Available online at www.sciencedirect.com

SCIENCE DIRECT

Cement and Concrete Research 35 (2005) 483–493

CEMENT AND CONCRETE RESEARCH

Prediction equation of drying shrinkage of concrete based on composite model

Kiyoshi Eguchi[a,*], Kohji Teranishi[b]

[a]*Technical Research Institute, Maeda Corporation, 1-39-16 Asahityo, Nerima-ku, Tokyo 179-8914, Japan*
[b]*Department of Architecture, Faculty of Science and Technology, Meijo University, 1-501, Shiogamaguchi, Tenpaku-ku, Nagoya 468-8502, Japan*

Received 16 July 2003; accepted 9 August 2004

Abstract

In this study, the prediction equation of drying shrinkage of concrete is obtained with two-phase composite model as aggregate and matrix. In order to obtain the input values for this prediction equation easily, the experimental formula of drying shrinkage and Young's modulus of cement paste are obtained, and the estimation method of Young's modulus of aggregate are proposed with easy test using cement paste, mortar and concrete. According to the experimental results, this equation can predict the drying shrinkage at any age in error by less than about 100 μm.

Keywords: Drying; Shrinkage; Prediction equation; Concrete; Elastic modulus

1. Introduction

The drying shrinkage of concrete is one of the main causes of the cracks generated in the RC structures. However, with regard to this point in the present concrete works, only the maximum value of the unit water content (185 kg/m^3) is specified in the JASS5, which is not considered to be sufficiently controlled. This is partly because of the lack of confirming the amount of drying shrinkage strain for the concrete used in an ordinary construction, as the drying shrinkage test (JIS A 1120) takes long time and many procedures.

As alternatives of the direct measuring by test, many researchers proposed prediction equations of drying shrinkage such as ones proposed by Bazant and Panula [1], by Branson [2] (ACI-209 equation), in CEB-FIP-70, 78, 90 equations. [3–5], and by Sakata [6]. These equations are practical in a sense that the drying shrinkage can be predicted from the figures written in mix proportion table, and are also useful for the preliminary study at the design stage because the external factors such as the relative humidity, and dimensions, shape of the member, or the age are considered. However, the accuracy of those predicted results are not very good and it is even reported to be around plus and minus 40% [7]. The reasons are considered to be that most of the prediction equations are empirical equations derived from the measured data which involves some errors generated in its statistical processing, and that those equations do not consider the effects of the material properties of which data is difficult to be obtained, in order to enhance its practicability.

Meantime, as a different approach from the above, many researchers proposed theoretical models in which the concrete or mortar was considered as two-phase material and its mechanism of drying shrinkage was derived from combination of the drying shrinkage of aggregate and that of matrix. The advantage of this approach is the clear rationale and the drying shrinkage strain can be accurately estimated by considering the impacts by the material properties only if necessary data were obtained. However, it is obviously distinguished from those practical and generic prediction equations in the following points. Those are that (1) three

* Corresponding author. Tel.: +81 3 3977 2242; fax: +81 3 3977 2251.
E-mail address: kegchi@jcity.maeda.co.jp (K. Eguchi).

0008-8846/$ - see front matter © 2004 Elsevier Ltd. All rights reserved.
doi:10.1016/j.cemconres.2004.08.002

types of material including fine aggregate and coarse aggregate cannot be considered because of the assumption of two-phase material, that (2) it is not a type of equation to directly consider the external factors or the age, and that (3) the input values for the theoretical equation cannot be obtained easily.

The ultimate goal of this study is developing the complex model to a practical prediction equation. Within the scope of this paper, as the first step, the internal factors of the concrete, i.e., factors regarding the mix proportion or material properties, are taken, and a prediction equation of the drying shrinkage is proposed under the condition that the temperature, humidity, and the dimension or shape of the member are kept constant. This becomes the core of the prediction equation to which the external factors will be added in the future. For the development of the prediction equation, problem (1) described above is intended to be solved by applying the complex model stepwise, first to the mortar component in the concrete, and second to the concrete as a whole. For problem (2), a time-dependent prediction equation is proposed, and for problem (3), the method of measuring the factors of the prediction equation, such as the drying shrinkage strain of the cement paste or the Young's modulus of aggregate, is studied.

2. Estimation of drying shrinkage strain by stepwise application of complex model

2.1. Application method of complex model

As the complex models for drying shrinkage strain were proposed by Ohno [8], Pickett [9], Shiire [10], Hobbes [11], or Baba, the equation proposed by Kishitani and Baba [12], which the authors consider to give the most accurate answer, is used here:

$$\frac{\varepsilon_{\mathrm{sc}}}{\varepsilon_{\mathrm{sm}}} = \frac{[1-(1-mn)V_{\mathrm{a}}][n+1-(n-1)V_{\mathrm{a}}]}{n+1+(n-1)V_{\mathrm{a}}} \tag{1}$$

where $n=E_{\mathrm{a}}/E_{\mathrm{m}}$, $m=\varepsilon_{\mathrm{sa}}/\varepsilon_{\mathrm{sm}}$, ε_{s}: drying shrinkage strain, E: Young's modulus (N/mm^2), V_{a}: aggregate volume ratio. The suffixes c, a, m, stand for the two-phase material, aggregate, matrix, respectively.

When the equation shown above is rewritten for the mortar component in the concrete (two-phase material consisting of fine aggregate and cement paste), and for the concrete (two-phase material consisting of coarse aggregate and mortar), the following equations (Eqs. (2), and (3), respectively) are derived.

$$\frac{\varepsilon_{\mathrm{s}}(V'_{\mathrm{s}},0)}{\varepsilon_{\mathrm{sp}}} = \frac{[1-(1-m_1n_1)V'_{\mathrm{s}}][n_1+1-(n_1-1)V'_{\mathrm{s}}]}{n_1+1+(n_1-1)V'_{\mathrm{s}}} \tag{2}$$

$$\frac{\varepsilon_{\mathrm{s}}(V'_{\mathrm{s}},V_{\mathrm{g}})}{\varepsilon_{\mathrm{s}}(V'_{\mathrm{s}},0)} = \frac{[1-(1-m_2n_2)V_{\mathrm{g}}][n_2+1-(n_2-1)V_{\mathrm{g}}]}{n_2+1+(n_2-1)V_{\mathrm{g}}} \tag{3}$$

where, $n_1=E_{\mathrm{s}}/E_{\mathrm{p}}$, $n_2=E_{\mathrm{g}}/E\ (V'_{\mathrm{s}},0)$, $m_1=\varepsilon_{\mathrm{ss}}/\varepsilon_{\mathrm{sp}}$, $m_2=\varepsilon_{\mathrm{sg}}/\varepsilon\ (V'_{\mathrm{s}},0)$, ε_{s}: drying shrinkage strain, E: Young's modulus (N/mm^2), V'_{s}: fine aggregate volume ratio (to the volume of mortar), V_{g}: coarse aggregate volume ratio (to the volume of concrete). The suffixes s, g, p stand for the fine aggregate, coarse aggregate, and cement paste, respectively.

