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Reliability analysis for the existing reinforced concrete pile corrosion of bridge substructure

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Abstract

This study investigated the failure probability for existing reinforced concrete (RC) pile corrosion due to carbonation, chloride ion ingress and sulfate attack combined with the load-carrying capacity for bridge substructures at different amounts of service times. The general corrosion resulted from carbonation, the pitting corrosion occurred from chloride ion ingress, and the chemical mechanism due to sulfate attack to the existing RC piles of a bridge substructure is described. Both the theory of structural joint failure probability and the measurement techniques of carbonation depth, chloride ion ingress depth, and sulfate expansion are also provided. An existing RC bridge pile with a service time of 60 years is offered as an example for illustrating the structural joint failure probability theory. The structural joint failure probability of existing RC bridge piles due to the combination of carbonation, chloride ion ingress, sulfate attack, and load-carrying capacity at a service time of 60 years had lower and upper bound values of .440382 and .502755, respectively. These results are provided as a critical decision-making for the repair, strengthening, or demolition of existing RC bridge substructure pile foundations.

Keywords: Carbonation; Corrosion; Degradation; Expansion; Reaction

1. Introduction

The corrosion damage mechanism in existing reinforced concrete (RC) bridge exhibits different corrosion properties depending on the environment. In RC bridge substructure pile corrosion, the corrosion factors involve carbonation, chloride ions, and sulfate. Because the clay in the earth consists of alkali sulfate and sulfuric solutions exist in the soil voids, the compressive strength of the concrete is decreased and the harmful factors penetrate easily into the RC piles. Somuah et al. [1] pointed out that the passive oxide film formed on the surface of the steel in concrete breaks down in the presence of contaminants such as chloride, CO_2, and sulfate ions. Careful detailed investigations must be performed to provide crucial references for the planning, design, repair, and reinforcement of RC piles.

Recently, a variety of analytical methods have been developed for the problem of bridge structure. Among these methods, the reliability analysis [2–8], the service life prediction method [9–14], and the field testing method [15–19] are the three main research approaches. The field testing method, accounting for both material and environmental factors, is comparatively more accurate, and its accuracy greatly depends on the exactness of the testing method and analytical technique employed.

Generally speaking, any structure could exhibit many failure modes during its service time. If each failure mode cannot be considered to evaluate the structural joint failure status, then the evaluated results will be not reliable. To solve this problem, many structural joint failure probability methods have been studied and provided. Cornell [20] furnished a bound formula for failure probability. A single failure model was only considered in this theory while the relativity between two failure models was not considered. This method has largely width between lower and upper bound under general condition and is roughly suitable to estimate the reliability index of structural system. Kounias [21] provided the failure probabilities of two failure models at the same time. As the obtained bound is rather narrow, it is called the narrow reliability bound method. Ditlevsen and Madsen [22] suggested the plan for the reliability design and evaluation of structural systems. Formerly concerning the investigation on

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corrosion damage, the greater part only performs analysis and study using a single failure model. Under this condition, the structural status is described using a limited state function or analyzed using an empirical formula to present each failure condition. With respect to the whole structure, the corrosion status cannot be represented definitely and integratedly. Accordingly, for the sake of evaluating the corrosion damage status in detail, in this paper the existing RC pile corrosion of bridge substructure is to evaluate the structural joint failure probability due to carbonation, chloride ion ingress, sulfate attack, and load-carrying capacity. The results from this study will provide a safe index reference for an existing bridge.

2. Major corrosion failure model for existing RC piles

Existing RC bridge substructures pile usually experience a combination of general corrosion (carbonation), pitting corrosion (chloride ion ingress), and alkali sulfate expansion and sulfuric solutions as shown in Fig. 1.

2.1. General corrosion caused by carbon dioxide

The amount of carbon dioxide dissolved in water or existing in the soil is not great. Nevertheless, if the existing RC pile suffers deterioration with a great number of organic compounds, large amounts of carbon dioxide will be released after the organic compound is resolved. Normally, carbon dioxide in the gas or liquid phase may be immediately reacted with chemical compounds in the porous concrete. This is called carbonation. The pH value of concrete reduces to below 9. It is thus harmful to the existing RC pile. The chemical reaction of concrete carbonation is depicted as follows [23]:

$$\mathrm{CO_2 + Ca(OH)_2 \rightarrow CaCO_3 + H_2O}$$

$$\mathrm{CaCO_3 + H_2O + CO_2 \rightarrow Ca(HCO_3)_2}$$

If the carbonation is very severe, the chemical reaction in concrete is carried out as follows [24]:

$$\mathrm{CO_2 + [Ca(OH)_2 + 3CaO \cdot Al_2O_3 \cdot CaSO_4 \cdot 18H_2O]}$$
$$\mathrm{\rightarrow 3CaO \cdot Al_2O_3 \cdot CaCO_3 \cdot 17H_2O + CaSO_4 \cdot 2H_2O}$$

$$\mathrm{3CO_2 + 3CaO \cdot 2SiO_2 \cdot 3H_2O \rightarrow 3CaCO_3 + 2SiO_2 \cdot 3H_2O}$$

gypsum products, $\mathrm{CaSO_4 \cdot 2H_2O}$, have high resolvability. This increases the permeability of concrete or resolves the

