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From mini-cone test to Abrams cone test: measurement of cement-based materials yield stress using slump tests

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Abstract

Stoppage tests in civil engineering consists in measuring the shape of a fresh material deposit after flow occurred. This measured geometrical value (slump, spread) is linked to the plastic yield value of these yield stress materials. The most famous example is the Abrams cone for concrete. In this work, the flow induced by a smaller cone test for cement pastes and grouts is studied. In a first part, the spread is theoretically linked to the plastic yield value. Experimental results on several cement pastes validate the obtained relation but also shows the necessity to take in account the surface tension effects for low yield stress materials. The modified relation allows the prediction of the plastic yield value from the measured spread. The proposed method is then applied to the Abrams cone and fresh concrete. It is demonstrated that it is only suitable for high slumps (>20 cm).

Keywords: Fresh concrete; Rheology; Cement paste; Slump test

1. Introduction and general rheological behaviour

It is now accepted that fresh behaviour of cementitious suspensions, such as concrete, mortar or cement pastes and grouts, may be approximated using a Bingham model (or even better an Herschell–Bulkley model). If the fresh material is assimilated to a Bingham fluid, two independent parameters are needed to describe the rheological behaviour: the plastic yield value: τ_0 and the plastic viscosity: μ_p. The behaviour law then writes in one dimension:

$$\tau = \mu_p \dot{\gamma} + \tau_0 \quad (1)$$

where τ is the shear stress and $\dot{\gamma}$ is the rate of strain.

If the fresh material is assimilated to a Herschell–Bulkley fluid, three independent parameters are then needed to describe the rheological behaviour.

$$\tau = \mu_p \dot{\gamma}^n + \tau_0 \quad (2)$$

Steady state simple shear data for various cement pastes, mortars or even concretes have often been well represented using these simple models [1–3]. Their general form is:

$$\dot{\gamma} = 0 \rightarrow \tau < \tau_0 \quad (3a)$$

$$\dot{\gamma} \neq 0 \rightarrow \tau = \tau_0 + f(\dot{\gamma}) \quad (3b)$$

f is a positive continuous increasing shear rate function with $f(0)=0$. It should be noted here that, despite their effectiveness in numerous cases, these models do not take in account the thixotropic aspect of many cementitious materials [4,5]. It should also be noted that the above models do not account for flow situations which are more complex than simple shear flow. Whatever model is chosen, the plastic yield value has to be measured. Traditional rheometry tests (Couette Viscometer for cement paste) or specific ones (BTRHEOM for concrete [6]) may be used but these tests are often expensive, time consuming and give often too many information when only the plastic yield value is needed. In situ, simpler and cheaper tests are preferred. One interesting category of in situ tests are what could be called the “stoppage tests”. They are based on the fact that if the shear stress in the tested sample becomes smaller than the plastic yield value

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(the plasticity criterion is not fulfilled any more), the flow stops. The shape at stoppage (spread, slump or height) is directly linked to the plastic yield value, which may even be calculated. The relation between plastic yield value and slump for the Abrams cone has been studied by Hu [7] or Christensen [8]. The present study deals with another "stoppage test" very suitable for cement pastes and grouts yield stress measurement, the mini-cone test.

2. Theoretical analysis of deposit form

Most flow models mainly concern confined flow. In the case of a cone test, a free surface exists and its shape is to be determined. The method developed here is similar to Coussot's work in Cartesian coordinates [9]. A finite volume of fluid is released on an horizontal plane. The initial shape of the fluid is assumed to be conical. The axial symmetry is assumed (no angular velocity). The cylindrical frame of reference (O, r, θ, z) is shown on Fig. 1. The motion is described within the frame of the long-wave approximation: the characteristic length of the contact surface in the horizontal plane is much larger than the characteristic length of the fluid depth. It could be noted that this assumption is not licit in the case of an Abrams cone test with a low slump value. We will come back on this point in Section 5. The radial velocity component are correspondingly much larger than the axial velocity component. The fluid depth is noted $h(r)$. In cylindrical coordinates, in the case of axial symmetry, when inertia is neglected and within the frame of the long wave approximation, the equilibrium equations simplifies to:

