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A new way of prediction elastic modulus of normal and high strength concrete—fuzzy logic

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Abstract

In this paper, the theory of fuzzy sets, especially fuzzy modeling is discussed to determine elastic modulus of both normal and high-strength concrete. A fuzzy logic algorithm has been devised for estimating elastic modulus from compressive strength of concrete. The main advantage of fuzzy models is their ability to describe knowledge in a descriptive human-like manner in the form of simple rules using linguistic variables only. On the other hand, many parameters will be effected and elastic modulus can be taken into account easily by using the proposed fuzzy model.

Keywords: Fuzzy logic; Concrete; Compressive strength; Elastic modulus

1. Introduction

Ideally, the elastic modulus is measured directly on concrete samples under compression by recording the load–deformation curve, but from an experimental point of view, this is not always easy. When compared with compressive strength measurements carried out to characterize the compressive strength f_c, this testing procedure is much more complicated and time-consuming. To avoid the demanding and time-consuming direct measurements of elastic modulus E_c, engineers and researchers have tried to find some shortcuts to enable them to predict the elastic modulus of concrete using either a theoretical or an empirical approach. In latter case, which is most widely used, the modulus of elasticity is usually expressed as a function of compressive strength. On the other hand, different national building codes propose various formulas for normal strength concrete (NSC) and high strength concrete (HSC). A lot of relationships for NSC are given as follows.

ACI 318-95 [1]

$$E_c = 4.73(f_c)^{1/2}$$

TS-500 [2]

$$E_c = 3.25(f_c)^{1/2} + 14$$

in which relationships f_c and E_c are expressed in MPa and in GPa, respectively. When HSC began to develop, a number of attempts were made to see whether the existing relationship could be used to predict HSC modulus of elasticity that had to be developed. For example American, European, and Norwegian committees on high strength concrete propose the following relationships:

ACI 363 [3]

$$E_c = 3.32(f_c)^{1/2} + 6.9$$

CEB90 [4]

$$E_c = 10(f_c + 8)^{1/3}$$

NS 3473 [5]

$$E_c = 9.5(f_c)^{0,3}$$

in which relationships f_c and E_c are expressed in MPa and in GPa, respectively.

Due to different characteristics of high strength concrete, for designing structures made of it, some design procedure traditionally used in normal strength concrete structures have to be changed. For this reason, determination of elastic properties of concrete has become very important from a

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design point of view when the deformations of the different structural elements of a structure have to be calculated. Many authors have pointed out the importance of the determination of the elastic modulus [6,7]. In addition, several studies [8–10] on high strength concrete have been developed with the objective of studying the effect of parameters on elastic properties of high strength concrete.

In this paper, a new fuzzy approach has been presented to predict elastic modulus of both NSC and HSC. Numerical investigation is carried out and the fuzzy results are compared with those of test data and some other available in literature. Numerical results reveal a good agreement between the test and fuzzy results. To have an objective comparison of the performance of the models against the experimental results, the error measure of the root mean square error (RMSE) was computed for each model. The variances were computed for each model and experiment.

2. Fuzzy sets and logic

The concept of "fuzzy set" was introduced by Zadeh [11], who pioneered the development of fuzzy logic instead of Aristotelian logic which has two possibilities only. Fuzzy logic concept provides a natural way of dealing with problems in which the source of imprecision is the absence of sharply defined criteria rather than the presence of random variables. Fuzzy approach considers cases where linguistic uncertainties play some role in the control mechanism of the phenomena concerned. Herein, uncertainties do not mean random, probabilistic and stochastic variations, all of which are based on the numerical data. Zadeh has motivated his work on fuzzy logic with the observation that the key elements in human thinking are not numbers but levels of fuzzy sets. Further he saw each linguistic word in a natural language as a summarized description of a fuzzy subset at a universe of discourse representing the meaning of this word. In consequence, he introduced linguistic variables as variables whose values are sentences in a natural or artificial language [12]. In this study, however, a simplified view of linguistic variables of concrete compressive strength and modulus of elasticity of concrete is adopted. The fuzzy logic definition in the following sequel is tailored to the application of elastic modulus of normal and high strength concrete modeling which in many ways is very similar to the established use of fuzzy logic in the control of dynamic systems, also known as "fuzzy logic control". In both contexts, fuzzy propositions, i.e. IF–THEN statements, are used to characterize the state of a system and the truth value of the proposition is a measure of how well the description matches the state of the system. Fuzzy logic has been developing since then and is now being used especially in Japan for automatic control for commercial products such as washing machines, cameras and robotics. Many textbooks provide basic information on the concepts and operational fuzzy algorithms [13–17]. In several research, fuzzy approach has been used [18,19].

