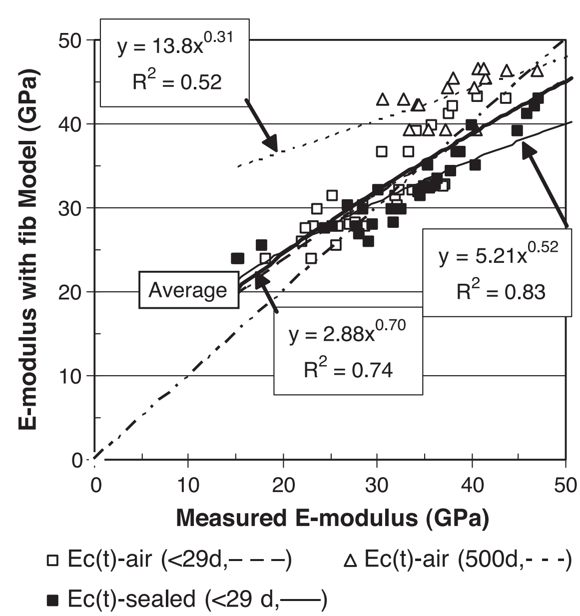

Fig. 1. Estimation with fib 2000 model versus measured E-modulus, 56-mm cylinder, d = day's age.

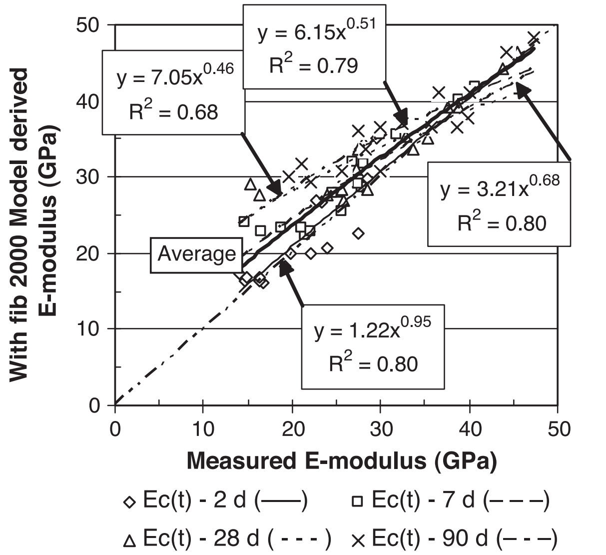

Fig. 2. Estimation with fib 2000 model versus measured E-modulus, 100-mm cylinder, d = day's age.

2000 model at 500 days age showed about 15% larger values compared with the measured value—decreasing with increasing strength, Fig. 3. At 1 day age, the elastic modulus was overestimated by about 13% also decreasing with increasing strength. At 28 days age, an overestimation by about 5% of the E-modulus occurred at lower strength also decreasing with increasing strength. Only at 28 days age the result showed sufficiently high significance (R^2>0.50). However, on average, the resisting results showed clear tendencies of the differences between the results estimated with the fib model and the results measured in the laboratory.

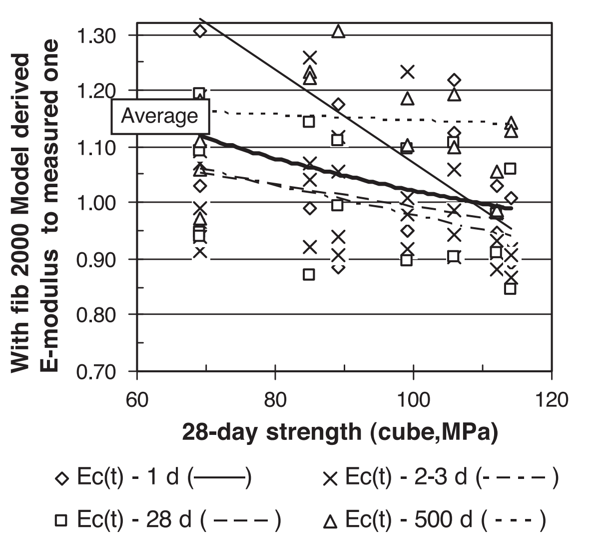

Fig. 3. Ratio of derived to measured elastic modulus versus cube strength, d = day's age.