In this study, Eq. (3) is applied to the concrete and the drying shrinkage strain $\varepsilon_{\mathrm{s}}(V'_{\mathrm{s}},V_{\mathrm{g}})$ is predicted. At this time, the value of the factor $\varepsilon_{\mathrm{s}}(V'_{\mathrm{s}},0)$ is estimated by applying Eq. (2) to the mortar component. This approach, the stepwise application of Eqs. (2) and (3), becomes the basis of the prediction equation to be proposed by the authors in Section 4 (these two equations together are called as the basic equation, hereinafter).

Fig. 1 shows the summary of the factors for the basic equation. Although these values have to be obtained in advance in order to predict the drying shrinkage strain, detail discussion will be made in Section 3 with regard to the drying shrinkage strain and the Young's modulus for the cement paste. The estimation method for the Young's modulus of the aggregate will be presented in Section 5.

2.2. Outline of verification experiment

In order to verify the reasonableness of the stepwise application of the complex model, an experiment was conducted by taking a concrete sample under the condition of the drying shrinkage test in accordance with JIS A 1129 (temperature of 20 °C, humidity of 60%, and specimen size of 10×10×40 cm), as an object for prediction.

2.2.1. Drying shrinkage test

In order to obtain data for verification of the basic equation, the drying shrinkage strain was measured for the concrete specimens (series I) and the mortar specimens (series II). The experimental parameters were the water–cement ratio at three levels, 30%, 45%, and 60%, and the aggregate volume ratio at five levels, in a range between 0 and 0.5, as shown in Table 1. It was executed in accordance with JIS A 1129 (dial gauge method) as shown in Table 2, and the size of specimens were 10×10×40 cm for the concrete and 4×4×16 cm for the mortar.

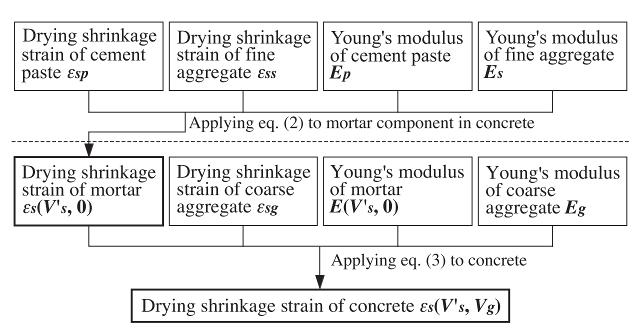

Fig. 1. Flow chart of prediction of drying shrinkage strain for concrete.

Table 1
Experimental factors and levels

Series	Specimen	Experimental factor	Level
I	Concrete	Water–cement ratio (%)	30, 45, 60
		Coarse aggregate volume ratio	0 (Mortar), 0.15, 0.3, 0.4, 0.5
II	Mortar	Water–cement ratio (%)	30, 45, 60
		Fine aggregate volume ratio	0 (Cement paste), 0.15, 0.3, 0.4, 0.5

The mix proportion of the concrete was determined so as to take the mortar mix shown in Table 3 as the matrix, and to keep its ratio to the coarse aggregate to be the level of the designated coarse aggregate volume ratio. The mix proportioning of the mortar was determined so as to take the cement paste obtained by subtracting the coarse aggregate from the mix proportion shown in Table 3, as the matrix, and to keep its ratio to the fine aggregate to be the level of the designated fine aggregate volume ratio.

In this series of experiment, the air content was not conditioned, and no special attention was paid to the bleeding. Impacts of these factors on the drying shrinkage strain have not been made clear yet and another study in detail would be necessary.

2.2.2. Test for Young's modulus

In order to obtain the factors of the basic equation, $E(V'_s,0)$, E_p, the Young's modulus was measured for the specimens of the series I with the coarse aggregate volume ratio of 0 (mortar), and for the specimens of the series II with the fine aggregate volume ratio of 0 (cement paste), respectively. In this measurement, considering that E_p and $E(V'_s,0)$ are the properties of the member subject to the drying shrinkage, the curing of the specimens was carried out under the same conditions as the drying shrinkage test (curing at a constant temperature and a constant humidity). The age at which test was conducted, was the same as the drying shrinkage test for the series I, and was only 28 days for the series II.

Table 2
Test items and outlines

Test item	Series	Test method	Test conditions		Test age[a] (day)
			Curing method	Specimen dimensions (cm)	
Drying shrinkage strain	I II	JIS A 1129	Standard water curing (7 days) ↓	10×10×40 4×4×16	0, 3, 7, 28, 56, 91, 182
Young's modulus	I II	ASTM C 469	Constant temperature and humidity[b]	10ϕ×20 5ϕ×10	28

[a] Not including standard water curing period.
[b] Temperature 20 °C, humidity 60%.

Table 3
Proportion of mortar component in concrete

W/C (%)	Air (%)	Sand–cement ratio	Fine agg. volume ratio	Unit weight (kg/m^3)			Ad.[a] (C×%)
				W	C	S	
30	4.2	1.73	0.451	247	882	1178	1.25
45	6.4	2.81	0.503	254	565	1312	0.90
60	6.4	3.65	0.522	271	452	1362	0.25

[a] High-performance AE water reducing agent (W/C 30, 45%) or AE water reducing agent (W/C 60%).

2.2.3. Material used and test contents

For applying the basic equation, the factors prescribing the aggregate ε_{ss}, ε_{sg}, E_s, E_g, shall be determined. Therefore, crushed sand and crushed stone were used for the test and their raw rocks (size: 200–300 mm) were obtained from the manufacturer. Cores of 35 mmϕ×70 mm were taken from these rocks and the drying shrinkage strain and the Young's modulus were measured on them. In the drying shrinkage test, two sheets of water-proof type wire strain gauges were attached on the sides of the core, and by setting the state well absorbed with water as the initial point, the strain was measured under the constant temperature and humidity environment (temperature 20 °C, humidity 60%). The cement used was an ordinary Portland cement.

2.3. Values of factors in basic equation

2.3.1. Drying shrinkage strain and Young's modulus of aggregate

Table 4 shows the quality test results of the aggregates and the test results on the cores taken from the raw rocks. Fig. 2 shows time-dependent change in the drying shrinkage strain for the aggregates (a result of water absorption test conducted after the drying shrinkage test is shown together for reference). Drying shrinkage strain of limestone (the fine aggregate) was small as pointed out in general. On the other hand, drying shrinkage strain of hard sandstone (the coarse aggregate) reached to the ultimate level around 35 days and its value was 2.41×10^{-4}. The Young's modulus of both fine and coarse aggregate was those of average values for concrete aggregates [12].

2.3.2. Young's modulus of matrix

Fig. 3 shows the Young's modulus of the mortar and the cement paste (test results of Section 2.2.2). According to the figure, the Young's modulus of the mortar cured under the constant temperature and constant humidity exhibits little change after age of 3 days, which is different from the case under the standard water curing. It thus implies that it would not cause any problem practically if the $E(V'_s,0)$ in Eq. (3) were regarded as the a factor independent on time as far as being conducted under the test condition in accordance with JIS A 1129. Although no time-dependent test was executed on the cement paste, it is considered that the same tendency would be observed for the E_p in Eq. (2).