The key idea in fuzzy logic is allowance of partial belongings of any object to different subsets of the universal set instead of belonging to a single set completely. Partial belonging to a set can be described numerically by a membership function which assumes values between 0 and 1 inclusive. For instance Fig. 1 shows a typical membership function for small, medium and large class size in universe, U. Hence, these verbal assignments are fuzzy subsets of the universal set. In this figure, set values less than 2 are definitely "small"; those between 4 and 6 are certainly "medium; while values larger than 8 are definitely "large". However, intermediate values such as 2.2 partially belong to the subsets "small" and "medium". In fuzzy terminology 2.2 has a membership value of 0.9 in "small" and 0.1 in "medium", but 0.0 in "large" subsets. The literature is rich with references concerning the ways to assign membership values or functions to fuzzy variables. Among these ways are intuition, inference rank ordering, angular fuzzy sets, neural networks, genetic algorithms, inductive reasoning, etc. [16]. Especially, the intuitive approach is used rather commonly because it is simply derived from capacity of humans to develop membership functions through their own innate intelligence and understanding. Intuition involves contextual and semantic knowledge about an issue; it can be also involve linguistic truth values about this knowledge [15].

Even if the measurements are carefully carried out as crisp quantities they can be fuzzified. Furthermore, if the form of uncertainty happens to arise because of imprecision, ambiguity or vagueness, then the variable is fuzzy and can be represented by a membership function. Unlike the usual constraint where, say, the variable in Fig. 1 must not exceed 2, a fuzzy constraint takes the form as saying that the same variable should preferably be less than 2 and certainly should not exceed 4. This is tantamount in fuzzy sets terms that values less than 2 have membership of 1 but values greater than 4 have membership of 0 and values between 2 and 4 would have membership between 1 and 0. In order to simplify the calculations, usually the membership function is adopted

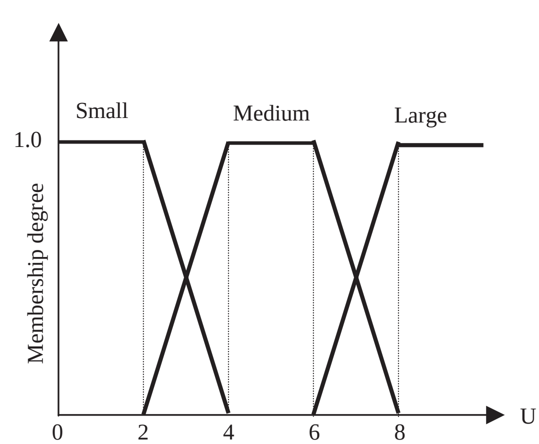

Fig. 1. Fuzzy subsets.

as linear in practical applications. The objective then can be formulated as maximising the minimum membership value, which has the effect of balancing the degree to which the objective is attained with degrees to which the constraints have to be relaxed from their optimal values [12].

3. Fuzzy rule base

In the fuzzy inference method sets of corresponding input and output measurements are provided to the fuzzy system and it learns how to transform a set of inputs to the corresponding set of outputs through a Fuzzy Associative Map (FAM) which is also called the Fuzzy Associative Memory [14]. Fuzzy logic does not provide a rigorous way for developing or combining fuzzy rules which can be achieved through many ways. The method adopted in this paper is outlined below.