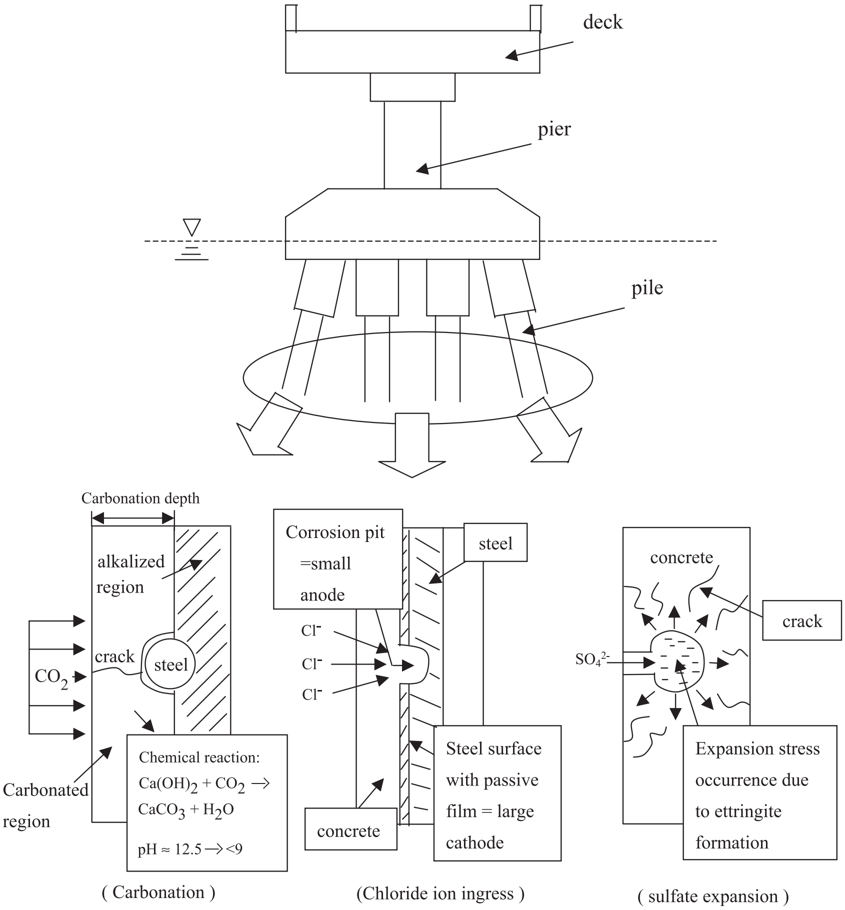


Fig. 1. Major corrosion deterioration models of existing RC pile of bridge substructure.

major gel material C-S-H (calcium silicate hydrate, $3CaO{\cdot}2SiO_2{\cdot}3H_2O$) in concrete. Both deteriorate the concrete properties.

2.2. Pitting corrosion caused by chloride ion ingress

When the chloride ion concentration in the concrete reaches a threshold value in units of 0.4% Cl^-/cement, the chloride ions concentrate on the steel surface and form $Fe(OH)_2$ and $FeCl_2$. Its chemical reaction can be stated as in the following [25]:

$$Fe^{2+} + 2Cl^- + 4H_2O \rightarrow FeCl_2 \cdot 4H_2O$$

$$FeCl_2 \cdot 4H_2O \rightarrow Fe(OH)_2 + 2Cl^- + 2H^+ + 2H_2O$$

In the above chemical reaction, $Fe(OH)_2$ settle on the surrounding of steel anode and release both H^+ and Cl^- backward to anode. The local acidification produces near anode. The Cl^- brings forward more Fe^{2+}. It is seen that the Cl^- is neither formed corrosion products nor consumed during corrosion process. However, the Cl^- provides enough catalysis in the corrosion process. Because of the nonhomogeneous characteristics of concrete itself, the pore solutions in the concrete have a high chloride concentration to break the locally passive film. At this time, the region of the steel under the protection of the passive film forms a cathode while the broken region forms an anode. Consequently, an electrochemical reaction is formed at the local corrosion. The electrochemical mechanism is denoted in Fig. 2.

2.3. Chemical mechanism for existing RC pile corrosion caused by alkali sulfate and sulfuric solution in soils

Concrete subjected to chemical corrosion can be divided into expansion and solvent actions.

2.3.1. Expansion action

The sulfate solution with respect to the corrosion action of concrete is classical expansion action. During the concrete suffered sulfate attack, the formation of ettringite, gypsum, and thaumasite is the principal reason of the occurrence of concrete corrosion damage.

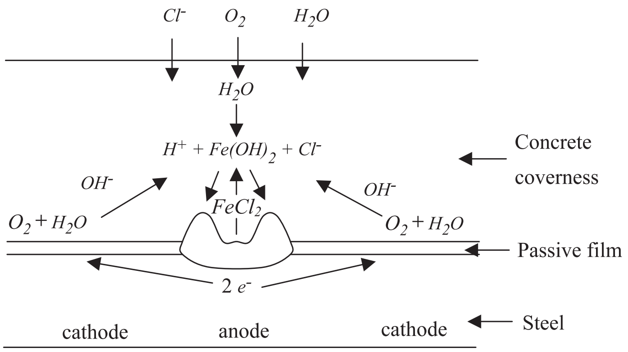
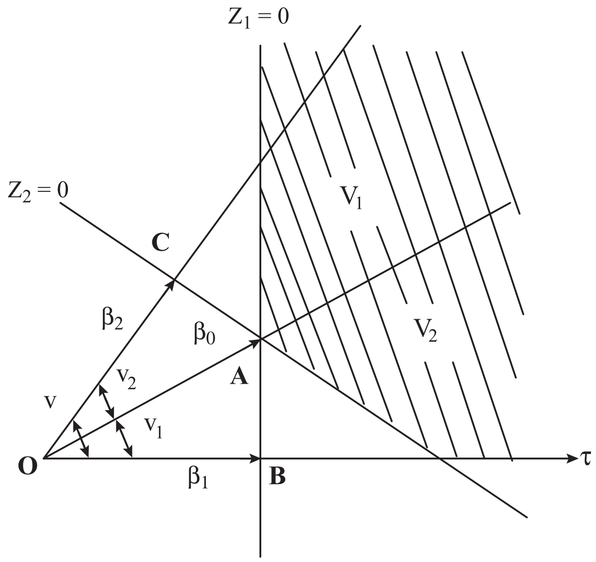
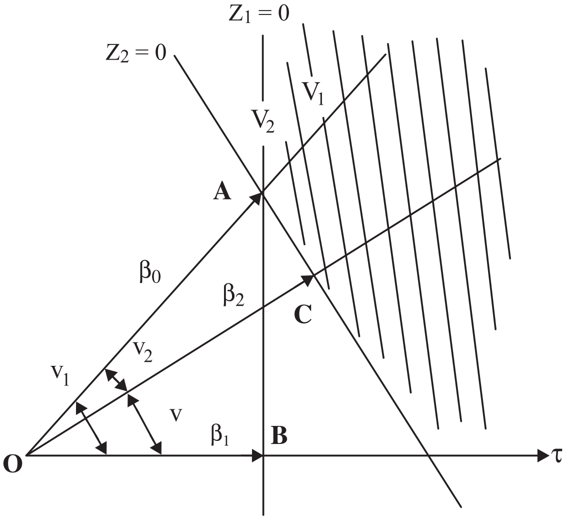


Fig. 2. Schematic diagram of pitting corrosion due to chloride ion ingress.



