



Effect of temperature and age on the relationship between dynamic and static elastic modulus of concrete

Sang-Hun Han^{a,*}, Jin-Keun Kim^b

^aCoastal & Harbour Engineering Laboratory, Korea Ocean Research & Development Institute, 1270 Sadong Ansan, Kyunggido 426-744, Republic of Korea

^bDepartment of Civil Engineering, Korea Advanced Institute of Science and Technology, 373-1 Kusong, Yusong, Taejon 305-701, Republic of Korea

Received 19 November 2002; accepted 11 December 2003

Abstract

This study investigates the effects of cement type, curing temperature, and age on the relationships between dynamic and static elastic moduli or compressive strength. Based on the investigation, new relationship equations are proposed. The impact-echo method is used to measure the resonant frequency of specimens from which the dynamic elastic modulus is calculated. Types I and V cement concrete specimens with water–cement ratios of 0.40 and 0.50 are cured isothermally at 10, 23, and 50 °C and tested at 1, 3, 7, and 28 days.

Cement type and age do not have a significant influence on the relationship between dynamic and static elastic moduli, but the ratio of static to dynamic elastic modulus approaches 1 as temperature increases. The initial chord elastic modulus, which is measured at low strain level, is similar to the dynamic elastic modulus. The relationship between dynamic elastic modulus and compressive strength has the same tendency as the relationship between dynamic and static elastic moduli for various cement types, temperatures, and ages.

© 2004 Elsevier Ltd. All rights reserved.

Keywords: Elastic moduli; Compressive strength; Impact echo; Temperature; Age

1. Introduction

To evaluate stability and durability of existing structures or monitor strength and elastic modulus development during construction, nondestructive evaluation (NDE) methods have been developed recently. Many NDE methods estimate concrete strength using the established relationship between the strength by uniaxial compression test and the property given by NDE method. But, there are few researches that investigate the relationship between the static elastic modulus and the dynamic elastic modulus. Various types of static elastic modulus can be calculated from the stress–strain curve obtained in an uniaxial compression test. The tangent elastic modulus is the slope of a line tangent to the stress–strain curve at any point on the curve. The secant elastic modulus is the slope of a line

drawn from origin to a point on the curve corresponding to 40% of ultimate strength. The chord elastic modulus is the slope of a line drawn from a point representing a longitudinal strain of 50×10^{-6} to a point at 40% of ultimate strength. Generally, the elastic modulus obtained by uniaxial compression test using ASTM C 469 is the chord elastic modulus. NDE methods measure dynamic elastic modulus based on the stress wave velocity [1]. The dynamic elastic modulus corresponds to initial tangent modulus because strains are very low during these dynamic tests.

Research on the impact-echo method used in this study to estimate dynamic elastic modulus was begun in the early 1980s at what is the National Institute of Standards and Technology by Nicholas J. Carino and Mary Sansalone. In addition to demonstrating the practicality of the method, they developed a theoretical understanding of stress wave propagation in concrete plates. Sansalone and her students at Cornell University have continued the work including innovations in signal analysis using artificial intelligence [2]. Pessiki and Carino [3] and Pessiki

* Corresponding author. Tel.: +82-31-400-6345; fax: +82-31-408-5823.

E-mail address: shhan@kordi.re.kr (S.-H. Han).

and Johnson [4] evaluated the feasibility of impact-echo method to determine setting time and to monitor strength development of concrete. Also, Pessiki and Johnson [4] used the impact-echo method to estimate the in-place strength of concrete in plate-like elements, such as slabs and walls.

The objectives of this study are to compare the dynamic elastic modulus measured by the impact-echo method with the static elastic modulus or compressive strength determined from a uniaxial compression test and to investigate whether the relationships between dynamic and static elastic moduli or compressive strength are affected by temperature and age.

2. Experimental program

2.1. Experimental variables

Experimental variables are cement type, water–cement ratio, curing temperature, and testing age; the details are tabulated in Table 1. This research was planned as a part of the project that investigated the material properties of concrete used in the containment building of nuclear power plant. Thus, dynamic elastic modulus of Type V cement concrete used in mass concrete structures, such as the containment building, was measured at different ages for curing temperatures of 10, 23, and 50 °C. Also, dynamic elastic modulus of Type I cement concrete cured at 23 °C were measured at different ages and the results were compared with those of Type V cement concrete.

2.2. Materials

Material properties of cement, sand, and coarse aggregate are shown in Table 2 and the chemical properties of the cements are provided in Table 3. River sand and crushed stone were used, and the maximum coarse aggregate size was 19 mm.

