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The Marsh cone: a test or a rheological apparatus?

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Abstract

The present study is concerned with the Marsh cone, a workability test used for specification and quality control of cement pastes and grouts. It is demonstrated here that, under several consistency and geometry conditions, the flow time reflecting "fluidity" may be calculated from the plastic viscosity and yield stress in the case of a Bingham fluid and from the cone geometry. A relation between the behaviour parameters and the flow time of fresh pastes tested is derived and experimentally validated. A practical application of these results is suggested. Lastly, the concept of a new test apparatus based on two cones is presented and tested on a single cement paste.

Keywords: Rheology; Cement paste; Physical properties; Marsh cone

1. Introduction

The Marsh cone test is a workability test used for specification and quality control of cement pastes and grouts. It belongs to the family of the orifice tests as the V-funnel [1] or the ORIMET developed by Bartos et al. [2]. The Marsh cone test standard varies from one country to another, but its principle is usually the same. The time needed for a certain amount of material to flow out of the cone is recorded. This measured flow time is linked with the so-called "fluidity" of the tested material. The longer the flow time, the lower is the fluidity. In the present study, it will be shown that the flow time can be directly linked to the material behaviour, namely, parameters such as yield stress and plastic viscosity for Bingham fluids. The flow time depends on the tested fluid but is also affected by the cone geometry.

The equations needed to solve the flow problem can be derived in the case of a Bingham fluid, which is a common and simple approximation of a fresh cement-based material behaviour. The flow problem can be solved, and the yield stress and plastic viscosity influences on the flow time are studied. The obtained relations are then used to predict the flow time of various cement pastes used in self-compacting concrete. The correlation between the flow time and the rheological behaviour of the cement pastes is experimentally validated. A practical application of these results is proposed that does not depend on the cone geometry measurements. Finally, a method using two cone differing by their nozzles is presented. This method could allow the determination of the two behaviour parameters from the results of the two different Marsh cone tests.

2. Experimental setup: the Marsh cone

The procedure is as follows:

– A Marsh cone is attached to a stand (Fig. 1). The cone geometry may vary. It should be noted that one of the main geometrical parameters is the nozzle radius, as it will be demonstrated in Section 5.
– Closing the nozzle, a certain amount of the tested material is poured into the cone (this amount may vary

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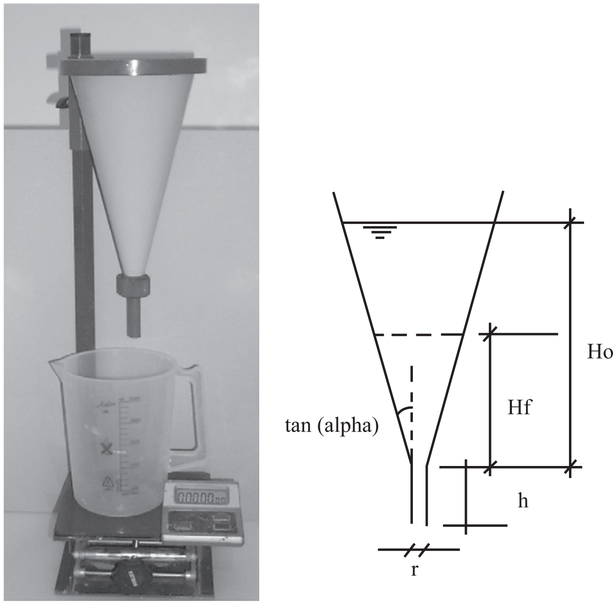

Fig. 1. Studied geometry and main notations. α, H_f, H_0, h and r depend on the test standard.

from 0.8 to 1.7 L depending on the authors or on the country standard test: ASTM C939-94a, EN 445 or EN12715). The initial filling height is noted H_0.
– The orifice is opened, and a stop watch is started.
– The time for a certain amount of the tested material to flow is recorded. This amount is always a fraction of the initial amount, 0.4 L for an 0.8-L initial amount, 1.0 L for a 1.7-L initial amount. The final filling height is noted H_f.

This test has already been used in the optimisation of high-performance concrete mix fitting [3,4]. The superplasticizer saturation point is defined as the superplasticizer dosage beyond which the flow time does not decrease appreciably. It is also used to control the poststressed cable injection grouts rheology. In the present study, different cone geometries are tested.