Table 4
Aggregate used

Items			Fine aggregate	Coarse aggregate
Source, specification			Crushed sand made in Nishi-Tama	Crushed stone made in Ryogami
Type of rock			Limestone	Hard sandstone
Maximum size (mm)			5	20
Quality test results	Fineness modulus		3.34	6.67
	Density (g/cm3)	Surface-dry	2.61	2.72
		Oven-dry	2.55	2.69
	Water absorption (%)		2.26	0.95
	Unit volume weight (kg/l)		1.61	1.63
	Solid content (%)		63	60.7
Test results on core of raw rock	Compressive strength[a] (MPa)		82.1 (49.5–129.3)	131.5 (92.2–167.9)
	Young's modulus[a] (GPa)		67.9 (59.9–73.1)	64.8 (53.1–73.1)
	Ultimate drying shrinkage strain[b] ($\times 10^{-4}$)		0.28	2.41

Figures in () stands for the range of test results.
[a] Average of about 10 specimens.
[b] Average of 3 specimens.

2.4. Verification of estimated value of drying shrinkage strain

Here at first, the accuracy of the equation proposed by Baba for the mortar and the concrete is studied, respectively. Then the drying shrinkage strain of the concrete is estimated based on the authors' theory, and the degree of fit with the experimental results is examined. For the estimation, the following values are assumed for the factors in the basic equation.

ε_{ss}, ε_{sg}: values of raw rocks shown in Fig. 2 (ultimate values), ε_{sp}: test values at V'_s=0 in the series II experiment, E_s, E_g: values of raw rocks shown in Table 4, E_p, $E(V'_s,0)$: values shown in Fig. 3 (drying period of 28 days).

2.4.1. Estimated values for mortar

Fig. 4 shows the relationship between the drying shrinkage strain of the mortar $\varepsilon_s(V'_s,0)$ (drying period of 182 days) and the volume ratio of fine aggregate, V'_s. The curve shown in the figure is the theoretical curve given by Eq. (2). According to the figure, the estimated value (value given by the theoretical curve) becomes larger than the test value for the mortar specimen because the curvature of the theoretical curve is small for any water–cement ratio case. The theoretical curve becomes straighter as n_1 (=E_s/E_p) in Eq. (2) is smaller, however the causes are not understood in detail as far as this study is concerned. It could be attributed to the errors induced by variation in the air content or in the bleeding. Meantime, the errors in the estimation do not appear to be significantly large compared to the verification results given by Eq. (1) originally proposed by Kishitani and Baba [12].

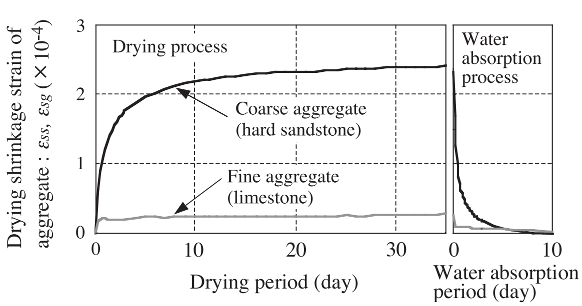

Fig. 2. Drying shrinkage strain of aggregate.

2.4.2. Estimation results for concrete

Fig. 5 shows a relation between the drying shrinkage strain for the concrete, $\varepsilon_s(V'_s,V_g)$ (drying period of 182 days), and the volume ratio of coarse aggregate, V_g. The theoretical curve shown in the figure is for verifying the fitness of the equation proposed by Baba. Therefore, the $\varepsilon_s(V'_s,0)$ in Eq. (3) is not the estimated value given by Eq. (2), but the test value obtained from the concrete specimen with V_g=0. According to the figure, the theoretical curve coincides well with the test values for the concrete specimens.

2.4.3. Estimated values for concrete by method proposed in this study

Fig. 6 shows the relationship between the $\varepsilon_s(V'_s,V_g)$ and V_g for the concrete, together with the relationship $\varepsilon_s(V'_s,0)$ and V'_s for the mortar shown in Fig. 4. The theoretical curve of the concrete is according to the method proposed in this study, in which the $\varepsilon_s(V'_s,0)$ in Eq. (3) is given by the estimated value obtained by Eq. (2). In this figure, the

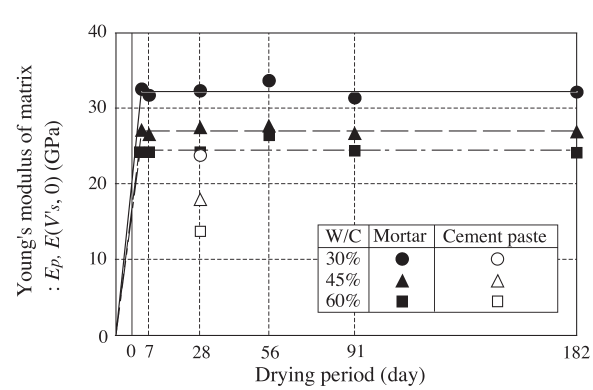

Fig. 3. Young's modulus of mortar and cement paste (cured at constant temperature and humidity).

theoretical curves appear to coincide with the tested values for the cases with the water–cement ratio of 45% and 60%. However, at the water–cement ratio of 30%, it becomes larger than the test values caused by the estimated value of $\varepsilon_s(V'_s,0)$ in Eq. (2) being larger than the actual data.

Based on the above discussion, as the fitness of the Baba's equation to the concrete is considerably well, if a precise value of the $\varepsilon_s(V'_s,0)$ were obtained by Eq. (2), the drying shrinkage strain would be estimated accurately by Eq. (3). However, as the drying shrinkage strain for the mortar is large compared to that for the concrete, a small estimation error for the mortar causes large impacts on the estimation accuracy for the concrete. Therefore, in order to increase the accuracy of the estimation for the concrete, more precise estimation of the drying shrinkage strain for the mortar is considered important.

2.4.4. Time-dependent change of estimated value for concrete

By applying the same estimation method at various ages as that applied at the unique age (182 days), and by connecting those estimated values, a time-dependent change curve of the drying shrinkage strain can be drawn. Fig. 7 shows the time-dependent change curve estimated in this manner, and the test values, comparatively.

According to the figure, the estimation values at early ages become greater than the test values at any water–cement ratio. These are the errors caused by taking the tested value for the 4×4×16-cm specimen as the ε_{sp} in Eq. (2), in the estimation process of $\varepsilon_s(V'_s,V_g)$ for the 10×10×40-cm specimen. The reason is that, since the drying shrinkage proceeds faster as the member dimensions are smaller, the $\varepsilon_s(V'_s,0)$ is estimated larger than the actual values at early ages. Therefore, when the drying shrinkage strain at early ages is questioned, it is necessary to study further on this point. But the member dimension is a factor that impacts only on the time-dependent change of the drying shrinkage strain [13,14], the point discussed above is not considered to be the factor inducing errors for the ultimate value.