2.3. Mixture proportions

Table 4 shows the mixture proportions. Fine aggregate–aggregate ratios were 0.39 and 0.42. The cement contents were 362 and 452 kg/m³ according to water–cement ratio, and the water content of all mixtures was 181 kg/m³. The quantities of air entraining agent and superplasticizer are given as the mass ratio of cement.

Table 1
Testing variables

Cement type	Water–cement ratio	Age (days)	Curing temperature (°C)
Type I	0.40, 0.50	1, 3, 7, 28	23
Type V	0.40, 0.50	1, 3, 7, 28	10, 23, 50

Table 2
Material properties

	Type I cement	Type V cement	Fine aggregate	Coarse aggregate
Type	–	–	river sand	crushed stone (granite) (max size: 19 mm)
Specific Surface (cm ² /g)	3315	3210	–	–
Specific Gravity	3.15	3.15	2.55	2.58
Fineness modulus	–	–	2.95	7.23
28-day compressive strength (MPa)	40	35	–	–

2.4. Test methods

After mixing the concrete, cylinders were cast in 100 × 200 mm paper molds and put into curing chambers within 30 min. Subsequently, cylinders including paper molds were cured in the chambers at 100% humidity and target temperatures of 10, 23, and 50 °C. Paper molds were removed after 24 h and specimens were cured in the chamber until test ages of 1, 3, 7, and 28 days. The cylinder ends were ground flat before testing. The results from three replicate specimens were averaged. Compressive strength was measured according to ASTM C 39 and elastic modulus was according to ASTM C 469. Concrete strain gages measured strains, and the strains and stresses were accumulated using the system 4000 produced by the Measurement Group.

Dynamic elastic modulus was estimated by the impact-echo method [2,3]. Fig. 1 shows a schematic of the impact-echo technique used in this study. A small diameter steel sphere (ϕ 8 mm) was used as the impact source. The sphere was dropped onto the top surface of a cylindrical specimen, and an accelerometer measured the vertical motion produced by the arrivals of reflected P wave. A dynamic signal analyzer was used to record and process the waveform. The pulse generated by impact undergoes multiple reflections between the top and bottom surfaces of cylinder, and the recorded waveform has periodic characteristic. The travel path of P wave between successive arrivals at the top surface is twice the length L of cylinder. The periodicity of the waveform was obtained by transforming the time domain waveform into the frequency domain using the fast

Table 3
Chemical composition of cements

	Type I cement (%)	Type V cement (%)
Calcium oxide, CaO	63.03	63.53
Silicon dioxide, SiO ₂	20.57	21.74
Aluminum oxide, Al ₂ O ₃	5.48	3.18
Magnesium oxide, MgO	3.41	3.11
Ferric oxide, Fe ₂ O ₃	3.18	4.71
Sulfur trioxide, SO ₃	2.23	1.87
Potassium oxide, K ₂ O	0.80	0.59
Loss on ignition	1.24	0.53

Table 4
Basic mixture proportions

Cement type	Water–cement ratio	Sand–aggregate ratio	Quantity (kg/m ³)					Admixture	
			Water	Cement	Sand	Gravel		AE agent (cement × %)	Superplasticizer (cement × %)
Type I, V	0.40	0.39	181	452	630	989		0.005	0.5
Type I, V	0.50	0.42	181	362	707	989		0.005	0.5

Fourier transform technique. The largest amplitude peak occurred at the fundamental resonant frequency, and this frequency was used to estimate the P wave velocity as follows.

$$V_c = f_1 2L \quad (1)$$

where V_c is P wave velocity, f_1 is the resonant frequency of first longitudinal mode, and L is the cylinder length. The dynamic elastic modulus is obtained from the following equation:

$$E_d = \rho V_c^2 \quad (2)$$

where E_d is the dynamic elastic modulus and ρ is the density of the specimen [2]. The density was obtained dividing mass of specimen with calculated volume.

3. Results and discussion

3.1. Relationship between static and dynamic elastic moduli

Experimental results of compressive strength, static, dynamic, initial chord elastic moduli, and density are shown in Table 5. Figs. 2–5 present the relationships between static and dynamic elastic moduli according to cement type,

curing temperature, and aging. The following equation is used to represent the relationship between static and dynamic elastic moduli.

$$E_c = E_d(1 - ae^{-bE_d}) \quad (3)$$

where E_c is the static elastic modulus (GPa), E_d is the dynamic elastic modulus (GPa), a and b are constants. r in figures is the coefficient of determination. Eq. (3) includes the following characteristics of the relationship between static and dynamic elastic moduli. If the strength of concrete increases, the dynamic elastic modulus increases, and the stress–strain curve in the range less than 40% of the compressive strength becomes more linear. As the linearity of the stress–strain curve increases, the difference between static and dynamic elastic moduli decreases. In other words, at high dynamic elastic modulus, there is less difference between static and dynamic elastic moduli. Secondly, static elastic modulus is zero if dynamic elastic modulus is zero. Based on these two characteristics, Eq. (3) was suggested.