Table 1
Comparison of values estimated with the values obtained from codes and using the experimental results reported by Wee et al. [22]

Compressive strength	Elastic modulus (E_c)	ACI 363	CEB	NS 3473	Wee [22]	Gesoglu [25]	Fuzzy
42.7	37.6	28.6	37.0	29.3	35.7	30.6	35.9
63.2	41.8	33.3	41.4	33.0	40.6	38.5	42.0
70.2	43	34.7	42.8	34.0	42.1	40.8	43.1
65.1	41.5	33.7	41.8	33.3	41.0	39.1	42.3
70.5	40.4	34.8	42.8	34.1	42.1	40.9	43.2
69.7	41.5	34.6	42.7	33.9	42.0	40.7	43.1
71.5	41.4	35.0	43.0	34.2	42.3	41.3	43.3
63.6	42.6	33.4	41.5	33.0	40.7	38.6	42.1
85.9	45	37.7	45.5	36.1	45.0	45.8	45.6
90.2	44.4	38.4	46.1	36.7	45.7	47	46.3
78.3	44.3	36.3	44.2	35.1	43.6	43.4	44.4
85.9	44.3	37.7	45.5	36.1	45.0	45.8	45.6
81.2	43.9	36.8	44.7	35.5	44.2	44.3	44.9
88.1	44.5	38.1	45.8	36.4	45.4	46.4	46.0
81.6	43.8	36.9	44.7	35.6	44.2	44.5	45.0
82.6	44.2	37.1	44.9	35.7	44.4	44.8	45.1
84.8	47.2	37.5	45.3	36.0	44.8	45.4	45.5
85.6	45.6	37.6	45.4	36.1	45.0	45.7	45.6
96.2	46.6	39.5	47.1	37.4	46.7	48.7	47.3
46.4	35.2	29.5	37.9	30.0	36.7	32.2	37.9
65.8	40.8	33.8	41.9	33.4	41.2	39.4	42.4
73.9	41.6	35.4	43.4	34.5	42.8	42	43.7
87.6	44.5	38.0	45.7	36.3	45.3	46.3	45.9
93.1	45.4	38.9	46.6	37.0	46.2	47.9	46.8
95.3	45.2	39.3	46.9	37.3	46.6	48.5	47.1
100.6	45.8	40.2	47.7	37.9	47.4	50	48.0
102.1	46.1	40.4	47.9	38.1	47.7	50.4	48.2
102.8	46.7	40.6	48.0	38.1	47.8	50.6	48.3
106.3	48.4	41.1	48.5	38.5	48.3	51.5	48.9
104.2	46.3	40.8	48.2	38.3	48.0	51	48.6
92.8	45.8	38.9	46.5	37.0	46.2	47.8	46.7
94.6	47.3	39.2	46.8	37.2	46.5	48.3	47.0
94	46.3	39.1	46.7	37.1	46.4	48.1	46.9
96.6	46.5	39.5	47.1	37.4	46.8	48.9	47.4
91.5	45.9	38.7	46.3	36.8	46.0	47.4	46.5
93.6	47.1	39.0	46.7	37.1	46.3	48	46.9
91.7	46	38.7	46.4	36.8	46.0	47.5	46.6
119.9	49.1	43.3	50.4	39.9	50.3	55.1	51.1
125.6	50.9	44.1	51.1	40.5	51.1	56.5	52.0

Table 2
Comparison of values estimated with the values obtained from codes and using the experimental results reported by Gesoglu et al. [25]