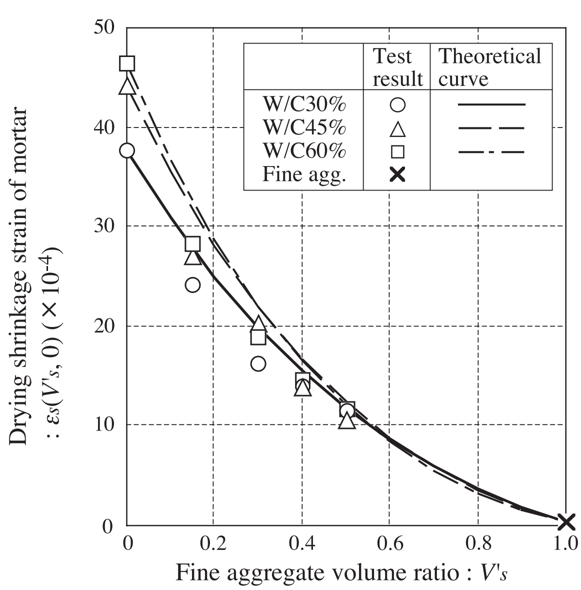

Fig. 4. Relationship between drying shrinkage strain and fine aggregate volume ratio for mortar (drying period of 182 day).

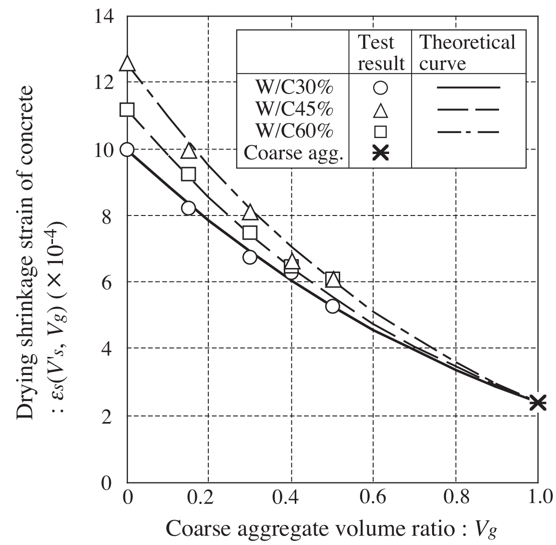

Fig. 5. Relationship between drying shrinkage strain and coarse aggregate volume ratio for concrete (drying period of 182 day).

3. Deriving equation of drying shrinkage strain and Young's modulus for cement paste

If the drying shrinkage strain for the cement paste, ε_{sp}, which is the factor in Eq. (2), were intended to be obtained always by tests, it would take the same period as that required for the drying shrinkage test for the concrete and not be feasible. Thus, it is discussed here on the derivation of the equation for the ε_{sp} from the test results. The same discussion is also made for the Young's modulus, E_p. However for the E_p, it is considered relatively easy to obtain data from the test.

3.1. Outline of test for data collection

By taking the cement paste as sample, the drying shrinkage strain and the Young's modulus (cured at a constant temperature and humidity, age 28 days) were measured. Table 5 shows the outline of the test. The water–cement ratios for the specimens were set at four levels of 30%, 35%, 45%, and 60%, and the air content was set at 0% (with allowance of +2%). The types of the cement tested were an ordinary Portland cement (N), a type B fly ash cement (FB), and a type B blast furnace slag cement (BB).

3.2. Derivation of equation for drying shrinkage strain

Fig. 8 shows plots of the drying shrinkage strain for the cement paste obtained by the experiment. In order to

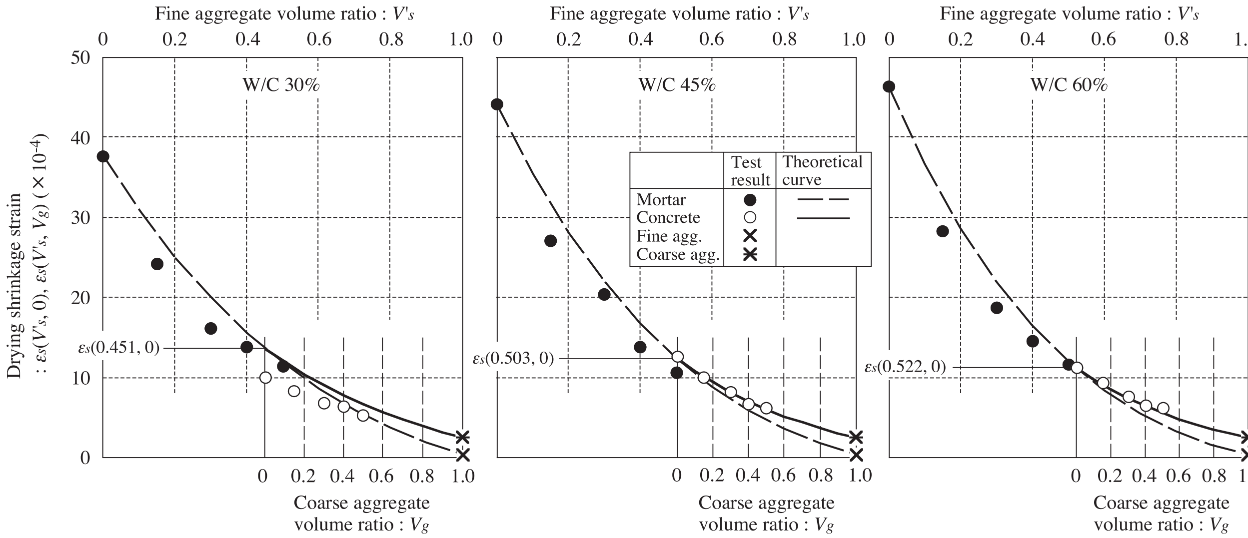

Fig. 6. Drying shrinkage strain for concrete obtained stepwise (drying period of 182 day).

approximate individual time-dependent change curve, simple approximation by Eq. (4) was carried out. This equation takes the same form as the Branson's equation [2] (the ACI-209 equation), or the Almudaihcem's equation [14] (modified Ross's equation), and it consists of the term representing the time-dependent change of the drying shrinkage strain and the term representing the ultimate shrinkage value.

$$\varepsilon_{sp}(t) = \frac{t}{\alpha + t}\varepsilon_{sp\infty} \tag{4}$$

where $\varepsilon_{sp}(t)$: drying shrinkage strain of the cement paste, t: drying period (day), $\varepsilon_{sp\infty}$: ultimate value of the drying shrinkage strain for the cement paste, α: constant determined by the type of cement and the water–cement ratio.

Table 6 shows the results of the simple approximation by the above equation. The regression coefficient R^2 shown in the table is close to 1 in most of the specimens and the approximation by Eq. (4) turns out to be accurate. In addition, when the relationship between the regression results of $\varepsilon_{sp\infty}$ and α, and the water–cement ratio is investigated, a proportionality is apparently observed between those as shown in Figs. 9 and 10. Then based on this observation, by replacing the $\varepsilon_{sp\infty}$ and α in Eq. (4) with direct functions of the water–cement ratio, respectively, and multiple regression by the following equation was carried out.

$$\varepsilon_{sp}(t) = \frac{t}{\alpha W/C + \beta + t}(\lambda W/C + \delta) \tag{5}$$

where $\varepsilon_{sp}(t)$: drying shrinkage strain of the cement paste, t: drying period (day), W/C: water–cement ratio (%), α, β, λ, δ: constants determined by the type of cement.