Fig. 2 shows the relationship between static and dynamic elastic moduli of concrete cured at 23 °C and made with two cement types. As shown in Fig. 2, the cement type does not have a significant effect on the relationship between static and dynamic elastic moduli. The conclusion can be confirmed by comparing the regression curve for Type V cement concrete from this study with the relationship obtained by other researchers for Type I cement concrete [5]. Fig. 3 shows the regression curves of Type V cement concrete obtained from this study, which includes data of concrete cured at 10 and 50 °C, and the relationship obtained by others for Type I cement concrete. The range of experimental data is indicated by the two dotted vertical lines. Fig. 3 shows that the relationship between static and dynamic elastic moduli of Type V cement concrete obtained in this study is similar to that of Type I cement concrete.

Fig. 4 shows experimental results and regression curves of Type V cement concrete for different curing temperatures. As shown in Fig. 4a, for the same dynamic elastic modulus, the static elastic modulus of concrete with 0.40 water–cement ratio increases slightly with increasing curing temperature. This trend is greater in concrete with 0.50 water–cement ratio. As shown in Fig. 4b, the ratio of static to dynamic

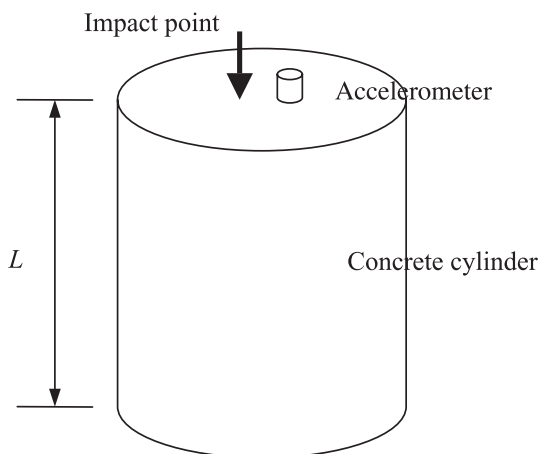


Fig. 1. Schematic of impact-echo test.

Table 5
Experimental results

Cement type	Water–cement ratio	Temperature (°C)	Age (days)	Compressive strength (MPa)	Static elastic modulus (GPa)	Dynamic elastic modulus (GPa)	Initial chord elastic modulus (GPa)	Unit weight (kg/m ³)
Type I	0.40	23	1	21	20	29	24	2330
				22	21	29	24	2380
				20	20	29	24	2380
			3	31	24	34	26	2360
				32	24	34	28	2360
				32	23	34	28	2380
			7	38	26	36	32	2360
				37	26	37	32	2360
				35	26	36	28	2320
			28	43	29	38	32	2340
				36	31	38	36	2350
				35	32	38	36	2350
	0.50	23	1	11	17	25	18	2370
				12	17	25	18	2350
				11	16	24	20	2350
			3	22	23	31	26	2340
				22	22	31	27	2330
				21	22	30	24	2340
			7	28	24	34	28	2340
				25	25	34	28	2350
				28	24	34	28	2350
			28	35	27	37	31	2350
				37	28	37	32	2370
				36	27	37	32	2360
Type V	0.40	10	1	4	6	13	10	2380
				3	7	14	12	2390
				4	7	14	12	2390
			3	16	17	28	24	2390
				16	18	28	24	2370
				17	20	28	26	2370
			7	25	25	34	31	2370
				24	22	34	29	2370
				25	23	32	30	2380
			28	32	25	36	31	2390
				35	26	36	32	2370
				33	26	37	35	2370
		23	1	11	15	25	22	2380
				12	15	24	22	2380
				13	17	25	23	2350
			3	22	24	33	30	2360
				24	23	32	27	2370
				25	21	32	26	2380
			7	29	26	35	30	2370
				29	27	35	31	2360
				30	28	35	33	2370
			28	38	29	38	35	2370
				40	29	39	35	2420
				38	28	39	36	2420
		50	1	22	21	30	29	2370
				20	20	29	27	2380
				22	21	29	26	2370
			3	30	25	34	30	2370
				31	25	34	33	2380
				32	24	34	29	2370
			7	36	25	34	30	2350
				37	26	35	36	2360
				35	26	35	35	2380
			28	45	28	35	35	2340
				45	27	36	36	2360
				45	26	35	34	2350

Table 5 (continued)