Compressive strength	Elastic modulus (E_c)	ACI 363	CEB	NS 3473	Gesoglu [25]	Wee [22]	Fuzzy
77.2	47.1	36.1	44.0	35.0	43.1	43.4	44.3
71.5	48	35.0	43.0	34.2	41.3	42.3	43.3
66.5	46.8	34.0	42.1	33.5	39.6	41.3	42.5
70.7	47.3	34.8	42.9	34.1	41.0	42.2	43.2
61.8	45.4	33.0	41.2	32.7	38.0	40.3	41.8
68.9	47.6	34.5	42.5	33.8	40.4	41.8	42.9
59.1	40.9	32.4	40.6	32.3	37.0	39.7	41.4
62.2	45.4	33.1	41.3	32.8	38.1	40.4	41.9
75.8	43	35.8	43.8	34.8	42.6	43.2	44.0
67.7	48.2	34.2	42.3	33.6	40.0	41.6	42.7
53.6	46.2	31.2	39.5	31.4	35.0	38.5	40.5
57.9	44.5	32.2	40.4	32.1	36.6	39.5	41.2
92.9	46.4	38.9	46.6	37.0	47.8	46.2	46.8
94	48.3	39.1	46.7	37.1	48.1	46.4	46.9
97.7	47	39.7	47.3	37.6	49.2	47.0	47.5
102	48.8	40.4	47.9	38.0	50.4	47.7	48.2
93.7	50.5	39.0	46.7	37.1	48.0	46.3	46.9
86.2	47.1	37.7	45.5	36.2	45.8	45.1	45.7
87.9	43	38.0	45.8	36.4	46.4	45.4	46.0
82.7	45.4	37.1	44.9	35.7	44.8	44.4	45.1
79.1	44.7	36.4	44.3	35.3	43.7	43.8	44.6
85.3	45	37.6	45.4	36.1	45.6	44.9	45.5
86.6	46.1	37.8	45.6	36.2	46.0	45.1	45.8
85.5	44.3	37.6	45.4	36.1	45.6	44.9	45.6
91.1	46.8	38.6	46.3	36.8	47.3	45.9	46.5
96.7	53.2	39.5	47.1	37.4	48.9	46.8	47.4
99.7	47.6	40.1	47.6	37.8	49.7	47.3	47.9
91.2	49.3	38.6	46.3	36.8	47.3	45.9	46.5
83.8	45.9	37.3	45.1	35.9	45.1	44.6	45.3
87.1	47.7	37.9	45.6	36.3	46.1	45.2	45.8
93.2	46.2	39.0	46.6	37.0	47.9	46.2	46.8
85.1	44.7	37.5	45.3	36.0	45.5	44.9	45.5
86.9	46.1	37.8	45.6	36.3	46.1	45.2	45.8
90.7	48.1	38.5	46.2	36.7	47.2	45.8	46.4
89.5	47.6	38.3	46.0	36.6	46.8	45.6	46.2
87.8	45.4	38.0	45.8	36.4	46.3	45.3	45.9
90.3	45	38.4	46.2	36.7	47.1	45.8	46.3
95.2	50.8	39.3	46.9	37.3	48.5	46.6	47.1
92.2	50	38.8	46.4	36.9	47.6	46.1	46.7
97.6	49.3	39.7	47.3	37.5	49.1	47.0	47.5
87.5	48.5	38.0	45.7	36.3	46.2	45.3	45.9
87.2	41.1	37.9	45.7	36.3	46.1	45.2	45.9
80.4	43.2	36.7	44.5	35.4	44.1	44.0	44.8
86.5	44.2	37.8	45.5	36.2	45.9	45.1	45.7
83.9	44.3	37.3	45.1	35.9	45.2	44.7	45.3
80.9	44.6	36.8	44.6	35.5	44.2	44.1	44.8
84.5	45.3	37.4	45.2	36.0	45.3	44.8	45.4
85.7	45.1	37.6	45.4	36.1	45.7	45.0	45.6