3. Theoretical investigation: flow time calculation

3.1. Tested materials behaviour

Various empirical and theoretical models have been used to describe fresh cement pastes behaviour. Among the most widely used are the Bingham and Herschel-Bulkley models, which take in account the pseudoplastic behaviour of those concentrated suspensions [5–8]. If the estimated yield stress is very small, as it is the case for cement grouts, then a purely viscous model, which is a particular case of a Bingham model, is often sufficient to describe correctly the grouts fresh behaviour. In the following theoretical study, the flow is modelled for the Bingham fluid approximation.

3.2. Flow equations

In a cylinder (radius R), if inertia effects are assumed to be negligible, and the flowing velocity is assumed to be equal to zero at the fluid/cylinder interface, the Buckingham–Reiner equation for a Bingham fluid with a yield stress K_i and a plastic viscosity μ_p writes

$$Q = \frac{\pi A R^4}{8\mu_p}\left(1 - \frac{4}{3}\left(\frac{2K_i}{AR}\right) + \frac{1}{3}\left(\frac{2K_i}{AR}\right)^4\right) \quad (1)$$

where Q is the rate of flow, and A is the pressure gradient motor of the flow.

It is worth noting that the flow will only occur if the cylinder radius is higher than a critical radius r_c:

$$R \geq r_c = \frac{2K_i}{A} \quad (2)$$

The flow criterion then writes:

$$\frac{2K_i}{AR} \leq 1 \quad (3)$$

Eq. (1) may be written for the two parts of the cone, the conical part and the cylindrical part. In each part, Eq. (1) gives a relation between the pressure gradient and the flow rate. As the flow rate is constant through the cone, a combination of these two equations gives the expression of the debit through the nozzle in terms of the cone geometric parameters and of the tested fluid behaviour. The debit may be linked to the cone level variations in time. The calculus details are given in Appendix A.

3.3. What happens if the tested fluid is purely viscous?

The integration of the rate of flow is possible in the case of a purely viscous fluid, and the analytical expression of the flow time is given in Ref. [9]. The fluid level decrease in the conical part accelerates through the test. The flow time for a purely viscous fluid t_v is proportional to the viscosity via a function of the cone geometry.

$$t_v = \frac{\mu}{\rho g}\,\text{function}\left(\alpha, r, H_0, h, H_f\right) \quad (4)$$

The function expression is reminded in Appendix A. Simulation and experimental results given by Ref. [9] are in good agreement for viscosity values higher than 200 mPa s for the EN445 cone. For lower viscosity values, the predicted flow time is lower than the measured one. The worst example is the case of pure water. The calculated flow time is 0.09 s (viscosity=1 mPa s), but the measured flow time is about 6 s. In these regimes, the assumption of a succession of quasi-steady states used in Ref. [9] is not valid any more, and the flow time is not any more proportional to the viscosity. As such, the flow time value is not a meaningful measurement on a rheological point of view.

3.4. What happens if the tested fluid behaves as a Bingham fluid?

The flow will not occur or will stop if criterion (Eq. (3)) is not fulfilled at any point in the sample. Moreover, it is obvious that if the flow occurs in the cylindrical part of the apparatus, it will occur everywhere in the sample as the corresponding cylinder radius is the smallest in the nozzle. The criterion to be fulfilled for the flow to occur is then

$$\frac{2K_i}{Ar} \leq 1 \quad (5)$$

The pressure gradient writes for Q=0 (critical situation where flow does not occur):

$$A = \rho g \frac{H}{h} + \frac{8K_i}{3h\tan(\alpha)} \mathrm{LN}\left(\frac{r}{(r + H\tan(\alpha))}\right) \quad (6)$$

For one-cone geometry, no flow will occur if the yield stress is higher than a critical yield stress. As $A(H)$ decreases through the test from $A(H_0)$ to $A(H_f)$, Eq. (5) may not be fulfilled during the entire test, and the flow will then stop.

If Eq. (5) is fulfilled, the equations can be solved numerically, and the fluid filling level can be plotted in terms of the flow time t for different yield stress values for a constant plastic viscosity value (Fig. 2). The presence of a yield stress slows down the cone emptying, as shown in Figs. 2 and 3.

The volume V increases linearly during approximately the first half of the test, as noticed in Refs. [9,10] and as shown in Fig. 3. During this phase, the debit through the cylinder is constant. Ref. [10] considered only the flowing of the first 0.7 l out of the tested 1.1 l and attributed this nonlinear evolution to an increased effect of friction, which is in a way correct. In fact, the friction stays the same, but the pressure gradient which is the “engine” of the flow in the cylindrical part relatively decreases through the test as the fluid level in the conical part decreases. The pressure gradient variations at the beginning of the test are small (small variations of the fluid filling height) and become stronger at the end of the test when the phenomenon accelerates. This leads to a constant debit during the first part of the experiment, which decreases only at the end of the total time needed to empty the cone.