Table 7 shows the results of the multiple regression by the above equation, and the regression results for $\varepsilon_{sp}(t)$ are shown in Fig. 8. As figured out by the regression coefficient

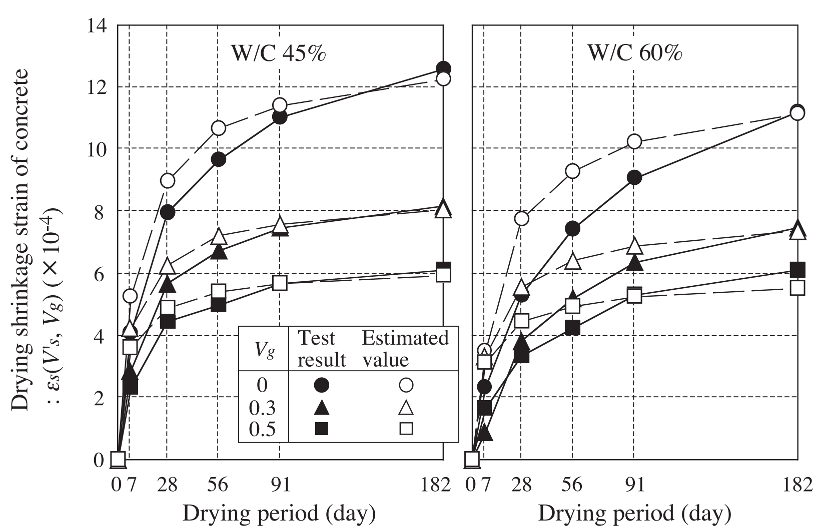

Fig. 7. Time-dependent changes in drying shrinkage strain for concrete.

Table 5
Outline of experiment for cement paste

Test item	Series	Experimental factors			Test method	Test conditions		Age at testing[a] (day)
		Type of cement	W/C (%)	Air (%)		Curing method	Size of specimen (cm)	
Drying shrinkage strain	III	N, FB, BB	30, 35, 45, 60	0	In accordance with JIS A 1129	Standard water curing (7 days) ↓	4×4×16	0, 7, 28, 56, 91, 182
Young's modulus					ASTM C 469	Constant temperature and humidity[b]	5ϕ×10	28

[a] Not including standard water curing period.
[b] Temperature 20 °C, humidity 60%.

R^2 shown in Table 7, the drying shrinkage strain for the cement pastes (air content of 0%) made of the various types of cement is approximated with a high accuracy by Eq. (5). When considered from the standpoint of the complex theory, impacts by the factors such as the air content, on the drying shrinkage strain for concrete shall be explained as impacts by the air content in Eq. (5), which will be considered in the future research.

3.3. Derivation of equation for Young's modulus

Fig. 11 shows the relationship between the Young's modulus and the cement–water ratio for the cement paste. Table 8 also shows values of the constants and the regression coefficients with which the regression curves in the figure are expressed by the following equation.

$$E_p = \gamma C/W + \eta \tag{6}$$

where E_p: Young's modulus of the cement paste (N/mm^2), C/W: cement–water ratio, γ, η: constants determined by the type of cement.

As it is confirmed by that the coefficient of regression, R^2 is close to 1, the Young's modulus of the cement paste (air content of 0%) cured at a constant temperature and humidity, can be expressed by the simple proportionality relation with the cement–water ratio. With regard to the impacts by factors such as the air content, it shall be considered in the same manner as the case for the drying shrinkage strain.

4. Proposal of estimation equation of drying shrinkage for concrete

Out of the factors in the basic equation, time-dependent factors are the drying shrinkage strain of cement paste and aggregate, ε_{sp}, ε_{ss}, and ε_{sg}. (E_p and $E(V'_s,0)$ are not considered as time-dependent factors based on the reason stated in the Section 2.3.2.) Thus, these factors will determine the time-dependent change in the drying shrinkage strain for the concrete.

Based on this consideration, when the basic equation is developed into a time-dependent prediction equation, the

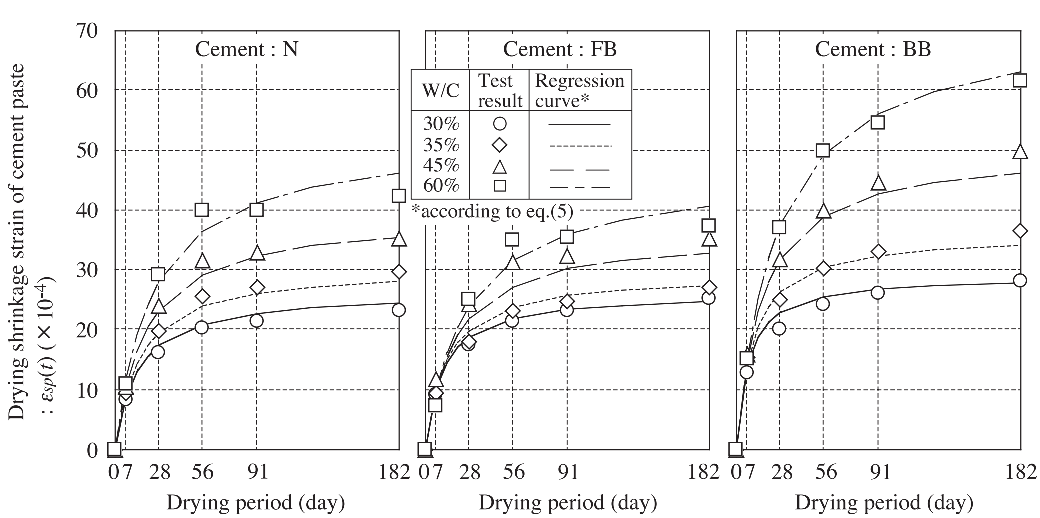

Fig. 8. Drying shrinkage strain for cement paste.

Table 6
Regression results by Eq. (4)

W/C (%)	N			FB			BB		
	α	$\varepsilon_{sp\infty}$ ($\times 10^{-4}$)	R^2	α	$\varepsilon_{sp\infty}$ ($\times 10^{-4}$)	R^2	α	$\varepsilon_{sp\infty}$ ($\times 10^{-4}$)	R^2
30	14.15	24.92	0.99	13.52	26.77	0.99	9.00	28.44	0.97
35	17.66	32.82	1.00	14.77	28.88	0.99	10.05	36.55	0.96
45	19.20	40.24	0.98	16.19	38.69	0.99	16.80	52.80	0.99
60	23.57	51.27	0.91	31.51	47.94	0.77	25.42	70.68	1.00

R^2: coefficient of regression.

following Eqs. (7) and (8) are derived. The factor $\varepsilon_{sp}(t)$ in Eq. (7) has already been obtained in Section 3. In addition, by considering the practicality in the application, V'_s in Eq. (2) is replaced with V_s in Eq. (7) by the relationship $V'_s=V_s/(1-V_g)$.