Cement type	Water–cement ratio	Temperature (°C)	Age (days)	Compressive strength (MPa)	Static elastic modulus (GPa)	Dynamic elastic modulus (GPa)	Initial chord elastic modulus (GPa)	Unit weight (kg/m ³)
Type V	0.50	10	1	2	2	7	5	2350
				2	4	7	10	2350
				2	5	8	8	2380
			3	10	13	24	21	2360
				11	14	25	22	2360
				12	15	26	23	2370
			7	18	19	31	28	2330
				18	18	31	27	2310
				16	18	30	26	2390
			28	22	21	33	27	2370
				26	23	35	32	2380
				26	24	33	33	2370
		23	1	7	13	20	16	2360
				7	12	21	17	2370
				7	12	19	18	2390
			3	17	21	30	31	2360
				17	19	30	28	2370
				17	19	30	27	2360
			7	22	23	32	29	2360
				22	22	33	30	2370
				22	24	34	31	2370
			28	29	27	37	34	2370
				31	27	36	32	2350
		50	1	10	16	25	21	2370
				10	15	24	20	2340
				11	16	24	23	2360
			3	17	18	28	26	2330
				17	21	29	30	2340
				18	19	29	28	2340
			7	21	24	30	29	2380
				23	22	31	28	2280
				24	25	32	30	2340
			28	26	24	33	28	2270
				28	25	32	31	2320

elastic moduli increases with increasing curing temperature and the slopes of concretes cured at high temperatures are larger than those of concrete cured at low temperature. Since the quantity of experimental data of this study is small and

there are few experimental researches investigating the effect of curing temperature on the relationship between dynamic and static elastic moduli, confirming the trend needs to accumulate more experimental data.

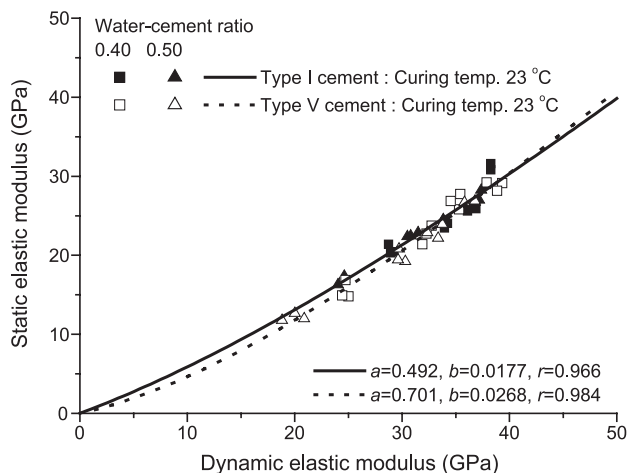


Fig. 2. Relationship between dynamic and static elastic modulus for two cement types and curing at 23 °C.

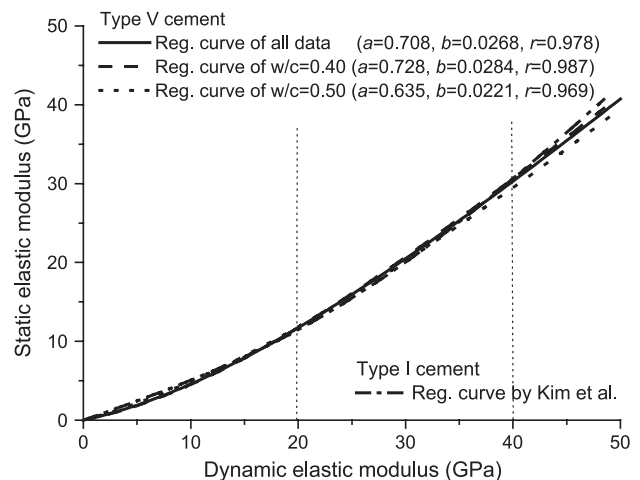


Fig. 3. Comparison of regression curves of static and dynamic elastic moduli for different cement types.