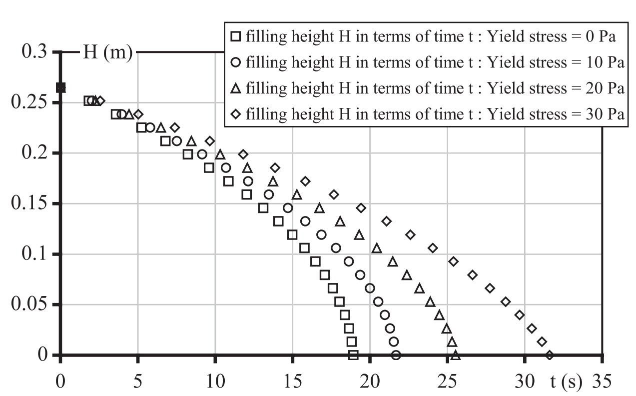

Fig. 2. Filling height in the conical part as a function of time for a Bingham fluid. h=0.06 m; tan(α)=0.253; H_0=0.273; μ_p=200 mPa s; ρ=2000 kg/m^3; r=0.005 m. The total time to empty the cone varies between 18 and 32 s.

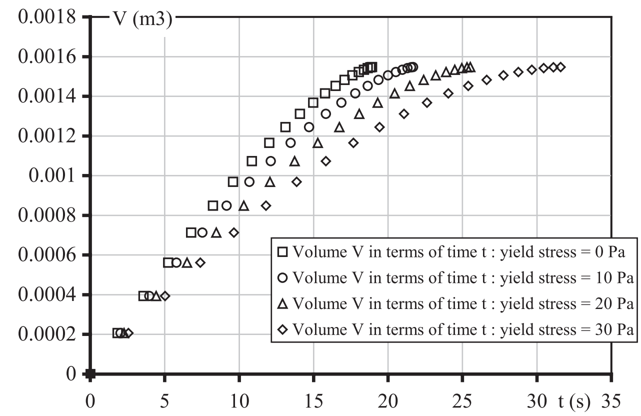

Fig. 3. [Simulation] Volume V as a function of time for a Bingham fluid. h=0.06m; tan(α)=0.253; H_0=0.273; μ_p=200 mPa s; ρ=2000 kg/m^3; r=0.005 m.

No analytical solutions can be found, as the integration of the debit expression is not analytically possible. But if the flow rate is considered constant during the first part of the test, Q can be calculated from the initial fluid filling level. Because of this approximation, the flow time is lightly underestimated. A volume V that has flown out of the cone during a time length t_V is then considered.

$$Q = \frac{V}{t_V} = -\frac{\mathrm{function_A}(r, H_0, \alpha, h)K_i + \mathrm{function_B}(r, H_0, \alpha, h)\rho g}{\mathrm{function_C}(r, H_0, \alpha, h)\mu_p} \quad (7)$$

The analytical expressions of $\mathrm{function_A}$, $\mathrm{function_B}$ and $\mathrm{function_C}$ are given in Appendix A.

For one given geometry and test procedure, t_v may be calculated in terms of the yield value and the plastic viscosity for one given volume V. The flow time is proportional to the viscosity, but the yield stress has to be taken into account to predict the flow time for one given fluid. The yield stress influence on the flow time can explain the fact that Ref. [10] or [11] does not find any correlation between flow time and plastic viscosity or flow time and yield stress for the cement pastes they tested. In fact, the correlation only exists between flow time and the two rheological parameters considered together, as the experimental results given in the next section will confirm.

4. Experimental results: application on self compacting concrete cement pastes and cement grouts

Different cement pastes with W/C around 0.56 and $W/(C+F)$=0.4 are studied, [12]. Their components are cement

Table 1
The 11 tested cement pastes compositions

Cement pastes N°	Cement (kg/m^{-3})	Filler (kg/m^{-3})	Water (kg/m^{-3})	Super plasticizer dry (kg/m^{-3})	Viscosity agent dry (kg/m^{-3})
0	984	365	473	3.44	1.41
1	985	366	473	2.70	1.41
2	986	366	473	2.03	1.41
3	988	367	473	1.36	1.41
4	989	367	473	0.68	1.42
5	985	366	473	0.68	2.82
6	983	365	473	2.03	2.81
7	981	364	473	3.43	2.80
8	979	364	473	0.68	5.59
9	977	363	473	2.01	5.58
10	974	362	473	3.41	5.57
11	972	361	473	0.67	8.33

(CEM I 52,5 N CE), filler, superplasticizer and viscosity agent. The chosen compositions are given in Table 1. The pastes are assumed to behave as Bingham fluids. Their rheological parameters are then identified using a viscometer test, (HAAKE ViscoTester VT550). Experimental flow curves are similar to the example shown in Fig. 4. Measured behaviour parameters for the tested pastes are gathered in Table 2.