$$\frac{\varepsilon_s(V_s,0,t)}{\varepsilon_{sp}(t)} = \frac{[1-V_g-(1-m_1n_1)V_s][(n_1+1)(1-V_g)-(n_1-1)V_s]}{[(n_1+1)(1-V_g)+(n_1-1)V_s](1-V_g)} \tag{7}$$

$$\frac{\varepsilon_s(V_s,V_g,t)}{\varepsilon_s(V_s,0,t)} = \frac{[1-(1-m_2n_2)V_g][n_2+1-(n_2-1)V_g]}{n_2+1+(n_2-1)V_g} \tag{8}$$

where, $n_1=E_s/E_p$, $n_2=E_g/E(V_s,0)$, $m_1=\varepsilon_{ss}(t)/\varepsilon_{sp}(t)$, $m_2=\varepsilon_{sg}(t)/\varepsilon(V_s,0,t)$, $\varepsilon_s(V_s,V_g,t)$: drying shrinkage strain of the concrete, V_s, V_g: fine aggregate volume ratio, coarse aggregate volume ratio (to the volume of concrete), t: drying period (day), $\varepsilon_{ss}(t)$, $\varepsilon_{sg}(t)$: drying shrinkage strain of fine aggregate and coarse aggregate, $\varepsilon_{sp}(t)$: drying shrinkage strain of the cement paste (obtained by Eq. (5) and Table 7), E_s, E_g: Young's modulus of fine aggregate and coarse aggregate (N/mm^2), E_p: Young's modulus of the cement paste (N/mm^2) (obtained by Eq. (6) and Table 8).

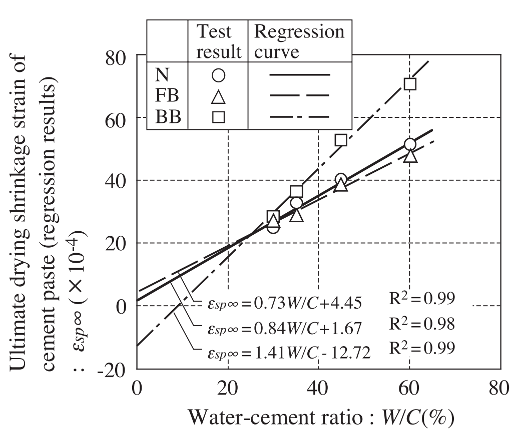

Fig. 9. Relationship between ultimate drying shrinkage strain and water–cement ratio for cement paste.

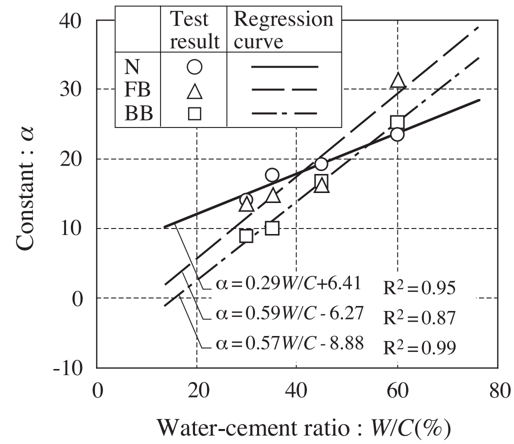

Fig. 10. Relationship between constant and water–cement ratio.

As shown above, Eqs. (7) and (8) with additional Eqs. (5) and (6), are the prediction equations of the drying shrinkage strain for the concrete proposed by the authors within the scope of the study presented in this paper. In this series of equations, the drying shrinkage strain of the concrete at a certain age is predicted by the factors regarding the mix proportions consisting of the water–cement ratio, fine aggregate volume ratio (unit volume of fine aggregate), and coarse aggregate volume ratio (unit volume of coarse aggregate), and the factors regarding the material properties consisting of the type of cement, drying shrinkage strain of fine aggregate and coarse aggregate, and their Young's modulus.

Fig. 12 shows the comparison of the predicted values of the drying shrinkage strain for concrete by Eqs. (7) and (8), and the actual test values. In the figure, data for the concrete specimens shown in Table 1 (three levels of water–cement ratio, five levels of coarse aggregate volume ratio), at the five levels of age are presented. According to the figure, the accuracy of the prediction equations is within about $\pm 1\times 10^{-4}$. However, there is some room left for improvement such as that the improvement in the accuracy can be expected in the range of small values shown in Fig. 12 by reducing the errors in the prediction values at early ages described in the Section 2.4.4.

5. Estimation of Young's modulus for aggregate

At present, there is no method to obtain the Young's modulus accurately by measuring the aggregate itself. Therefore, in the discussion in Section 2, the test values of the Young's modulus obtained by the raw rocks were

Table 7
Regression results by Eq. (5)

Type of cement	α	β	λ	δ	R^2
N	0.322	4.77	0.863	0.54	0.95
FB	0.518	−4.72	0.678	5.81	0.82
BB	0.608	−10.77	1.437	−14.08	0.98

R^2: coefficient of regression.

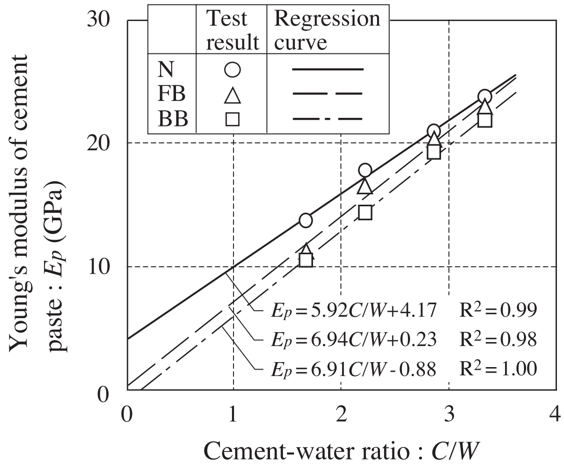

Fig. 11. Relationship between Young's modulus and cement–water ratio for cement paste (curing at constant temperature and humidity, age 28-days).

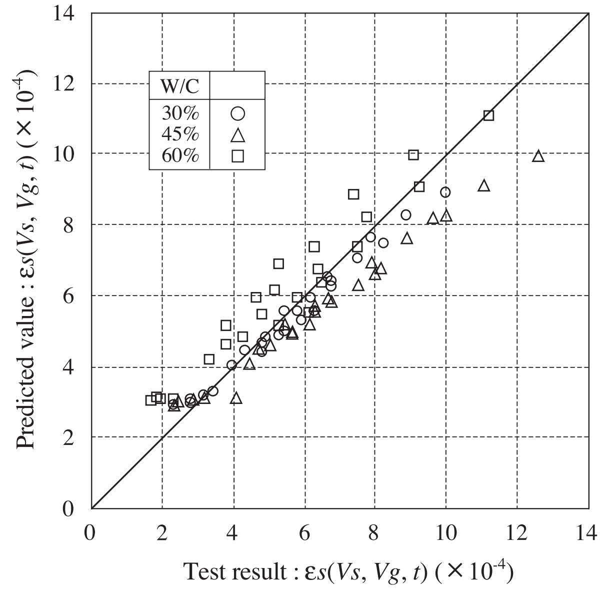

Fig. 12. Comparison of predicted values and test results of drying shrinkage strain for concrete.

used for the ε_{ss} in Eq. (2) and the ε_{sg} in Eq. (3). But it is not practical to adopt this method in real situations, and there is no raw rock for gravel or sand. Therefore, a method is hereby proposed to estimate the Young's modulus of the aggregate by conducting a simple test and applying the complex model for the Young's modulus to the test result, and its applicability is discussed.