(a) $\beta_1 / \beta_2 \geq \rho$



(b) $\beta_1 / \beta_2 < \rho$

Fig. 3. Geometric representation of probability calculation for P_{fij}.

When the ettringite is surrounded with calcium carbonate and absorbent (activated) silicon, after a long time, it is transformed into thaumasite ($CaCO_3{\cdot}CaSO_4{\cdot}CaSiO_2{\cdot}15H_2O$) without gelatification. This is another major factor of concrete expansion under sulfate action. At present, the expansion character of concrete is the same as ettringite. The surface of concrete subjected to corrosion failure exhibits the drum bubble phenomena, expansion cracks, and protuberance. The concrete becomes slack soft with decreasing strength.

2.3.2. Solvent action

The major factors involved in the concrete solvent action include the sulfate medium and carbon dioxide, as mentioned above. The concrete corrosive action due to the sulfate medium is a classic solvent action. Its chemical reaction can be expressed as [26]:

$$\begin{cases} \mathrm{Ca(OH)_2 + H_2SO_4 \rightarrow CaSO_4 + 2H_2O} \\ \mathrm{2CaO \cdot SiO_2 + 2H_2SO_4 \rightarrow 2CaSO_4 + Si(OH)_4} \\ \mathrm{CaSO_4 + 2H_2O \rightarrow CaSO_4 \cdot 2H_2O} \end{cases}$$

Sulfate can directly destroy the interior construction of the concrete and forms gypsum with the nongel substance. The concrete fails from the interior layers out to the exterior layers. Because of the strong sulfuric acid action, most of the $\mathrm{Ca(OH)_2}$ is neutralized so that the alkalinity of the interior concrete pores is decreased. This results in the decomposition of both calcium silica hydrate and calcium aluminate hydrate. Therefore, the gel pore structure in the concrete is destroyed and the mechanical performance deteriorates.

3. Theory of structural joint failure probability

Cornell [20] and Kounias [21] used the narrow reliability bound concept to obtain the approximate structural joint failure probability calculation. The equation for the narrow reliability bounds [27,28] is:

$$P_{\mathrm{f}1} + \sum_{i=2}^{k}\left[\max\left\{P_{\mathrm{f}i} - \sum_{j=i}^{i-1} P(E_iE_j)\right\}, 0\right] \le P_{\mathrm{f}} \le \sum_{i=1}^{k} P_{\mathrm{f}i} - \sum_{i=2}^{k} \max_{j<i} P(E_iE_j) \tag{1}$$

where the failure probability $P_{\mathrm{f}i}$ can be calculated from the reliability index β_i and expressed as:

$$P_{\mathrm{f}i} = \Phi(-\beta_i) \tag{2}$$

in which Φ is the standardized normal distribution function.

The term $P(E_iE_j)$ in Eq. (1) represents the joint failure probability which expresses two models that are failing at the same time. This is not convenient owing to the $P(E_iE_j)$ numerical integration calculation. For solving this calculation, Yao et al. [29] suggested an approximate method to calculate the $P(E_iE_j)$ value. For the convenience of expression, let β_1 and β_2 represent the reliability index of two failure models and let ρ_{ij} denote the correlation coefficient of the two failure models.

Assume that $0 \le \beta_1 \le \beta_2$ and $|\rho| < 1$. Fig. 3 is a schematic diagram of the probability calculation of $P(E_iE_j) = P_{ij}$ in the two-dimensional standard normal random vector space. Some parameters in Fig. 3 are:

$$\beta_0 = \sqrt{\frac{\beta_1^2 - 2\rho\beta_1\beta_2 + \beta_2^2}{1-\rho^2}} \tag{3}$$

$$v_i = \arccos(\beta_i/\beta_0) \quad (i = 1, 2)$$

$$v = \arccos(\rho)$$

According to the geometrical relation in Fig. 3, one knows

$$P_{\mathrm{f}ij} = \begin{cases} V_1 + V_2 & \beta_1/\beta_2 \ge \rho \\ \Phi(-\beta_2) + V_1 - V_2 & \beta_1/\beta_2 < \rho \end{cases} \tag{4}$$

where V_i stands for the probability of the region determined by the angle $(\pi/2) - v_i$ between the extent line OA and line $Z_i = 0$.

One can determine this from Fig. 3

$$\left\{\Phi^2(-\beta_0/\sqrt{2}) - \left[\frac{\pi}{4} - \left(\frac{\pi}{2} - v_i\right)\right] V_i\right\} \times \frac{2(\pi/2 - v_i)}{\pi/2} \approx 2V_i \quad v_i \ge \frac{\pi}{4} \tag{5}$$