The first step in the model validation is to consider whether or not the flow can occur through the entire test knowing the plastic yield value and the test geometry. Eq. (5) has to be fulfilled at any time point during the test.

For the test geometry used in this part, r=4 mm, h=6 mm, Tan(α)=0.253. The filling volume is 0.8 L, and the passing volume is 0.4 L. The initial height in this test geometry is H_0=0.212 m, and the final height is H_f=0.165 m. As A decreases through the test, the critical state is when H is the smallest, namely 0.165 m. The criterion values for each cement pastes may then be calculated. They are given in Table 3 with experimental comments on the test outcome.

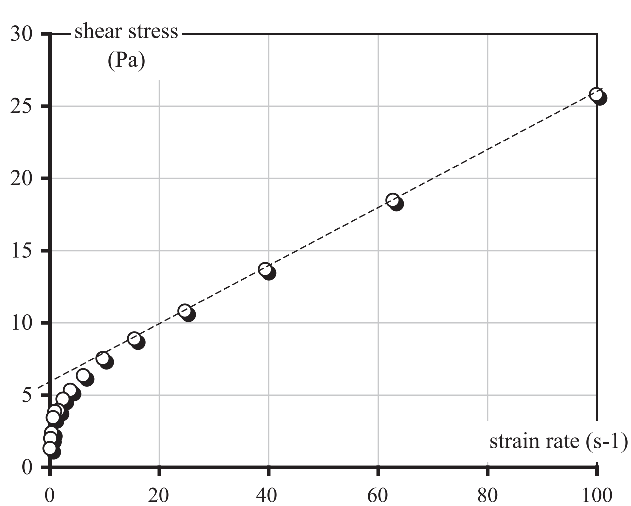

Fig. 4. [Experimental results] Shear stress in terms of strain rate during a viscometer test. Cement paste N°2. If the paste is considered as a Bingham fluid, the plastic yield value K_i=6 Pa, and the plastic viscosity μ_p=200 mPa s.

Table 2
[Experimental results] Cement pastes rheological parameters identified using a viscometer test

Cement pastes N°	Plastic viscosity (mPa s) μ_p	Plastic yield value (Pa) K_i
0	170	0
1	170	5
2	200	6
3	220	10
4	265	19
5	290	28
6	200	7
7	170	5
8	500	40
9	240	18
10	240	8
11	610	60

The relation between shear stress and strain rate writes in the case of a Bingham fluid: $\tau=K_i+\mu_p\dot{\gamma}$.

Eq. (5) seems adequate to predict whether or not the flow will occur. On a practical point of view, this criterion allows the prediction of the limits of a given test geometry. With this test, it will be impossible to test materials with a yield value higher than 50 Pa. If the cone nozzle radius is increased, this limit increases as well. It should be noted that the influence of the nozzle radius on this flow criterion is strong.

Using the measured rheological parameters and Eq. (7), one may then try to predict the time flow and compare it to the experimental time flow. This comparison is plotted on Fig. 5. If the yield stress is taken in account, there is a correlation between the fresh behaviour parameters and the flow time.

Table 3
[Experimental results and theoretical values] Cement pastes flow criterion for the tested geometry

Cement pastes N°	Criterion Eq. (5)	Flow (experimental comment)
0	0.01<1	Normal
1	0.01<1	Normal
2	0.02<1	Normal
3	0.04<1	Normal
4	0.19<1	Normal
5	0.32<1	Normal
6	0.03<1	Normal
7	0.01<1	Normal
8	0.71<1	Difficult
9	0.07<1	Normal
10	0.03<1	Normal
11	2.60>1	Impossible

If the criterion value is higher than 1, the flow will not occur or will stop before 0.4 L has passed through the nozzle.