5.1. Estimation method of Young's modulus for aggregate

The complex models of the Young's modulus were proposed by many researchers [15,16]. Here, the Hashin-Hansen's equation [17] (Eq. (9)), of which several papers pointed out high fitness [12,15,16], is used.

$$\frac{E_c}{E_m} = \frac{n+1+(n-1)V_a}{n+1-(n-1)V_a} \tag{9}$$

where $n = E_a/E_m$, E: Young's modulus (N/mm^2), V_a: aggregate volume ratio. The suffixes c, a, m, stand for two-phase material, aggregate, and aggregate, respectively.

At first, the following equation is obtained by rearranging it for E_a.

$$\frac{E_a}{E_m} = -\frac{h-1+(h+1)V_a}{h-1-(h+1)V_a} \tag{10}$$

where $h = E_c/E_m$.

Next, the above equation is rewritten in forms separately for the mortar component of the concrete and for the concrete, and the following Eqs. (11) and (12) are obtained.

$$\frac{E_s}{E_p} = -\frac{h_1-1+(h_1+1)V_s'}{h_1-1-(h_1+1)V_s'} \tag{11}$$

$$\frac{E_g}{E(V_s',0)} = -\frac{h_2-1+(h_2+1)V_g}{h_2-1-(h_2+1)V_g} \tag{12}$$

where $h_1 = E(V_s',0)/E_p$, $h_2 = E(V_s',V_g)/E(V_s',0)$, E: Young's modulus (N/mm^2), V_s': fine aggregate volume ratio (to the mortar volume), V_g: coarse aggregate volume ratio (to the concrete volume). The suffixes s, g, p, stand for fine aggregate, coarse aggregate, and cement paste, respectively.

In the real application, the V_s' in Eq. (11) can be replaced with the fine aggregate volume ratio (to the concrete volume), V_s, by the relationship $V_s' = V_s/(1-V_g)$, and expressed as follows.

$$\frac{E_s}{E_p} = -\frac{(h_1-1)(1-V_g)+(h_1+1)V_s}{(h_1-1)(1-V_g)-(h_1+1)V_s} \tag{13}$$

where $h_1 = E(V_s,0)/E_p$.

Fig. 13 shows the flow of the estimation of the Young's modulus for aggregate. In the method assumed in this study, at least one test for the Young's modulus is executed for cement paste, mortar, and concrete, respectively. Then, the Young's modulus of the fine aggregate is estimated by

Table 8
Regression results by Eq. (6)

Type of cement	γ	η	R^2
N	5.92	4.17	0.99
FB	6.94	0.23	0.98
BB	6.91	−0.88	1.00

R^2: coefficient of regression.

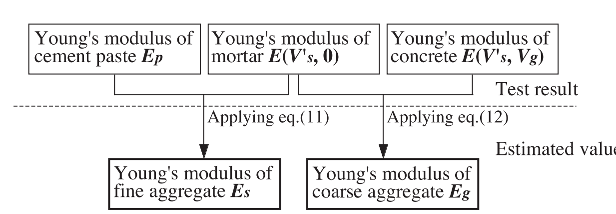

Fig. 13. Flow chart of estimation of Young's modulus for aggregate.

applying Eq. (11) to the test results, E_p and $E(V'_s,0)$, and in the same manner, the Young's modulus of the coarse aggregate is estimated by applying Eq. (12) to the test results $E(V'_s,0)$ and $E(V'_s,V_g)$. (The V'_s, V_g described here are taken freely, and are not necessarily the same values as those by Eqs. (2) and (3).)

5.2. Outline of verification test

In order to obtain the verification data for the estimation equation of the Young's modulus for aggregate, tests for the Young's modulus were executed by using the same specimens as those for the concrete (series I) and for the mortar (series II) as shown in Table 1. The outline of the test is summarized in Table 9. The specimens were cured at the standard water curing condition, and the test was conducted at age 7 and 28 days.

5.3. Verification of estimated value of Young's modulus for aggregate

5.3.1. Estimated values for fine aggregate

Fig. 14 shows the relationship between the Young's modulus of the mortar, $E(V'_s,0)$, and the fine aggregate volume ratio, V'_s. The following two types of curves are shown in the figure, (1) the regression curve obtained by Eq. (11) with its input parameters being the tested value for the mortar specimens, (2) a theoretical curve obtained by Eq. (11) with E_s being the tested value for the raw rock of the aggregate, E_p being the tested value of the mortar specimen at $V'_s=0$. Here, the estimation value for the Young's modulus of the fine aggregate, E_s, becomes the value given by the regression curve (1) at $V'_s=1$, that is, $E(1,0)$. (In the original method, $E(0,0)$ is obtained by one of the tested values for the mortar sample, but it is estimated here by regression in order for verification of the accuracy.)

According to the figure, the tested values for the mortar specimens derail downward from the theoretical curve as the V'_s increases at any water–cement ratio. Due to this effect, the gradient of the regression curve becomes flat, and the estimated value for the fine aggregate, E_s, becomes 65–73% (average 67.5%) of the tested value of the raw rock.

5.3.2. Estimated values for coarse aggregate

Fig. 15 shows the relationship between the Young's modulus of concrete, $E(V'_s,V_g)$, and the coarse aggregate volume ratio, V_g. In the figure, the same two curves in Fig. 14 are shown. As read from the figure, the tendency of the regression curve becoming flat is basically the same as the case for the fine aggregate. However, the estimated value of the coarse aggregate, E_g (=$E(V'_s,1)$) obtained by the regression curve is 72–90% (average 82.3%) of the tested value for the raw rocks, which is closer to the raw rock value than the case with the fine aggregate.

5.3.3. Issues in estimation method

The tendency that the estimated values by the complex model does not fit with the tested values in a range of high

Table 9
Outline of test for Young's modulus

Test item	Series	Test method	Test conditions		Age at testing (day)
			Curing method	Size of specimen (cm)	
Young's modulus	I II	ASTM C 469	Standard water curing	10ϕ×20 5ϕ×10	7, 28

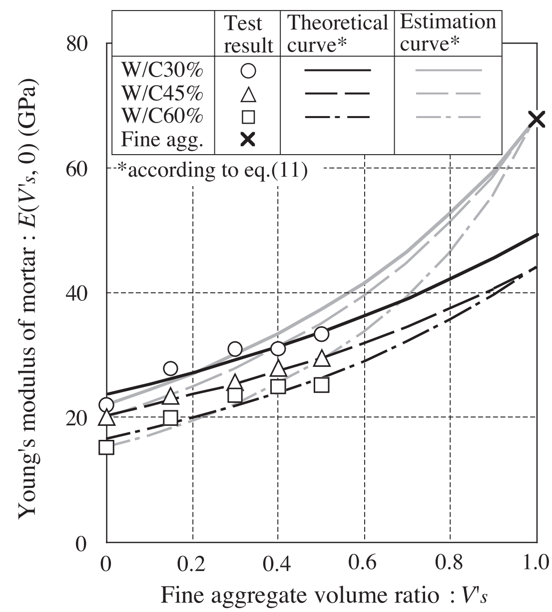

Fig. 14. Relationship between Young's modulus of mortar and fine aggregate volume ratio (age 28 day).