$$\frac{\partial p}{\partial r} = \frac{\partial \tau}{\partial z} \tag{4a}$$

$$\frac{\partial p}{\partial z} = -\rho g \tag{4b}$$

with

$$\tau = \tau_0 + f(\dot{\gamma}) \tag{5}$$

ρg is the studied fluid volumic weight. Within the frame of the long-wave approximation, the strain rate simplifies to:

$$\dot{\gamma} = \frac{\partial V_r}{\partial z} \tag{6}$$

p is the pressure. Its distribution is assumed to be hydrostatic, as usually found for various types of free surface flow, even with yield stress fluids. Equations are licit for viscous fluids. We assume here that they can be extended to yield stress fluids. Integration of Eq. (4b) gives:

$$p = \rho g(h(r) - z) \tag{7}$$

The pressure for z=$h(r)$ equals the atmospheric pressure which is taken as the reference pressure. In the first part of this work, surface tension effects are assumed to be small

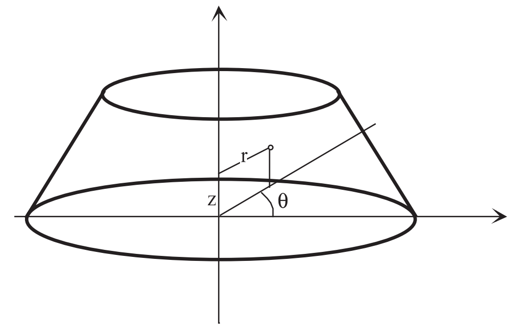

Fig. 1. Initial cone shape and cylindrical coordinates.

compared to the yield stress consequences. Eqs. (4a) Eqs. (4b) may then be integrated over the fluid depth between z=0 and z=h. It writes:

$$\rho g h(r)\frac{\mathrm{d}h(r)}{\mathrm{d}r} = -\tau_0 - f(\dot{\gamma})|_{z=0} \tag{8}$$

At stoppage, the Eq. (8) simplifies to:

$$\rho g h(r)\frac{\mathrm{d}h(r)}{\mathrm{d}r} = \tau_0 \tag{9}$$

as the strain rate tends towards zero.

$h(r)$ may then be integrated using the boundary condition $h(r$=$R)$=0. The cone shape at stoppage is not conical any more but is described by:

$$h(r) = \left(\frac{2\tau_0(R-r)}{\rho g}\right)^{\frac{1}{2}} \tag{10}$$

The tested sample volume V is equal to:

$$V = \int_0^{2\pi}\int_0^R h(r)r\mathrm{d}r\mathrm{d}\theta = \frac{8\sqrt{2}\pi\tau_0{\frac{1}{2}}R^{\frac{5}{2}}}{15(\rho g)^{\frac{1}{2}}} \tag{11}$$

Using relation (11), the measured spread flow may then be predicted in terms of the tested sample volume, the plastic yield value and the volumic weight and for a given spread, the associated yield stress is given by:

$$\tau_0 = \frac{225\rho g V^2}{128\pi^2 R^5} \tag{12}$$

3. Experimental results

Eleven different cement pastes with W/C around 0.56 and W/(C+F)=0.4 are studied [10]. Their components are:

– C: Cement: CEM I 52.5 N CE
– F: Filler: Betocarb P2 MEAC
– Superplasticizer: Cimfluid Adagio 2019
– Viscosity agent: Collaxim L4

Table 1
The 11 tested cement paste compositions

Cement pastes no.	Cement (kg m^{-3})	Filler (kg m^{-3})	Water (kg m^{-3})	Superplasticizer dry (kg m^{-3})	Viscosity agent dry (kg m^{-3})
0	984	365	473	3.44	1.41
1	985	366	473	2.70	1.41
2	986	366	473	2.03	1.41
3	988	367	473	1.36	1.41
4	989	367	473	0.68	1.42
5	985	366	473	0.68	2.82
6	983	365	473	2.03	2.81
7	981	364	473	3.43	2.80
8	979	364	473	0.68	5.59
9	977	363	473	2.01	5.58
10	974	362	473	3.41	5.57
11	972	361	473	0.67	8.33

The chosen compositions are given in Table 1. The pastes are assumed to behave as Bingham fluids. Their rheological parameters are then identified using a viscometer test (HAAKE ViscoTester® VT550). A coaxial cylinder geometry is used. The plastic yield is then identified as shown on Fig. 2. Experimental results for the 11 tested pastes are gathered in Table 2.