5. Experimental procedure: Marsh cone practical use as a rheological apparatus

From Eq. (7), the following relation may be derived:

$$t_{\mathrm{V}} = \frac{a_{\mathrm{V}}\mu_{\mathrm{p}}}{\rho - b_{\mathrm{V}}K_{\mathrm{i}}} \tag{8}$$

where a_{V} and b_{V} are constants depending on the cone geometry and on the observed flowing volume *V*. They can either be calculated using the analytical expressions of $\mathrm{function}_{\mathrm{A}}$, $\mathrm{function}_{\mathrm{B}}$ and $\mathrm{function}_{\mathrm{C}}$ given in Annex 1 or calibrated using known materials. Both methods should give similar results, but the estimation has one advantage; the coefficients are very sensitive to *h* and *r*, which are sometimes difficult to measure or, as shown on Fig. 6, even difficult to define for some test geometry. It can be demonstrated from Eq. (7) that, for slow flows and small *r*, a_1 and b_1 are respectively proportional to h/r^4 and h/r. For the test geometry chosen in part 4., $a_{0.4\ \mathrm{L}}$=95000 s^2/m^2 and $b_{0.4\ \mathrm{L}}$=25.6 s^2/m^2.

This method may also be applied to [13] flow time results. In that study, the used cone geometry is not given. The only information available is that the poured volume is 1.1 L, and the flow time at 0.7 L is measured. For all the tested cement grouts, the yield value and the plastic viscosity were measured using a coaxial rotating cylinder viscometer.

The coefficients may be identified for the cone [13] used.

$$t_{0.7\ \mathrm{L}} = \frac{a_{0.7\ \mathrm{L}}\mu_{\mathrm{p}}}{\rho - b_{0.7\ \mathrm{L}}K_{\mathrm{i}}} \tag{9}$$

with $a_{0.7\ \mathrm{L}}$=1.65.10^6 s^2/m^2 and $b_{0.7\ \mathrm{L}}$=24.0 s^2/m^2.

Predicted flow time and measured flow time are plotted in Fig. 7.

Other cone calibration examples are given for other cone and nozzle geometries and for a 1-L tested volume on Fig. 8(a) and (b). The proportionality of the *a* and *b* coefficients to respectively h/r^4 and h/r is confirmed on these examples.

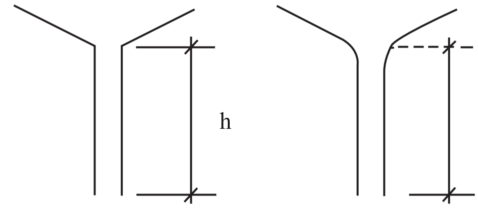

Fig. 6. The difficulty in measuring the nozzle *h* value.

6. Experimental procedure: two cones test

A Marsh cone, even calibrated, does not provide enough data to measure the values of two parameters (plastic yield value and plastic viscosity). The idea here is to use two cones only differing in their nozzle shape to get two different flow time values. These two values will allow the user to calculate the two behaviour parameters.

The procedure is the following:

For cone 1, flow time is linked to the behaviour parameters through

$$T_1 = \frac{a_1\mu_{\mathrm{p}}}{\rho - a_2K_{\mathrm{i}}} \tag{10}$$

For cone 2, flow time is linked to the behaviour parameters through

$$T_2 = \frac{b_1\mu_{\mathrm{p}}}{\rho - b_2K_{\mathrm{i}}} \tag{11}$$

The first cone geometry is the same as in Sections 4 and 5: *h*=0.06 m, *r*=0.005 m, a_1=95000 s^2/m^2 and a_2=25.6 s^2/m^2. The *s* cone has a different nozzle: $h \cong 0.08$ and *r*=0.004 m. This second cone coefficients are calculated from experimental results: b_1=310000 s^2/m^2 and b_2=42.7 s^2/m^2. The times T_1 and T_2 for a certain amount of the tested material to flow are recorded.

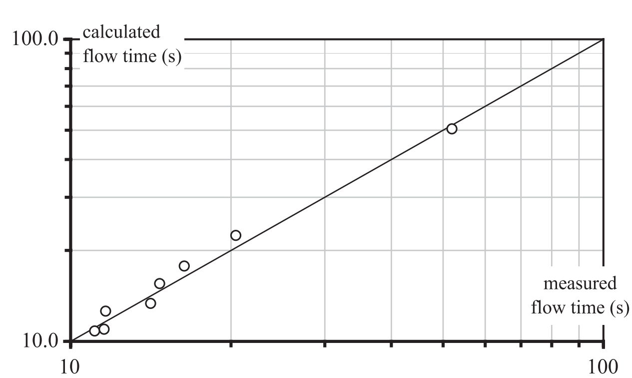

Fig. 5. [Experimental results and simulation] Comparison between the measured flow time and the predicted flow time using Eq. (7) and the rheological parameters in Table 2. The *x*=*y* curve is also plotted.