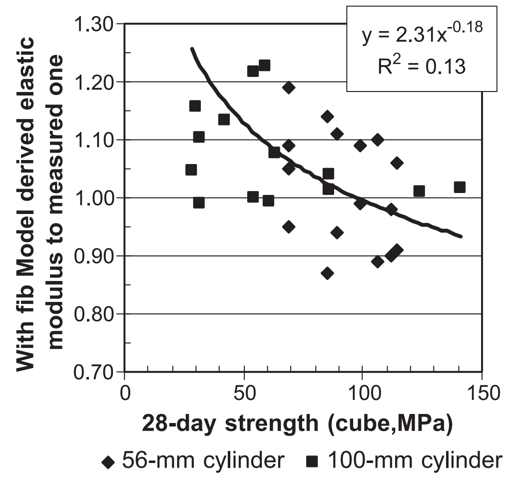

Fig. 4. Elastic modulus with the fib model to measured versus 28-day strength.

3.2. Analyses

Previously, a comparison was performed between measured shrinkage of HPC and shrinkage estimated by the fib 2000 model [18]. It was shown that the drying shrinkage estimated with the fib 2000 model correlated well with the measured one, but that the derived autogenous shrinkage only was about 50% of the measured one. Creep, both basic and drying, correlated well estimated with the fib 2000 model with measured one [19]. In this study on the elastic modulus, the observed overestimation by the fib model may be affected by too a large exponent for the time dependence of strength, 0.5 or, in contrast, by too a low exponent for the strength dependence, 1/3. The ratio of the elastic modulus with the fib 2000 model, $E_{c(t)}$, and measured, E_m, is shown in Figs. 4 and 5. The number of specimens is given in Table 5. The following 28-day strength, f_{ccube} (MPa), and loading

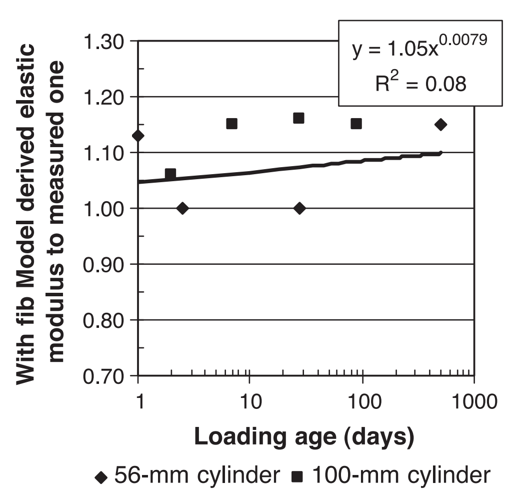

Fig. 5. Elastic modulus with the fib model to measured versus loading age.

Table 5
Elastic modulus with the fib 2000 model and measured number of specimens [8,9]

d (mm)	E_m (GPa)	E_c (GPa)	f_{cm} (MPa)	$E_{c(t)-1d}$	$E_{c(t)-2-3d}$	$E_{c(t)-7d}$	$E_{c(t)-28d}$	$E_{c(t)-90d}$	$E_{c(t)-500d}$	$E_{c(t)-\text{total}}$
56	33	45	93	16	32	–	16	–	16	80
100	27	37	60	–	16	16	16	16	–	64
Total			153	16	48	16	32	16	16	144

age, t (days), dependence were calculated (GPa) however with low significance ($R^2 < .50$):

$$E_{c(t)}/E_m = 2.31 f_{\text{ccube}}^{-0.18} \tag{2}$$

$$E_{c(t)}/E_m = 1.05 t^{0.0079} \tag{3}$$

where f_{ccube} denotes the 28-day strength (cube, MPa); t denotes the loading age (days); $E_{c(t)}$ denotes the elastic modulus with the fib 2000 model (GPa); and E_m denotes the measured elastic modulus (GPa).

Even though the results of Eqs. (2) and (3) show low significance, the tendencies of the results were clear:

- An overestimation of the elastic modulus when the fib model was used that decreased from about 25% at 30 MPa 28-day strength to an underestimation about 5% for concrete with 140 MPa 28-day strength, Fig. 4.
- An overestimation of the elastic modulus when the fib model was used that increased slightly with the age of loading the concrete, Fig. 5.

From Fig. 5, it also was observed that little influence existed between the estimated elastic modulus and the size of the specimen.

4. Summary and Conclusions

An experimental and theoretical comparison between the elastic modulus estimated with the fib 2000 model for NC and HPC, and 144 laboratory tests on 16 concretes was performed. Two dimensions of specimen were studied (diameters 56 and 100 mm) and 2 climates (20 °C, sealed or ambient 60% RH). The following conclusions were drawn [20]:

- The age dependence did not seem to be well correlated between the derived elastic modulus and the measured elastic modulus.
- The fib 2000 model overestimated the elastic modulus of mature concrete and in contrast slightly underestimated the elastic modulus of young concrete.
- As a whole, the derived modulus with the fib model was about 110% of the elastic modulus measured at loading.
- It was found that the overestimation of the E-modulus increased at low strength.

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