Table 1
Concrete carbonation depth (mm) of pile at different service times

Tested points	Service time (years)											
	5	10	15	20	25	30	35	40	45	50	55	60
1	24.5	34.5	41.6	47.9	53.7	58.8	64.2	67.8	71.9	77.3	80.4	83.4
2	23.9	32.2	41.2	46.8	52.7	57.3	62.9	67.4	71.0	74.1	79.3	82.6
3	22.8	31.9	40.9	45.2	52.1	56.2	60.8	66.5	69.9	73.4	78.2	80.1
4	22.7	30.8	38.2	45.1	51.4	54.3	59.4	65.2	69.3	72.3	77.0	78.7
5	21.9	29.7	37.3	44.3	50.5	53.5	57.3	63.1	68.4	71.5	73.2	77.1
6	21.8	29.0	37.0	42.4	49.2	53.2	56.9	62.1	65.2	69.7	72.5	75.5
7	21.2	28.8	35.9	40.8	48.3	50.4	55.6	60.2	63.3	68.2	70.6	74.3
8	20.7	28.7	35.2	40.7	47.7	50.2	54.7	58.1	61.9	66.0	69.1	73.2
9	19.9	27.9	35.1	40.2	45.9	50.1	53.8	56.2	61.7	64.2	68.7	70.8
10	19.6	27.3	34.4	39.3	45.6	49.4	52.5	55.4	60.8	64.1	64.9	69.9
11	19.5	26.8	33.9	39.2	43.3	47.6	51.9	53.9	59.1	62.7	63.8	65.9
12	19.3	26.7	32.7	37.3	41.0	46.3	48.2	52.7	58.4	60.4	62.1	64.3
13	18.6	25.5	31.5	36.9	40.4	44.4	47.7	51.3	53.2	59.0	61.5	64.2
14	18.4	25.4	30.0	35.3	39.6	43.8	45.1	48.2	52.6	54.2	61.2	61.3
15	17.6	24.9	27.3	34.1	38.8	42.1	44.9	48.0	51.7	53.3	58.4	60.7

Table 2
Chloride ion ingress depth (mm) of pile at different service times

Tested points	Service time (years)											
	5	10	15	20	25	30	35	40	45	50	55	60
1	19.9	28.9	39.4	41.9	48.2	51.0	56.2	58.8	63.1	67.2	68.9	72.3
2	19.2	28.6	35.3	41.2	44.9	50.3	53.2	57.9	60.7	63.4	68.1	70.1
3	18.7	27.4	33.9	39.6	44.7	49.8	53.1	56.3	60.1	62.7	67.2	68.5
4	18.6	27.0	33.5	38.9	44.0	48.4	52.3	55.8	57.7	61.0	65.4	68.1
5	18.6	26.9	32.8	38.8	39.6	48.1	51.8	54.3	56.7	60.4	64.8	67.3
6	17.2	26.5	31.9	37.2	39.6	46.6	50.4	53.9	56.7	58.1	63.9	65.4
7	17.1	25.8	30.7	36.1	39.4	45.9	49.5	52.5	54.4	57.8	63.2	63.9
8	16.9	25.7	30.6	35.4	39.3	43.5	47.9	50.2	53.9	57.4	62.9	63.1
9	16.8	25.4	29.7	34.9	38.8	43.2	47.2	50.1	53.8	57.1	59.2	62.7
10	16.6	24.8	29.1	34.7	38.7	42.7	45.7	49.7	53.7	56.8	58.2	61.6
11	16.3	24.3	29.1	33.9	37.4	41.9	44.0	48.6	52.5	55.5	56.3	58.3
12	15.8	23.8	28.2	32.5	37.3	41.0	43.8	48.1	51.6	52.8	54.3	58.0
13	15.4	23.5	27.3	32.1	37.0	40.3	42.9	46.9	50.3	52.7	54.2	57.5
14	15.0	23.3	27.0	31.4	36.9	39.4	42.0	44.4	48.8	52.4	53.9	56.2
15	14.7	22.9	27.0	30.3	36.8	38.1	40.6	43.9	48.4	50.2	53.2	54.7

Eq. (4) can be rewritten as

$$V_i \approx \begin{cases} \Phi(-\beta_i)\Phi\left(-\sqrt{\beta_0^2-\beta_i^2}\right) - \Phi^2(-\beta_0/\sqrt{2})\dfrac{4v_i}{(2+v_i)\pi-4v_i^2} & v_i < \dfrac{\pi}{4} \\ \Phi^2(-\beta_0/\sqrt{2})\dfrac{4u_i}{(2+u_i)\pi-4u_i^2} & v_i \geq \dfrac{\pi}{4} \end{cases} \tag{6}$$

where $u_i = (\pi/2) - v_i$.

Employing Eqs. (3) and (5), one can find the joint failure probability, $P_{\mathrm{f}ij}$. If substituting both the single failure probability, $P_{\mathrm{f}i}$ and the joint failure probability, $P_{\mathrm{f}ij}$ into Eq. (1), then one can seek the upper and lower bound of the structural joint failure probability.

4. Testing techniques

In this article, the method for structural joint failure probability mentioned above will be used to evaluate the substructural RC pile of the Chung-shan bridge, in Taipei, Taiwan. This bridge was completely constructed in 1933. The bridge site is located on the connected position between Yuan-shen third section and Jiann-tarn-shen and is stridden over the waterway downstream of the Keelung river. Many well-known parameters of the existing substructural RC pile should be described as follows: pile diameter $D = 60$ cm, pile length $L = 30$ m, concrete cover thickness $c = 7.5$ cm, concrete compressive strength $f_c' = 245$ kgf/cm^2, water–cement ratio W/C = 0.40, concrete creep coefficient $\varphi_{\mathrm{cr}} = 2.0$, Poisson's ratio of concrete $v_c = 0.18$, steel diameter $\phi = 19$ mm, yielding strength of steel $f_y = 4200$ kgf/cm^2, allowable tensile stress of steel $f_s = 1680$ kgf/cm^2.

Test cores were extracted every 5 years from the existing RC pile of the Chung-shan bridge substructure in Taipei, Taiwan. For each core, numerous cores of 55 mm diameter with 110 mm height were taken and tested for carbonation depth and chloride ion ingress depth. The other cores of 76.2 mm (3 in.) diameter with 152.4 mm (6 in.) height were taken and tested for sulfate expansion. The cores were cut using a rotary steel cutting machine with diamond bits.

4.1. Carbonation depth measurement

The carbonation depth was assessed by splitting the concrete cores and spraying the split specimens uniformly with phenolphthalein pH indicator [a solution of 1% phenolphthalein ($C_{20}H_{14}O_4$) in 70% ethyl alcohol (C_2H_5OH) [30,31,32]]. The carbonation depth was measured with steel ruler to the nearest 1 mm. Several measurements were taken on given specimens as indicated in Table 1.