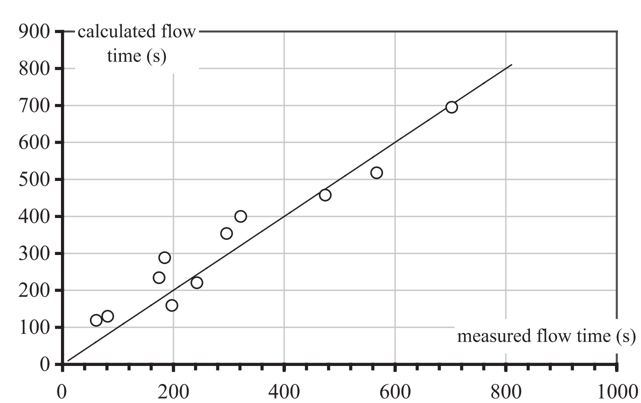

Fig. 7. [Experimental results and simulation] Comparison between the measured flow time and the predicted flow time using Eq. (9) and the rheological parameters measured by Ref. [12]. The *x*=*y* curve is also plotted.

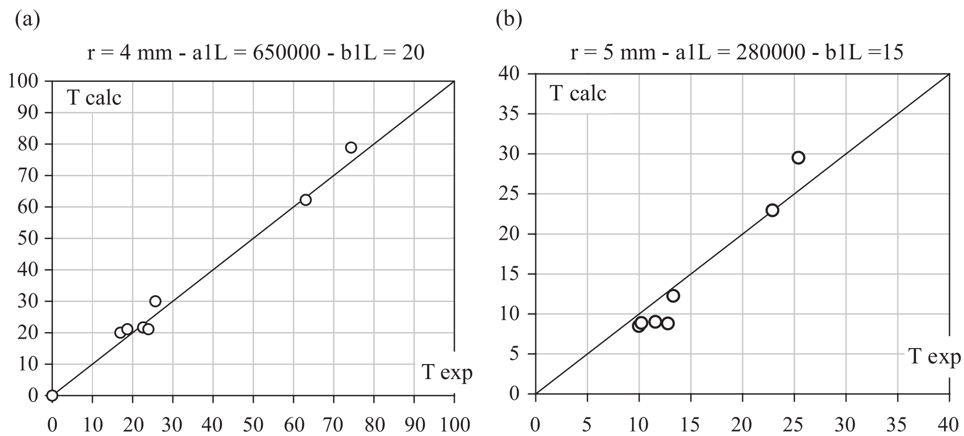

Fig. 8. [Experimental and theoretical results] Predicted flow time versus measured flow time for different tested cement pastes and for two cone nozzle radii. The x=y curve is also plotted.

Using Eqs. (12) and (13), the tested material behaviour parameters may be calculated

$$K_i = \rho \frac{(a_1 T_2 - b_1 T_1)}{a_1 b_2 T_2 - a_2 b_1 T_1} \quad (12)$$

$$\mu_p = \rho \frac{T_1 T_2 (b_2 - a_2)}{a_1 b_2 T_2 - a_2 b_1 T_1} \quad (13)$$

The measured flow times are

$T_1 = 17$ s

$T_2 = 79$ s

The tested material (mixture 4) behaviour parameters may then be calculated using Eqs. (12) and (13):

$K_i = 22$ Pa

$\mu_p = 235$ mPa s

Knowing that the average difference between measured and predicted flow time values on Fig. 6 is around 9%, the uncertainty on the yield value and plastic viscosity calculation may then be estimated,

$\Delta K_i = 6.4$ Pa $\cong 29\%$

$\Delta \mu_p = 35$ mPa s $\cong 16\%$

15 Pa$<K_i<$28 Pa for a K_i measured value (viscometer)

$= 19$ Pa

200 mPa s$<\mu_p<$270 mPa s for a μ_p measured value

(viscometer) $=$ 265mPa s

This uncertainty estimation is too brief to ensure the test efficiency, but in the opinion of the present author, this double-cone test may be worth a deeper study.