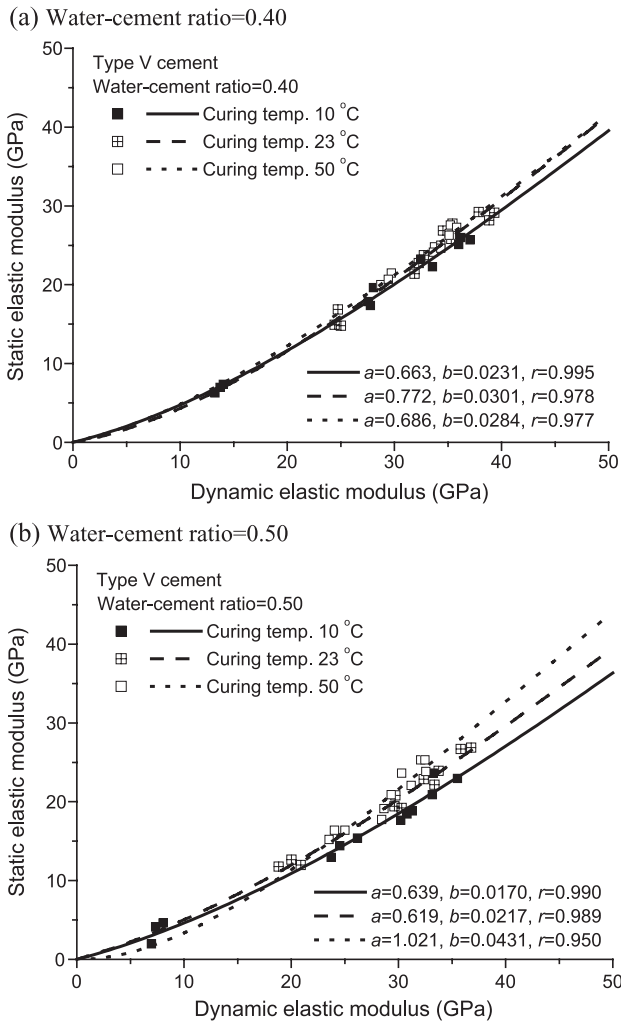


Fig. 4. Relationships between dynamic and static elastic moduli for different curing temperatures. (a) Water–cement ratio=0.40. (b) Water–cement ratio=0.50.

Fig. 5 compares dynamic with static elastic modulus at different ages. Since experimental data have dissimilar ranges for different ages, the comparison between dynamic and static elastic moduli cannot be accurately carried out according to ages. Therefore, evaluating the influence of age on the relationship between dynamic and static elastic moduli needs more experimental researches.

3.2. Relationship between initial chord and dynamic elastic moduli

Because dynamic elastic modulus can be assumed to be the tangent elastic modulus at low strain, static elastic modulus measured at low strain may have values similar to dynamic elastic modulus. It has been reported that the impact-echo test measures dynamic elastic modulus at a strain of about 6×10^{-6} [6]. To confirm the assumption, the “initial chord elastic modulus” was defined as the

slope of a line from a point representing a longitudinal strain of 10×10^{-6} to a point having a strain of 50×10^{-6} , and this definition was given considering the strain resolution of concrete strain gage. This initial chord elastic modulus was compared with the dynamic elastic modulus.

Fig. 6 compares the dynamic with the static and initial chord elastic moduli. As expected, the difference between initial chord and dynamic elastic moduli is smaller than that between static and dynamic elastic modulus.

As previously done for static elastic modulus, the relationship between initial chord and dynamic elastic moduli was analyzed for various cement types, curing temperatures, and ages. Fig. 7 compares initial chord with dynamic elastic modulus according to cement type. As shown in Fig. 7, the initial chord elastic modulus of Type I cement concrete is slightly larger than those of Type V cement concrete. Figs. 8 and 9 show the relationships between initial chord and dynamic elastic moduli for

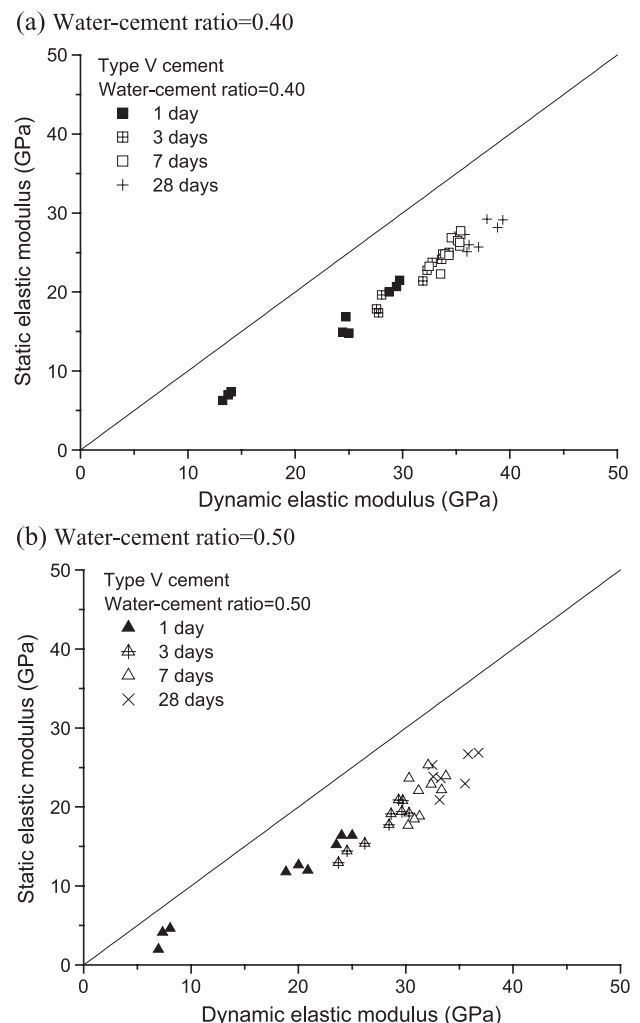


Fig. 5. Dynamic and static elastic moduli for different test ages. (a) Water–cement ratio=0.40. (b) Water–cement ratio=0.50.