4.2. Chloride ion ingress depth measurement

In accurately measuring the chloride ion ingress depth into existing RC structures, a simple and practical problem

Table 3
Correlation coefficient between carbonation depth and chloride ion ingress depth to pile at different service times

Service time (years)	5	10	15	20	25	30	35	40	45	50	55	60
ρ	.91	.92	.86	.93	.83	.92	.92	.95	.92	.91	.93	.94

$$^*:\ \rho(x,y) = \frac{\dfrac{1}{n}\displaystyle\sum_{i=1}^{n}[X_i-\bar{X}][Y_i-\bar{Y}]}{\sqrt{\dfrac{1}{n}\displaystyle\sum_{i=1}^{n}[X_i-\bar{X}]^2}\sqrt{\dfrac{1}{n}\displaystyle\sum_{i=1}^{n}[Y_i-\bar{Y}]^2}}$$

Table 4
Reliability index (β) and failure probability (P_{fi}) of pile subjected to individual carbonation and chloride ion ingress

Service time (years)	Carbonation		Chloride ion ingress	
	β	P_{fi}	β	P_{fi}
5	7.39	7.35×10^{-14}	7.12	5.4×10^{-13}
10	6.12	4.68×10^{-10}	5.71	5.65×10^{-9}
15	4.80	7.94×10^{-7}	4.92	4.33×10^{-7}
20	4.11	1.98×10^{-5}	4.25	1.07×10^{-5}
25	3.24	5.98×10^{-4}	3.69	1.12×10^{-4}
30	2.77	.0028	3.13	8.74×10^{-4}
35	2.16	.01539	2.74	.00307
40	1.66	.04847	2.36	.00914
45	1.24	.10752	2.32	.0102
50	0.84	.20051	1.67	.0475
55	0.52	.30161	1.35	.0885
60	0.24	.4053	1.11	.1335

Remark: concrete coverness obeys normal distribution $N(75,7^2)$.

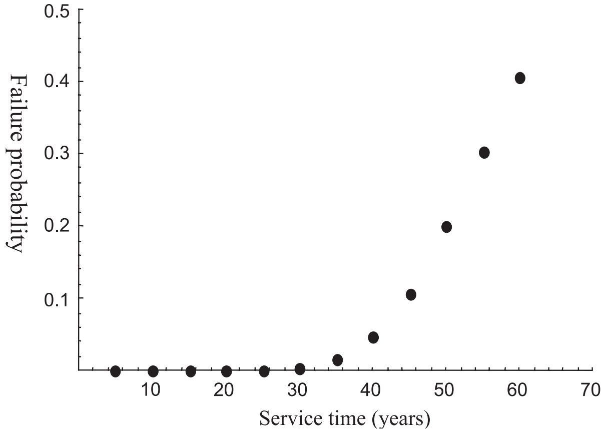


Fig. 4. Trend analysis of structural joint failure probability of pile subjected to both carbonation and chloride ion ingress at different service times.

in engineering should be considered. Collepardi [33] and Hong [34] suggested using the colourimetric method to quickly and directly measure the chloride ion ingress depth from a cylindrical concrete specimen cored from a field RC structure. The cored cylindrical concrete specimen was split into two half cylindrical column, along the direction of chloride ions diffusion. Half of the cylindrical column is placed into a baking oven at 105 °C for over 30 min until the split surface was dried. After taking the half cylindrical column from the baking oven, immediately and repeatedly (3–5 times) spread a fluorescent yellow solution with 1 g/l concentration on the surface of the half cylindrical column until the surface of the half cylindrical column turns brown in color. $AgNO_3$ solution with 0.1 M/l concentration was uniformly spread onto the surface. At this time, in the region of Cl^-, the $AgNO_3$ will react with Cl^- to product AgCl. The surface of the half cylindrical column will show a brown color. However, in the area without Cl^-, under light, the Ag^+ will be dissolved from $AgNO_3$ and the surface of the half cylindrical column will appear black in color. The chloride ion ingress depth can thus be measured from the concrete surface penetrated by chloride ion to the intersection of the brown and black colors.

Table 5
Failure probability (P_f) of pile under the combination of failure model of carbonation and chloride ion ingress

Service time (years)	Failure probability P_f
5	5.94×10^{-13}
10	5.86×10^{-9}
15	1.13×10^{-6}
20	2.48×10^{-5}
25	6.53×10^{-4}
30	.002991
35	.015623
40	.048531
45	.10752
50	.20116
55	.30196
60	.40538

4.3. Sulfate expansion measurement

The sample standard determined by the U.S. Bureau of Reclamation (USBR) [35] were 76.2 × 152.4 mm (3 × 6 in) cylinders. The samples were cured for 14 days in a fog room, followed by 14 days at 50% relative humidity for a total of 28 days. Following the curing, the samples were immersed in a 0.15 M sodium sulfate (Na_2SO_4) solution. For the first 5 years of the experiment, the samples were removed from the solution every 4 weeks, and their expansion was measured. After 5 years of exposure, the frequency of measurement was gradually reduced to once every year, which was continued until the sample failed or until the experiment was finished. The USBR used the criterion of 0.5% expansion to define failure.