7. Summary and conclusions

On a practical point of view, it should first be noted that the Marsh cone has two limits. On one hand, if the tested fluid viscosity is too low, there is no linear relation between the viscosity and the flow time. Water is about 1000 times less viscous than glycerol, but water flow time is only about 10 times lower than glycerol flow time. As such, for low viscosity fluids (flow time lower than 12 s for the EN440 cone geometry), the flow time value is not a meaningful measurement on a rheological point of view.

On the other hand, if the tested fluid has a yield stress, the flow may not occur. The pressure gradient created by the fluid weight above the nozzle may not be sufficient for the shear stress to overcome the yield stress in the nozzle. In this case, the marsh cone becomes useless. In this study, a criterion for the flow to occur is given in terms of the tested fluid yield stress and volumic weight and the cone geometry.

To link the measured flow time with the tested material rheological behaviour, a relation between flow time and plastic viscosity and plastic yield value is written allowing, for cement pastes and grouts, the flow time prediction. This relation is validated for different cone geometries in the case of various self-compacting concrete cement pastes and literature data.

The interest of using two cones is discussed in the last part of the present work. This method allows the calculation of both the plastic viscosity and the plastic yield value. It would be an handy on site control tool to check cement pastes and cement grouts fresh properties.

More work is of course still required. But this study is a step towards a better understanding of the general relationships between behaviour and flow time in orifice rheometers. The methods proposed here could be declined to describe the flows generated by the V-funnel or the ORIMET.

Notations

A	Pressure gradient, engine of the flow (Pa m^{-1})
g	Gravitational constant (m s^{-2})
h	Nozzle height (m), (Fig. 1)
K_i	Yield stress (Pa)
$p(z)$	Pressure at a distance z above the nozzle (Pa)
Q	Rate of flow (m^3/s)
r	Nozzle radius (m)
V	Volume that has flown out of the cylinder during a time t (m^3)
α (or alpha)	Cone angle (rad.; Fig. 1).
ρ	Tested material volumic mass
μ	Newtonian viscosity (Pa s)
μ_p	Bingham fluid plastic viscosity (Pa s)

Appendix A

If only the predominant terms are written, Eq. (1) becomes

$$Q=\frac{\pi AR^4}{8\mu_p}\left(1-\frac{4}{3}\left(\frac{2K_i}{AR}\right)\right) \tag{A1}$$

In the conical part, Eq. (A1) writes for small α values, and if the flow is considered as a succession of steady states (slow flow),

$$Q=\frac{\pi}{8\mu_p}\left(\frac{\partial p}{\partial z}+\rho g\right)(r+z\tan(\alpha))^4-\frac{\pi K_i}{3\mu_p}(r+z\tan(\alpha))^3 \tag{A2}$$

where p is the pressure at a distance z above the nozzle, ρ is the tested material volumic mass, and g is the gravitational constant.

From Eq. (A2) comes

$$\frac{\partial p}{\partial z}=\frac{8\mu_p Q}{\pi(r+z\tan(\alpha))^4}+\frac{8K_i}{3(r+z\tan(\alpha))}-\rho g \tag{A3}$$

with $p(H)$=0 if the atmospheric pressure is the reference pressure.

The pressure then writes

$$p(z)=-\frac{8\mu_p Q}{3\pi\tan(\alpha)}\left(\frac{1}{(r+z\tan(\alpha))^3}-\frac{1}{(r+H\tan(\alpha))^3}\right)+\frac{8K_i}{3\tan(\alpha)}\mathrm{LN}\left(\frac{(r+z\tan(\alpha))}{(r+H\tan(\alpha))}\right)+\rho g(H-z) \tag{A4}$$

and

$$p(0)=\rho gH-\frac{8\mu_p Q}{3\pi\tan(\alpha)}\left(\frac{1}{r^3}-\frac{1}{(r+H\tan(\alpha))^3}\right)+\frac{8K_i}{3\tan(\alpha)}\mathrm{LN}\left(\frac{r}{(r+H\tan(\alpha))}\right) \tag{A5}$$