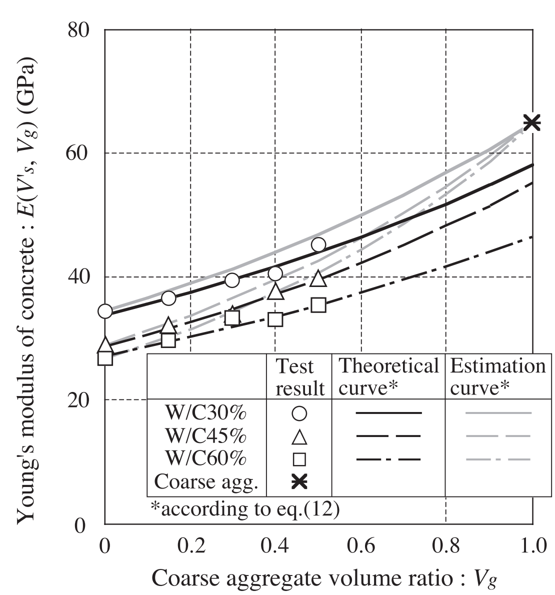

Fig. 15. Relationship between Young's modulus of concrete and coarse aggregate volume ratio (age 28 day).

aggregate volume ratio shown in Figs. 14 and 15, was also reported by Chou and Kobayashi [15], or by Kosaka et al. [16]. As a cause of this phenomenon, the increase in the entrapped air due to the increase in the aggregate volume ratio was pointed out. It is also considered that as weak part is generated inside of the aggregate in the process of crushing raw rocks down to the size for aggregate, it is possible to induce some reduction in the Young's modulus for the aggregate. However, it has not been cleared by this study which causes impacted on the errors in the estimation, which remains as future research topics.

Among the factors in Eqs. (7) and (8), the drying shrinkage strain for aggregate, ε_{ss}, ε_{sg}, is the same case as that a test using the aggregate itself is difficult. In this regard, there is no effective method for measurement and a simple test or estimation method is required to be proposed in the future.

Although it has not been mentioned on how to determine a factor in Eq. (8), $E(V_s,0)$, this value can be estimated by using Eq. (11). However, the value needed for Eq. (8) is that on the member subjected to the drying shrinkage (cured at a constant temperature and humidity), and it cannot be estimated from this test using the specimen subjected to the standard water curing condition. Thus, in order for practical application of the test for estimating the Young's modulus for aggregate, it is reasonable to specify the curing at a constant temperature and humidity in which the value of $E(V_s,0)$ can be obtained at the same time.

6. Conclusions

In this study, the prediction equation of the drying shrinkage for concrete based on the complex model was proposed and its applicability was examined. In addition, the method to determine the factors for the prediction equation was investigated. The findings throughout this study are listed as follows.

(1) The drying shrinkage strain of the concrete at any age can be predicted by Eqs. (7) and (8), in which the existing complex model is applied stepwise to the mortar component of the concrete and to the concrete, and by the additional Eqs. (5) and (6).
(2) The factors of the prediction equations are the water–cement ratio, fine aggregate volume ratio (unit fine aggregate volume), coarse aggregate volume ratio (unit coarse aggregate volume), type of cement, drying shrinkage strain and Young's modulus for fine aggregate and coarse aggregate.
(3) The accuracy of the prediction equations is about within $\pm 1\times10^{-4}$. For further improvement in the accuracy, it is important to minimize the prediction error for the mortar component. In addition, for minimizing the prediction errors generated at early ages, the dimensions of the member shall be adjusted.
(4) The Young's modulus, which is one of the factors for the prediction equations, can be obtained relatively easily by at least one measurement of the Young's modulus for the cement paste, mortar, and concrete, respectively, and by applying the results to Eqs. (12) and (13). However, the Young's modulus estimated by this method tends to be smaller than the Young's modulus of the raw rock.

Major issues which remain unsolved are, at first, that a simple method to determine the drying shrinkage strain of the aggregate shall be considered. Then, in order to extend the prediction equations to the member level, the factors such as the relative humidity, the size or shape of the member shall be involved into the prediction equations.

References

[1] Z.P. Bazant, L. Panula, Practical prediction of time-dependent deformation of concrete, Part I: Shrinkage, Materiaux et Constructions 11 (65) (1978) 307–316.
[2] ACI committee 209, Prediction of creep, Shrinkage and temperature effects in concrete structures, ACI Spec. Publ. (SP-76) (1982) 193–300.
[3] CEB-FIP: International Recommendations for the Design and Construction of Concrete Structures, Comite Europeen du beton-Federation International de la Precontrainte, Cement and Concrete Association, London, 1970, pp. 27–29.
[4] CEB-FIP: Model Code for Concrete Structures (International System of Unified Standard Codes of Practice for Structures), Comite Euro-International du Beton, Lausanne, Switzerland, 1978, pp. 331–338.
[5] CEB-FIP: Model Code 1990, Comite Euro-International du Beton, Thomas Telford Services, 1990, pp. 53–58.
[6] K. Sakata, Prediction of drying shrinkage and creep of the concrete, Concrete Journal, JCI 31 (2) (1993) 5–14.
[7] Japan Concrete Institute, Proceedings of symposium committee for the time-dependent deformation due to the creep and shrinkage of the concrete structures, JCI, Tokyo, 2001, pp. 101–121.
[8] K. Ohno, A study on drying shrinkage and crack prevention for the mortar and concrete, Bulletin of the Engineering, Hokkaido University (9) (1953) 49–110.
[9] G. Pickett, Effect of aggregate on shrinkage of concrete and a hypothesis concerning shrinkage, Journal of the ACI (1) (1956) 581–590.
[10] T. Shiire, Experimental study on the amount of cement paste and the drying shrinkage of the concrete, Proceedings of the Architectural Research Meetings of Kanto Chapter, AIJ 58 (6) (1961) 13–16.
[11] D.W. Hobbs, The dependence of the bulk modulus, Young's modulus, creep, shrinkage and thermal expansion of concrete upon aggregate volume concentration, Materiaux et Constructions 4 (20) (1971) 107–114.
[12] K. Kishitani, A. Baba, Mechanism of drying shrinkage for construction material, Cement and Concrete (346) (1975) 30–40.
[13] K. Goto, Recent research on the prediction equations of drying shrinkage for concrete, Concrete Journal, JCI 33 (4) (1995) 62–67.
[14] J.A. Almudaiheem, W. Hansen, Effect of specimen size and shape on drying shrinkage of concrete, ACI Materials Journal 84 (2) (1987) 130–135.
[15] R.Z. Chou, K. Kobayashi, Complex properties in the modulus of elasticity for the concrete, Concrete Journal, JCI 12 (7) (1974) 27–34.
[16] Y. Kosaka, Y. Tanigawa, F. Ohta, Effects of aggregate on the Young's modulus of concrete, JCA Proceedings of Cement and Concrete (22) (1968) 260–265.
[17] T.C. Hansen, Influence of aggregate and voids on modulus of elasticity of concrete, cement mortar, and cement paste, Journal of the ACI (2) (1965) 193–216.