5. Illustrative example

Consider the Chung-shan bridge with RC pile over 60 years of service life. Core specimens were taken onsite every 5 years. The carbonation depth and chloride ion ingress depth were measured as shown in Tables 1 and 2. The coefficient of correlation between the carbonation depth and chloride ion ingress depth is listed in Table 3. The reliability index, β_i, and failure probability, P_{fi}, of concrete

Table 6
Degradation model [after Mori and Ellingwood, 1993]

Shape of the degradation function $g(t)$	Degradation rate $g(40)$	Corresponding degradation mechanism
Linear: $g(t)=1-at$	0.7, 0.8, 0.9	Corrosion
Parabolic: $g(t)=1-at^2$	0.8, 0.9	Sulfate attack
Square root: $g(t)=1-a\sqrt{t}$	0.8, 0.9	Diffusion-controlled degradation

Table 7
Reliability index (β) and failure probability (P_{fi}) of pile subjected to individual sulfate and load-carrying capacity at different service times

Service time (years)	Sulfate		Load-carrying capacity	
	β	P_{fi}	β	P_{fi}
5	7.89	1.51×10^{-15}	2.88	.0019674
10	6.26	1.92×10^{-10}	2.75	.0029355
15	5.75	4.46×10^{-9}	2.66	.0038347
20	4.02	2.91×10^{-5}	2.57	.0050224
25	3.68	1.16×10^{-4}	2.47	.0066418
30	2.42	.00776025	2.43	.0075868
35	1.72	.0427162	2.33	.0099187
40	1.11	.1335	2.28	.0113511
45	0.82	.206108	2.23	.0129968
50	0.61	.270931	2.17	.0147985
55	0.31	.37828	2.12	.0169621
60	0.15	.440382	2.07	.019448

carbonation were predicted using linearly unbiased estimation (the reader can refer to Liang et al. [36] for the linearly unbiased estimation theory) combined with normal distribution. The reliability index and failure probability of the chloride ion ingress depth to concrete were predicted using the Poisson distribution in conjunction with Fick's second law [37,38]. That is:

$$P(X_t = x) = \frac{(\lambda\sqrt{t})^x}{x!} e^{-\lambda\sqrt{t}} \qquad (7)$$

where P is the failure probability, X_t is the chloride ion ingress depth, x is a prescribed value, t is the time, λ is the average amount of chloride ion ingress depth per unit time, that is, the corrosion occurrence rate, $t=t_1+t_2+\cdots+t_n$, $\lambda=(\lambda_1+\lambda_2+\cdots+\lambda_n)/n$, in which n stands for n-term data measured at different times, $t_1,+t_2,\cdot\cdot,+t_n$, the value of λ_n corresponds to the value at t_n.

The complete results are listed in Table 4. Let the service time be $t=60$ years as an illustrative example. Substituting $\beta_1=\beta_{CO_2}=0.24$, $\beta_2=\beta_{Cl^-}=1.11$ and $\rho=.94$ into Eqs. (3) and (6), one obtains

$$\beta_0 = \sqrt{\frac{\beta_1^2 - 2\rho\beta_1\beta_2 + \beta_2^2}{1-\rho^2}} = 2.603331$$

$$v_1 = \arccos(\beta_1/\beta_0) = 1.47847$$

$$v_2 = \arccos(\beta_2/\beta_0) = 1.13031$$

$$u_1 = \frac{\pi}{2} - v_1 = 0.09233$$

$$u_2 = \frac{\pi}{2} - v_2 = 0.44049$$

$$V_1 = \Phi^2(-\beta_0/\sqrt{2})\frac{4u_1}{(2+u_1)\pi - 4u_1^2} = 6.08458 \times 10^{-5}$$

$$V_2 = \Phi^2(-\beta_0/\sqrt{2})\frac{4u_2}{(2+u_2)\pi - 4u_2^2} = 2.7549 \times 10^{-4}$$

As $\beta_1/\beta_2=0.216216<\rho=.94$, one has from Eq. (4)

$$P_{f12} = \Phi(-\beta_2) + V_1 - V_2 = .133285$$

Substituting these values for $P_{f1}=\Phi(-\beta_1)=.405165$, $P_{f2}=\Phi(-\beta_2)=.1335$ (Table 4) and $P_{f2}=.133285$ into Eq. (1), one obtains the failure probability $P_f=.40581$. Likewise, the failure probability obtained from the carbonation with chloride ion ingress combination under different

Table 8
Structural joint failure probability (P_{fij}) of pile subjected to the combination of carbonation, chloride ion ingress, sulfate, and load-carrying capacity

Joint failure probability P_{fij}	Service time t (years)					
	$t=5$	$t=10$	$t=15$	$t=20$	$t=25$	$t=30$
P_{f21}	5.39648×10^{-13}	5.64881×10^{-9}	7.93327×10^{-7}	2.90047×10^{-5}	5.52796×10^{-4}	.004129
P_{f31}	7.34353×10^{-14}	4.67877×10^{-10}	4.32721×10^{-7}	1.97459×10^{-5}	1.15304×10^{-4}	.004790
P_{f32}	1.58008×10^{-14}	2.05965×10^{-10}	2.29791×10^{-7}	1.0619×10^{-5}	3.20058×10^{-4}	.004741
P_{f41}	1.50591×10^{-15}	1.92489×10^{-10}	4.46217×10^{-9}	1.06805×10^{-5}	1.10927×10^{-4}	.002089
P_{f42}	8.70352×10^{-16}	1.05566×10^{-10}	1.58037×10^{-7}	1.26048×10^{-5}	3.13823×10^{-4}	.002091
P_{f43}	5.15549×10^{-16}	4.20782×10^{-11}	1.23936×10^{-7}	7.44925×10^{-6}	4.3166×10^{-5}	.001709

Joint failure probability P_{fij}	Service time t (years)					
	$t=35$	$t=40$	$t=45$	$t=50$	$t=55$	$t=60$
P_{f21}	.227365	.0448976	.0974535	.173039	.264154	.343627
P_{f31}	.009056	.011249	.012862	.047194	.088085	.132665
P_{f32}	.010548	.010399	.012700	.046485	.087126	.132125
P_{f41}	.003021	.009109	.010165	.014999	.017003	.019226
P_{f42}	.005897	.008583	.010083	.014981	.017000	.019226
P_{f43}	.005478	.006808	.007948	.022506	.016352	.018956

Table 9
Lower and upper bounds of structural joint failure probability for a pile at different service times

Failure probability	Bound	Service time t (years)					
		$t=5$	$t=10$	$t=15$	$t=20$	$t=25$	$t=30$
P_f	Lower bound	.001988	.00298	.003907	.005083	.006396	.007760
	Upper bound	.001988	.00298	.003907	.005085	.006756	.007977
	Bound	$t=35$	$t=40$	$t=45$	$t=50$	$t=55$	$t=60$
P_f	Lower bound	.031917	.1335	.206108	.270931	.37828	.440382
	Upper bound	.042716	.13799	.216159	.291109	.416081	.502755

Remark: $P_{f1} + \sum_{i=2}^{k}\left[\max\left\{P_{fi} - \sum_{j=i}^{i-1} P(E_iE_j)\right\}, 0\right] \le P_f \le \sum_{i=1}^{k} P_{fi} - \sum_{i=2}^{k} \max_{j<1} P(E_iE_j)$

service times for existing RC pile are calculated and listed in Table 5. Moreover, the increasing failure probability trend is denoted in Fig. 4.