In the cylindrical part, Eq. (A1) becomes

$$Q=\frac{\pi r^4}{8\mu_p}\left(\frac{\partial p}{\partial z}+\rho g\right)-\frac{\pi K_i r^3}{3\mu_p}=\frac{\pi r^4}{8\mu_p}\left(\frac{p(0)}{h}+\rho g\right)-\frac{\pi K_i r^3}{3\mu_p} \tag{A6}$$

$$\frac{8\mu_p h}{\pi}\left(\frac{1}{r^4}+\frac{1}{3h\tan(\alpha)}\left(\frac{1}{r^3}-\frac{1}{(r+H\tan(\alpha))^3}\right)\right)Q=\rho g(H+h)+\frac{8K_i}{3}\left(\frac{1}{\tan(\alpha)}\mathrm{LN}\left(\frac{r}{(r+H\tan(\alpha))}\right)-\frac{h}{r}\right) \tag{A7}$$

with

$$Q=-\pi(r+H\tan(\alpha))^2\frac{dH}{dt} \tag{A8}$$

A.1. Resolution in the purely viscous fluid case

In the case of a purely viscous fluid (Newtonian viscosity μ), the approximation used to obtain Eq. (A1) is not necessary, and the obtained solution is an exact one. Eq. (A7) becomes, for K_i=0 and μ_p=μ,

$$\frac{8\mu h}{\pi}\left(\frac{1}{r^4}+\frac{1}{3h\tan(\alpha)}\left(\frac{1}{r^3}-\frac{1}{(r+H\tan(\alpha))^3}\right)\right)Q=\rho g(H+h) \tag{A9}$$

or

$$\left(\left(1+\frac{3h\tan(\alpha)}{r}+\right)\frac{(r+H\tan(\alpha))^2}{(H+h)}-\frac{r^3}{(H+h)(r+H\tan(\alpha))}\right)dH=-\frac{3\rho g r^3\tan(\alpha)}{8\mu}dt \tag{A10}$$

If H_0 is the initial conical part filling height

$$t_v=\frac{8\mu}{3\rho g r^3\tan(\alpha)}\left(\left(1+\frac{3h\tan(\alpha)}{r}\right)(\tan^2(\alpha)(H_0-H)\times\left(\frac{H_0+H}{2}+h\right)-2\tan(\alpha)(h\tan(\alpha)-r)(H_0-H)+(h\tan(\alpha)-r)^2\mathrm{LN}\left(\frac{H_0+h}{H+h}\right)\right)-\frac{r^3}{h\tan(\alpha)-r}\times\left(\mathrm{LN}\left(\frac{H_0\tan(\alpha)+r}{H\tan(\alpha)+r}\right)-\mathrm{LN}\left(\frac{H_0+h}{H+h}\right)\right)\right) \tag{A11}$$

with

$$H=\frac{1}{\tan(\alpha)}\left(\left((H_0\tan(\alpha)+r)^3-\frac{3V\tan(\alpha)}{\pi}\right)^{\frac{1}{3}}-r\right) \tag{A12}$$

where V is the volume that has flown out of the cylinder during a time t. The fluid level decrease in the conical part accelerates through the test. Eq. (A9) may be written as

$$t_v = \frac{\mu}{\rho g}\,\text{function}(\alpha, r, H_0, h, H) \tag{A13}$$

A.2. Resolution in the Bingham fluid case

No analytical solutions can be found as the integration of Eq. (A10) is not analytically possible.

However, if the debit is considered constant through the first part of the test, Q may be calculated for H=H_0. A volume V that has flown out of the cone during a time length t_V is then considered.

$$Q = \frac{V}{t_V} = -\frac{\pi r^3 (8K_i r \mathrm{LN}(H_0\ \tan(\alpha) + r) - 8K_i r \mathrm{LN}(r) - \tan(\alpha)(3\rho g r(h + H_0) - 8hK_i))(H_0\ \tan(\alpha) + r)^3}{8\mu_p \tan(\alpha)\left(3h(H_0\ \tan(\alpha) + r)^3 + H_0 r\left(H_0^2 \tan^2(\alpha) + 3H_0 r\ \tan(\alpha) + 3r^2\right)\right)} \tag{A14}$$

The flow time is expressed in terms of the material characteristics:

$$t_V = \frac{a_V \mu_p}{\rho - b_V K_i} \tag{A15}$$

where a_{1L} and b_{1L} are constants depending on the cone geometry.

$$a_V = \frac{8V\ \tan(\alpha)\left(3h(H_0\ \tan(\alpha) + r)^3 + H_0 r\left(H_0^2 \tan^2(\alpha) + 3H_0 r\ \tan(\alpha) + 3r^2\right)\right)}{(3\pi r^3\ \tan(\alpha) g r(h + H_0))(H_0\ \tan(\alpha) + r)^3} \tag{A16}$$

$$b_V = \frac{\pi r^3 (8r\ \ln(H_0\ \tan(\alpha) + r) - 8r\ \ln(r) + 8h\ \tan(\alpha))}{3\pi r^3 \tan(\alpha) g r(h + H_0)} \tag{A17}$$

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