The spread flow is measured using a mini-cone test. Its dimensions are given on Fig. 3(a). It is a smaller version of the Abrams cone for cement pastes and grouts. On Fig. 3(b), the final shape of a cement paste tested sample is shown. The tested volume is 0.287 l. The final spread is plotted in terms of the plastic yield value on Fig. 4. Of course, the time needed to reach this final spread increases with the plastic viscosity. In our experiments, the final spread is measured on two perpendicular diameters 2 min after cone lifting.

There is a good agreement between experimental and theoretical spread flow values but when the plastic yield value tends towards zero, the correlation between the model and the measurements decreases. The predicted spread tends towards infinity for a viscous fluid with no yield stress whereas the experimental spread flow for water is between 45 and 55 cm. Surface tension effect for low or no yield stress fluid cannot be neglected in the stress equilibrium.

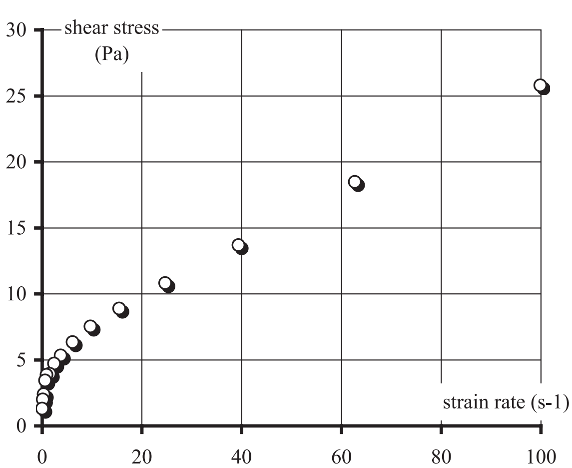

Fig. 2. Shear stress in terms of strain rate during a viscometer test for cement paste no. 2. If the cement paste fresh behaviour is approximated using a Herschell–Bulkley fluid model, the plastic yield value τ_0=1.57 Pa.

Table 2
Cement paste plastic yield values identified using a viscometer test assuming a fresh behaviour following the Herschell–Bulkley model

Cement pastes no.	Plastic yield value (Pa), τ_0
0	0.68
1	0.97
2	1.57
3	3.62
4	16.3
5	25
6	2.547
7	0.86
8	43.9
9	6.1
10	2.7
11	80.5

4. Surface tension influence for low plastic yield value cement pastes

4.1. Theoretical analysis

The Young's equation [11] describes the equilibrium of forces at the three-phase boundary shown on Fig. 5.

The equilibrium at the three phases interface writes:

$$A_{sl} - A_s + A_l \cos\theta = 0 \quad (13)$$

At equilibrium, if a small perturbation is considered (dh fluid depth variation and dR spread radius

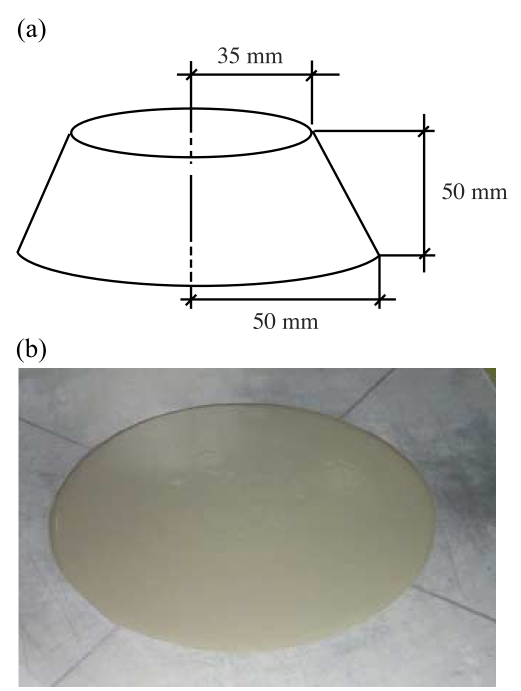

Fig. 3. (a) Mini-cone geometry; (b) spread sample at stoppage.