The expansion failure resulting from sulfate attack and the load-carrying capacity failure are the major factors. Accordingly, with relation to the same existing RC pile, excluding both the carbonation and chloride ion ingress, this study also considered the expansion due to sulfate attack and the failure due to the load-carrying capacity. Corr et al. [39] suggested an empirical formula with 0.5% expansion E_0 for computing the single failure probability. The limit state function is

$$g(\theta,\gamma) = E_0 - e^{\gamma_1+\gamma_3 \mathrm{WC}\sqrt{t}}(\mathrm{C_3A}+1)^{\gamma_2} \qquad (8)$$

where $\gamma=\langle\gamma_1,\gamma_2,\gamma_3\rangle$ is a vector for the normal random variables, $\theta=\langle E_0,T,\mathrm{WC},\mathrm{C_3A}\rangle$ is a vector for the deterministic material parameters, WC (W/C) is the water–cement ratio, C_3A is the percentage of C_3A content, and t is the exposure time (year). Mori and Ellingwood [40] and Stewart and Val [41] pointed out that the strength of the component deteriorates with service time due to environmental stressors according to

$$r(t) = R_0 \times g(t) \qquad (9)$$

where R_0 is the initial resistance of the component and $g(t)$ is the resistance degradation function. Their concrete values are represented in Table 6. In this study, the experimental results under the $E_0=0.5\%$ status, the concrete consisted of $C_3A=4\%$ [42] and W/C (WC) = 0.4 was adopted to estimate the failure probability. In the case of load-carrying capacity, assume that the strength after reducing $g(t)=1-at^2$ [40] still obeys the logarithm normal probability distribution while the impressed exterior load obeys the I-type extreme distribution. Based on the suggested method stated above, both the reliability index and failure probability for an existing RC pile subjected to both sulfate attack and load-carrying capacity at different service times $t=5$–60 years are listed in Table 7. To obtain the computed results with conservation, the coefficients of correlation among these four different failure models assumed ρ=.9. Table 8 shows the structural joint failure probability for a pile subjected to a combination of carbonation, chloride ion ingress, sulfate, and load-carrying capacity. Table 9 indicates the lower and upper bound for the structural joint failure probability of a pile at different service times. To understand the failure status for the evaluated pile, Fig. 5 denotes the trend analysis for the lower and upper bounds for the structural joint failure probability of an existing RC pile.

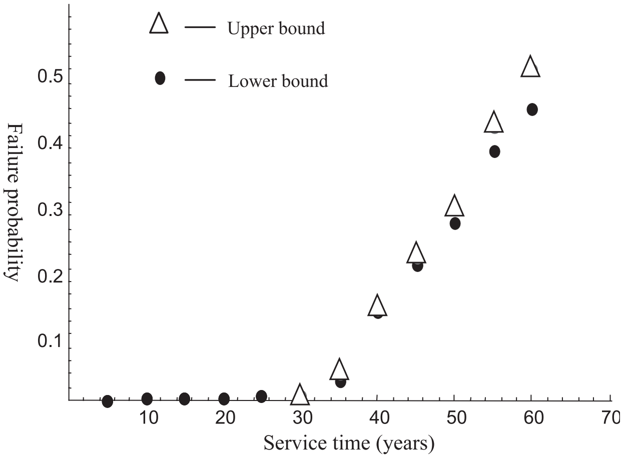


Fig. 5. Trend analysis of lower and upper bounds for the structural joint failure probability of pile subjected to among carbonation, chloride ion ingress, sulfate, and load-carrying capacity at different service times.

6. Discussion

The aim of this study was to predict the service status of an existing RC pile subjected to multiple failure models. The combination of failure status for each single failure model is presented as the structural joint failure probability. It was discovered from the preciously theoretical derivation that the structural joint failure probability was not obtained by the summation or product of each single failure probability. During the computation, possible repeat failure must be removed. Therefore, in the whole computation process, the main point is how to compute the joint failure probability, $P_{\mathrm{f}ij}$, between two failure models.

In actually seeking the $P_{\mathrm{f}ij}$ values, the traditional numerical integral method is adopted. However, the deficiency is not only that the calculation is too complicated but also that the application is not very convenient. The approximate methods for finding $P_{\mathrm{f}ij}$ are suggested by Melchers [43], Ditlevsen [27], and Ang and Tang [28]. An approximate computation formula is accepted and applied widely by many researchers and is expressed as

$$\max[P(A),P(B)]\le P_{\mathrm{f}ij}\le P(A)+P(B) \qquad \rho_{ij}\ge 0$$

$$0\le P_{\mathrm{f}ij}\le \min[P(A),P(B)] \quad \rho_{ij}\le 0 \tag{10}$$

where $P(A)=\Phi(-\beta_i)\Phi\left(-\dfrac{\beta_j-\rho_{ij}\beta_i}{\sqrt{1-\rho_{ij}^2}}\right)$

$$P(B)=\Phi(-\beta_j)\Phi\left(-\frac{\beta_i-\rho_{ij}\beta_j}{\sqrt{1-\rho_{ij}^2}}\right)$$

Therefore, the approximate computation formula was adopted to predict the $P_{\mathrm{f}ij}$ values for the existing RC pile of Chung-shan bridge substructure mentioned above. The predicted results were compared with results obtained from the proposed method and are listed in Table 10. If the results from Table 10 are substituted into Eq. (1) to compute the failure probability, P_f, the interval of the lower and upper bounds for the $P_{\mathrm{f}ij}$ values is increased, corresponding to the internal lower and upper bounds for the P_f values which also increase when the single failure probability of two failure models are larger and approach one another. Some errors may occur in evaluating the structural service status. If the proposed method is adopted, the errors can be avoided. The proposed method is better than the approximate computation technique.