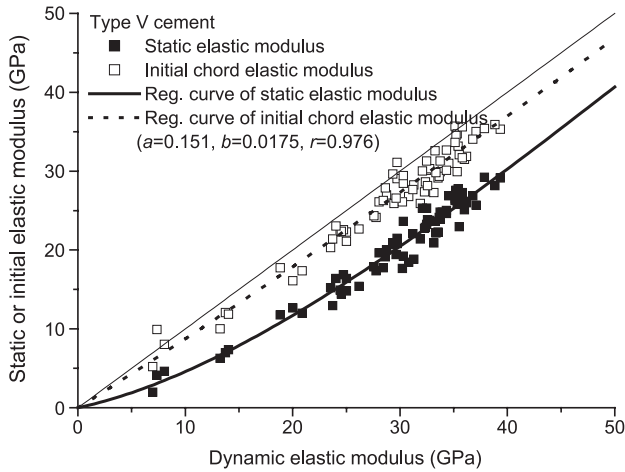


Fig. 6. Relationships among dynamic, static, and initial chord elastic moduli.

different curing temperatures and ages. Although the figures show that curing temperature and age do not have a significant influence on the relationship, confirming the trend needs more experimental researches.

3.3. Relationship between compressive strength and dynamic elastic modulus

Compressive strength is an important material property in design and construction of concrete structures. Using compressive strength, design codes estimate many material properties, such as splitting tensile strength, creep, shrinkage, etc. Thus, it is valuable to investigate the relationship between compressive strength and dynamic elastic modulus. Figs. 10, 11, and 12 show the variation of the relationships between compressive strength and dynamic elastic modulus according to cement type, curing temperature, and age, respectively. The following equations were used to analyze the relationships.

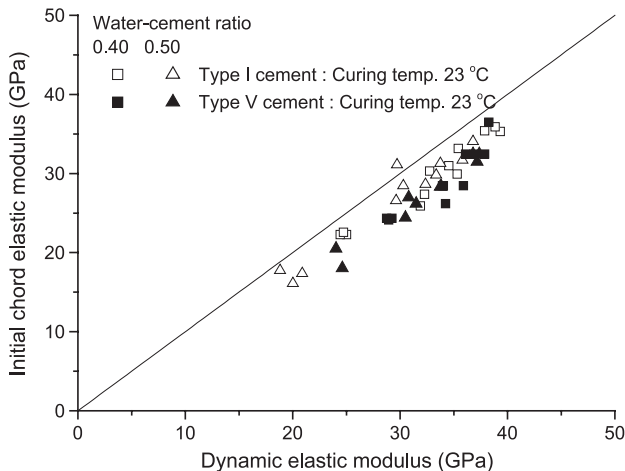


Fig. 7. Dynamic and initial chord elastic moduli for different cement types and curing at 23 °C.

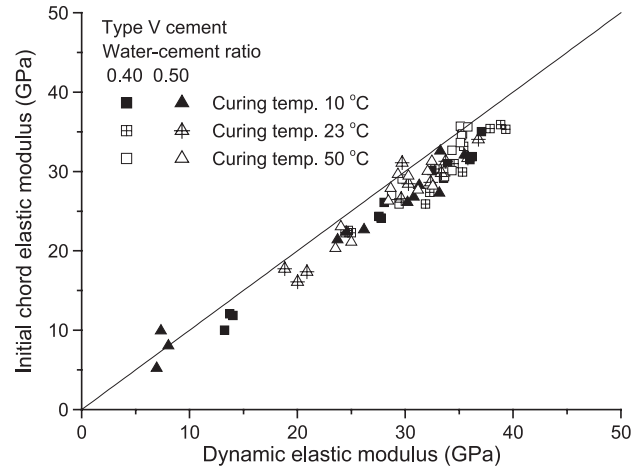


Fig. 8. Dynamic and initial chord elastic moduli for different curing temperatures.