variation), the associated energy variations are equal to:

$$\pi R^2 h\rho g \frac{dh}{2} + 2\pi R dR(A_{sl} - A_s + A_l) = 0 \tag{14}$$

Substituting Eq. (13) and considering that $dh/h = -2dR/R$ then

$$\rho g h - 2\frac{A_l}{h}(1 - \cos\theta) = 0 \tag{15}$$

For a purely viscous fluid, the stoppage depth is:

$$h = \sqrt{2\frac{A_l}{\rho g}(1 - \cos\theta)} \tag{16}$$

A_l=72.10 mN/m for distilled water and contact angle on wetted Teflon is around 30° [12]. If the final shape is assumed to be cylindrical, then the stoppage depth and the final spread for distilled water may be calculated using Eq. (16): h=1.4 mm and the final spread is 52 cm. This is in accordance with the order of the experimental measurements, 45 to 55 cm. (The variation in the experimental results may be explained by the fact that inertia seems to be not negligible for low viscosity fluids. The speed at which the mini-cone is lift becomes an influent parameter on the sample shape at stoppage.)

In order to take into account the surface tension effect in the case of a yield tress fluid, such as a cement paste, the theoretical analysis given in Section 2 has to be corrected. The shear stress that fulfils the plasticity criterion has to be reduced by the effect of the surface tension. Without surface tension effect, the hydrostatic pressure is equal to $\rho g h$. If the surface tension effect is taken into account, it can be noted that, on an experimental point of view, the equivalent yield stress seems to be reduced by a function of the height h similar to Eq. (15). The yield stress may then be approximated by:

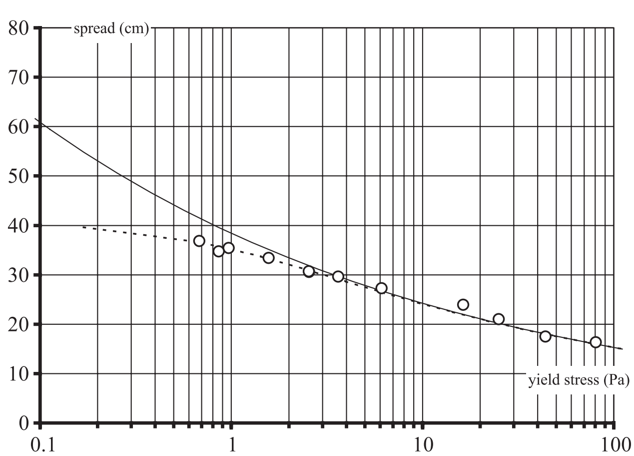

Fig. 4. Measured (circles) and predicted (line) spreads in terms of plastic yield value for the tested cement pastes. The dashed line is the predicted spread if the surface tension effect is taken into account for low plastic yield values cement pastes.

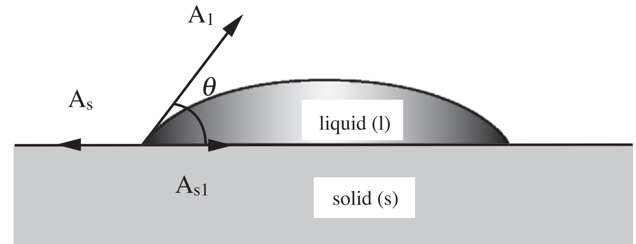

Fig. 5. Description of the wetting state A_s is the surface tension of the solid, A_l is the surface tension of the liquid and A_{sl} is the interfacial tension solid/liquid.

$$\tau_0 = \frac{225\rho g V^2}{128\pi^2 R^5} - \frac{\text{const.}}{h} \tag{17}$$

The constant in the above relation takes into account the influence of the unknown fluid surface tension and contact angle. It can also write if the volume is once again approximated by a cylinder:

$$\tau_0 = \frac{225\rho g V^2}{128\pi^2 R^5} - \text{const.}\frac{\pi R^2}{V} \tag{18}$$

A practical application of the above relation writes:

$$\tau_0 = 1.747\rho V^2 R^{-5} - \lambda\frac{R^2}{V} \tag{19}$$

The λ coefficient is a function of both the unknown tested fluid surface tension and contact angle. It is assumed that this coefficient is about the same for any cementitious material for a given test surface. It depends on the chosen test surface and have to be identified for a given apparatus. In the present study for our testing surface, it is equal to 0.005. Tests were carried out with different superplasticizers and viscosity agents with the same testing surface. The changes in the value of this coefficient were not measurable.