When calculating the structural joint failure probability, $P_{\mathrm{f}ij}$, it was found that besides reliability index and single failure probability, the coefficient of correlation among each different failure model plays an important role. In this study, ρ=.9 was conservatively adopted. Under practical conditions, both the interior material factor and the exterior environment factor were changed followed with different service times.

7. Conclusions

This study investigated the corrosion of existing RC bridge substructure piles. Existing RC piles are usually subjected to carbonation, chloride ion ingress, and sulfate

Table 10

Compared values for the structural joint failure probability of a pile at different service times

Joint failure probability	Bound	Service time t (years)					
		$t=5$	$t=10$	$t=15$	$t=20$	$t=25$	$t=30$
$P_{\mathrm{f}21}$	Lower bound	5.4×10^{-13}	5.65×10^{-9}	7.93×10^{-7}	2.89×10^{-5}	5.06×10^{-4}	2.24×10^{-3}
	Upper bound	5.4×10^{-13}	5.65×10^{-9}	7.93×10^{-7}	2.91×10^{-5}	5.72×10^{-4}	4.42×10^{-3}
$P_{\mathrm{f}31}$	Lower bound	7.34×10^{-14}	4.68×10^{-10}	4.33×10^{-7}	1.97×10^{-5}	1.14×10^{-4}	1.59×10^{-3}
	Upper bound	7.34×10^{-14}	4.68×10^{-10}	4.33×10^{-7}	1.98×10^{-5}	1.16×10^{-4}	2.26×10^{-3}
$P_{\mathrm{f}32}$	Lower bound	1.04×10^{-14}	1.5×10^{-10}	8.51×10^{-8}	4.56×10^{-6}	6.6×10^{-5}	1.56×10^{-3}
	Upper bound	1.69×10^{-14}	2.19×10^{-10}	1.52×10^{-7}	8.33×10^{-6}	8.98×10^{-5}	2.25×10^{-3}
$P_{\mathrm{f}41}$	Lower bound	1.51×10^{-15}	1.92×10^{-10}	4.46×10^{-9}	1.07×10^{-5}	1.09×10^{-4}	7.16×10^{-4}
	Upper bound	1.51×10^{-15}	1.92×10^{-10}	4.46×10^{-9}	1.07×10^{-5}	1.12×10^{-4}	8.28×10^{-4}
$P_{\mathrm{f}42}$	Lower bound	7.27×10^{-16}	8.29×10^{-11}	3.59×10^{-9}	3.50×10^{-6}	6.43×10^{-5}	7.1×10^{-4}
	Upper bound	9.09×10^{-16}	1.12×10^{-10}	4.00×10^{-9}	5.64×10^{-6}	8.7×10^{-5}	8.25×10^{-4}
$P_{\mathrm{f}43}$	Lower bound	3.82×10^{-16}	2.55×10^{-11}	3.22×10^{-9}	2.74×10^{-6}	2.3×10^{-5}	4.75×10^{-4}
	Upper bound	5.46×10^{-16}	4.53×10^{-11}	3.74×10^{-9}	4.78×10^{-6}	4.55×10^{-5}	6.76×10^{-4}

Joint failure probability	Bound	Service time t (years)					
		$t=35$	$t=40$	$t=45$	$t=50$	$t=55$	$t=60$
$P_{\mathrm{f}21}$	Lower bound	.01071	.03929	.08077	.12652	.19343	.22696
	Upper bound	.01414	.04793	.10648	.19485	.3032	.40524
$P_{\mathrm{f}31}$	Lower bound	.00799	.01113	.01283	.04650	.08683	.13007
	Upper bound	.00954	.01135	.01290	.04787	.08948	.13564
$P_{\mathrm{f}32}$	Lower bound	.00438	.00922	.01237	.04442	.08360	.12805
	Upper bound	.00727	.01095	.01294	.04803	.09009	.13621
$P_{\mathrm{f}41}$	Lower bound	2.94×10^{-3}	9.05×10^{-3}	.01015	.01499	.01700	.01923
	Upper bound	3.07×10^{-3}	9.17×10^{-3}	.01018	.01502	.01701	.01923
$P_{\mathrm{f}42}$	Lower bound	2.33×10^{-3}	7.83×10^{-3}	9.91×10^{-3}	.01492	.01699	.01922
	Upper bound	2.85×10^{-3}	8.96×10^{-3}	.01022	.01504	.01701	.01923
$P_{\mathrm{f}43}$	Lower bound	1.91×10^{-3}	3.29×10^{-3}	3.79×10^{-3}	.01113	.01530	.01842
	Upper bound	2.61×10^{-3}	6.00×10^{-3}	6.83×10^{-3}	.01412	.01697	.01935

attack. These corrosion mechanisms on the existing RC pile were described. The structural joint failure probability theory was also stated. An existing bridge with RC pile within 60 years service time was illustrated for their structural joint failure probability. The failure probability of existing bridge RC piles suffering carbonation, chloride ion ingress, and sulfate attack have values of .4053, .1335, and .440382 at a service time of 60 years. The failure probability for the existing bridge RC pile under load-carrying capacity and a combination of carbonation with chloride ion ingress has values of .019448 and .40538 at a service time of 60 years. The structural joint failure probability for an the existing bridge RC pile due to the combination of carbonation, chloride ion ingress, sulfate attack, and load-carrying capacity has lower and upper bound values of .44038 and .502755 at a service time of 60 years. The proposed method can reasonably and reliably predict the structural joint failure probability for existing bridge RC piles due to corrosion and load-carrying capacity. The proposed method can be extended to caissons subjected to corrosion attack and load-carrying capacity.

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