For Type I portland cement concrete,

$$f'_c = \alpha E_c^\beta = \alpha [E_d (1 - 0.492e^{-0.0177E_d})]^\beta \quad (4)$$

For Type V portland cement concrete,

$$f'_c = \alpha E_c^\beta = \alpha [E_d (1 - 0.708e^{-0.0268E_d})]^\beta \quad (5)$$

where f'_c is compressive strength (MPa) and α and β are constants. Many researchers [7] and design codes [8] used the equation $f'_c = \alpha E_c^\beta$ to analyze the relationship between static elastic modulus and compressive strength. Therefore, Eqs. (4) and (5) are formulated to obtain the relationship between compressive strength and dynamic elastic modulus.

As previously done for static and dynamic elastic moduli, the relationship between compressive strength and dynamic elastic modulus is investigated for various

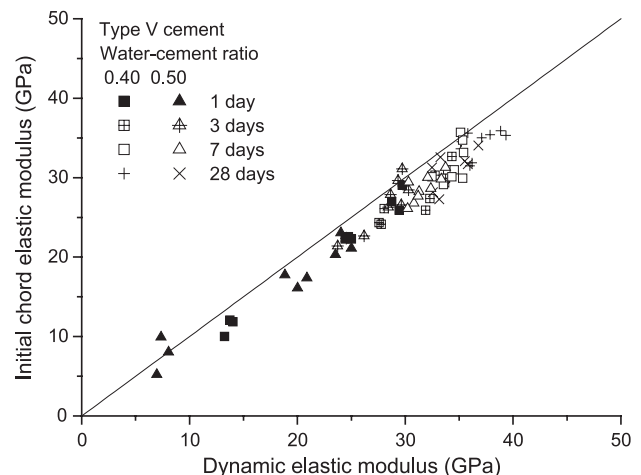


Fig. 9. Dynamic and initial chord elastic moduli for different test ages.

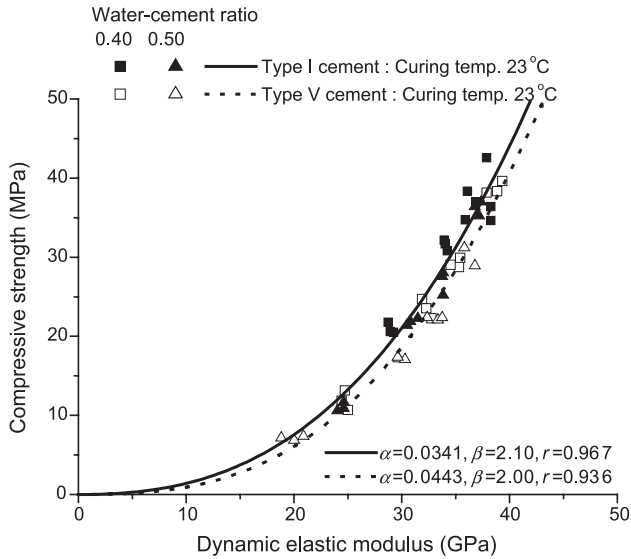


Fig. 10. Relationships between dynamic elastic modulus and compressive strength for different cement types.

cement types, curing temperatures, and ages. Fig. 10 compares the relationships for the cement types. As shown in Fig. 10, for the same dynamic elastic modulus, compressive strengths of Type I cement concrete are slightly larger than those of Type V cement concrete, but the differences are not greater than 5%.

Fig. 11 shows the relationship between compressive strength and dynamic elastic modulus for different curing temperatures. The regression curve of concrete cured at 10 °C is similar to that of concrete cured at 23 °C, but concrete cured at 50 °C has a different relationship. Fig. 11 also compares the experimental results with the American Concrete Institute (ACI) equations. The ACI proposes two different equations for the relationship between static elastic modulus and compressive strength according to compressive

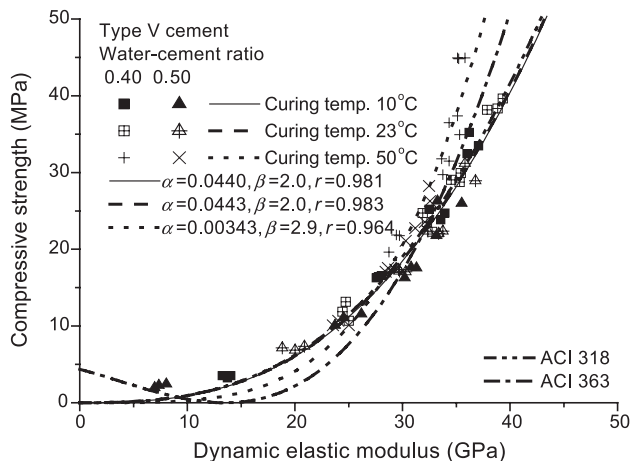


Fig. 11. Relationships between dynamic elastic modulus and compressive strength for different curing temperatures.