4.2. Comparison with the experimental results

The predicted corrected spread is plotted using the dashed line on Fig. 4. The parameter λ is obtained by fitting the predicted spread to the measured spread. If the spread is smaller than 35 cm, the correction is useless as the surface tension effects are negligible compared to the plastic yield value effect.

5. Application to the Abrams cone

Calculating a slump knowing the spread value or the other way round imposes an assumption about the sample shape at stoppage [13]. Following the calculus given in Section 2, the slump may be calculated in terms of the concrete plastic yield value. The results are compared with

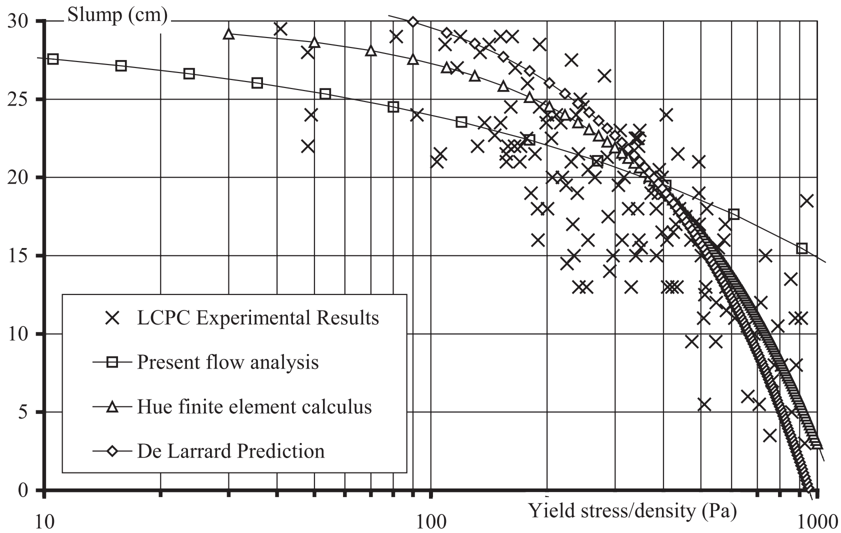

Fig. 6. Measured and predicted slump in terms of plastic yield value for the tested cement pastes (the experimental values given here are gathered from numerous internal experimental studies by C. Hu, D. Kaplan, F. de Larrard and L. Nachbaur).

various experimental and theoretical results from literature on Fig. 6 [14].

The long wave approximation is not valid for low slump as the sample height does not fulfil $h \ll 2R$. The tested volume should be high enough or the yield stress low enough to ensure that these inequalities are true. Moreover, the sample zone where the plastic criterion is fulfilled is not sufficient for the flow to be well developed. The quasi-static equilibrium approach developed by Hu [7] and corrected by de Larrard and Ferraris [14] is a better approach than the flow analysis presented in this study. However, for low plastic yield values, a developed flow occurs during the test. The sample height becomes small compared to the sample spread and the long-wave approximations becomes licit. The flow analysis given here is then suitable to predict the slump. This validity range is unfortunately the range in which the sample height is closed to the size of the gravel, which strongly reduces the meaningfulness of the measured slump.

6. Discussion and conclusions

The calculus developed here and its experimental validation in the case of the mini-cone test on cement pastes has two main interests. On the one hand, it upgrades the mini-cone empirical test to a real but simple rheological tool that allows its user to access an intrinsic property of the tested material instead of measuring a test geometry and material density dependant slump value. As the tested volume is small, the test complexity is low and its duration short, the relation (19) provides a quick way to estimate a cement paste or grout plastic yield value in a laboratory or in situ if the surface tension correction parameter λ has been previously identified. On the other hand, this calculus shows that surface tension often neglected in civil engineering flow studies may sometimes be the engine of the flow or of its stoppage. It is the case when low plastic yield value and low viscosity such as cement grouts are studied. For these low-concentration suspensions, the surface tension effect may be of the same order as the yield stresses (1–2 Pa).

This calculus is not valid in the case of the flow induced by a slump test on firm concrete. The assumptions needed to solve the flow equations are not fulfilled for low slump tested concretes. The already existing approaches are more efficient to link measured slump and plastic yield value. But, for high slump or high spread (SCC), the proposed relation could be tested as an alternative interpretation of the slump flow test.

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