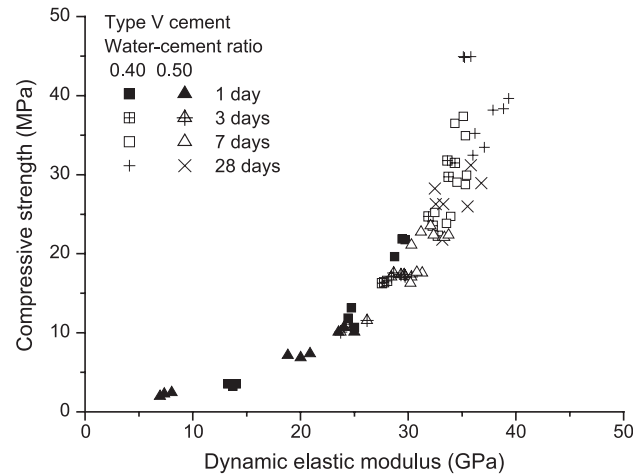


Fig. 12. Dynamic elastic modulus and compressive strength for different test ages.

sive strength. The ACI 318 model ($E_c = 4700\sqrt{f'_c}$) is suggested for ordinary strength concrete and, the ACI 363 model ($E_c = 3300\sqrt{f'_c} + 6900$) for high-strength concrete [8,9]. The dynamic elastic modulus curves for the ACI models were obtained by converting static to dynamic elastic modulus using Eq. (3). The relationship between dynamic elastic modulus and compressive strength of concrete cured at 50 °C is similar to the ACI 363 model and those of concrete cured at 10 and 23 °C are similar to the ACI 318 model. But, it is difficult to generalize the tendency because there are few experimental researches investigating the effect of curing temperature on the relationship between dynamic elastic modulus and compressive strength.

Fig. 12 shows the relationship between compressive strength and dynamic elastic modulus for different ages. Since experimental data have dissimilar ranges for different ages, the comparison between compressive strength and dynamic elastic modulus cannot be accurately carried out according to ages.

4. Conclusions

In this study, dynamic and static elastic moduli and compressive strength were experimentally measured as a function of curing temperature, age, and cement type. The relationships between dynamic and static elastic modulus or compressive strength were estimated using new prediction equations. The following conclusions can be made from the results.

1. Cement type (Types I and V cement) and age (less than 28 days) do not have a significant effect on the relationship between static and dynamic elastic moduli.
2. The ratio of initial chord elastic modulus, which is measured at lower strain level of stress–strain curve, to

dynamic elastic modulus is close to 1. Curing temperature, age, and cement type do not have a large influence on the relationship between initial chord and dynamic elastic moduli.

3. The relationship between dynamic elastic modulus and compressive strength of concrete cured at 50 °C is different than for concrete cured at 10 and 23 °C. However, the relationship is not significantly influenced by cement type and age.
4. Experimental equations are proposed for relationships between dynamic and static elastic modulus or compressive strength.

Acknowledgements

The authors would like to thank Korea Ocean Research & Development Institute (KORDI) for its financial support of this study through PE84000 project.

References

- [1] P.K. Mehta, P.J.M. Monteiro, *Concrete*, 2nd ed., Prentice-Hall, Englewood Cliffs, New Jersey 07632, USA, 1993.
- [2] M. Sansalone, Impact-echo: The complete story, *ACI Struct. J.* 94 (6) (1997) 777–786.
- [3] S.P. Pessiki, N.J. Carino, Setting time and strength of concrete using the impact-echo method, *ACI Mater. J.* 85 (5) (1988) 389–399.
- [4] S.P. Pessiki, M.R. Johnson, Nondestructive evaluation of early-age concrete strength in plate structures by the impact-echo method, *ACI Mater. J.* 93 (3) (1996) 260–271.
- [5] J.K. Kim, H. Kim, J.H. Noh, Estimation of mechanical properties of concrete in early age by resonance frequency test, *KCI Concr. J.* 7 (5) (1995) 164–171.
- [6] J.A. Bay, K.H. Stokoe, Field and laboratory determination of elastic properties of portland cement concrete using seismic techniques, *Transp. Res. Rec.* 1355 (1992) 67–74.
- [7] J.K. Kim, S.H. Han, Y.C. Song, Effect of temperature and aging on the mechanical properties of concrete: Part I. Experimental results, *Cem. Concr. Res.* 32 (7) (2002) 1087–1094.
- [8] ACI, ACI 318-95, *Building Code Requirements for Reinforced Concrete and Commentary*, American Concrete Institute, Detroit, MI, 1995.
- [9] ACI Committee 363, *State-of-the Art Report on High-Strength Concrete*, American Concrete Institute, Detroit, MI, 1992.