First the input and output variables are divided into a number of subsets with simple triangular fuzzy membership functions. Generally, there are m^n fuzzy rules where m and n are the numbers of subsets and input variables, respectively. In the case, say, of two inputs X_1 and X_2 with m subsets each, the rule base takes the form of an output Y_k (k=1,2,. . .,m^2). If there are two input variables as X_1 with

Table 3
Comparison of values estimated with the values obtained from codes and using the experimental results reported by Ozturan [23]

Compressive strength	Elastic modulus (E_c)	ACI 318	TS 500	Fuzzy
14	15.6	17.7	26.2	19.9
16.9	20.5	19.4	27.4	21.7
16.2	23.3	19.0	27.1	21.3
17.1	26.3	19.6	27.4	21.8
18	28.8	20.1	27.8	22.4
18.5	30.1	20.3	28.0	22.7
21.8	20.9	22.1	29.2	24.9
23.2	23.9	22.8	29.7	25.8
25.8	28.6	24.0	30.5	27.4
27.3	32.9	24.7	31.0	28.0
30.3	35.9	26.0	31.9	29.4
29.6	36.8	25.7	31.7	29.1
17.9	18.0	20.0	27.8	22.3
19.6	23.1	20.9	28.4	23.4
19.4	30.3	20.8	28.3	23.3
20.9	23.9	21.6	28.9	24.3
21.2	26.5	21.8	29.0	24.5
23.9	30.5	23.1	29.9	26.3
23.6	32.1	23.0	29.8	26.1
24.2	33.6	23.3	30.0	26.5
31.8	25.5	26.7	32.3	30.1
32.2	27.4	26.8	32.4	30.3
27.1	24.7	24.6	30.9	27.9
30.6	28.6	26.2	32.0	29.6
29.6	31.6	25.7	31.7	29.1
35	35.6	28.0	33.2	31.7
32.8	36.7	27.1	32.6	30.6
36.6	39.3	28.6	33.7	32.5
38.4	26.6	29.3	34.1	33.5
35.7	30.1	28.3	33.4	32.1
42.7	34.1	30.9	35.2	35.9
36.8	29.3	28.7	33.7	32.6
37.5	32.6	29.0	33.9	33.0
40.1	28.4	30.0	34.6	34.4
47.7	29.6	32.7	36.4	38.6

“very small” and “small” fuzzy subsets and X_2, say, “medium” and “large” 2 subsets, then consequently there will be four rules as

R^1 IF X_1 is very small and X_2 is medium THEN Y_1
R^2 IF X_1 is very small and X_2 is large THEN Y_2
R^3 IF X_1 is small and X_2 is medium THEN Y_3
R^4 IF X_1 is small and X_2 is large THEN Y_4.

For each triggered rule the membership degrees for both X_1 and X_2 are computed and these are multiplied to give the weight W_k to be assigned to the corresponding output Y_k. Hence, the weighted average of the outputs from four rules is a single output, y, as

$$y = \frac{\sum_{k=1}^{4} W_k Y_k}{\sum_{k=1}^{4} W_k} \qquad (1)$$

Thus once the rule base is set up, values of the output can be computed from Eq.(1) for any combination of input variables fuzzy subsets. A very common method in deciding about the fuzzy rule base is to use sample data and derive the necessary rule base by the fuzzy inference procedure. This involves computing the weight of each rule triggered, accumulating weights and outputs for each rule and finally computing the weighted output for each rule.

A fuzzy rule base can be achieved step-by-step from sets of input and output data as follows:

1. try to model the problem with minimum number of input variables.
2. divide the range of each input variable into a number (usually 4–8 in practice but into m in general) parts to give fuzzy subsets each with a triangular membership function. Theoretically, the optimum number of fuzzy subsets can be found by minimising the total squared

Table 4
Comparison of values estimated with the values obtained from codes and using the experimental results reported by Turan [26]

Compressive strength	Elastic modulus (E_c)	ACI 318	TS 500	Fuzzy
31.4	30.4	26.5	32.2	30.1
27.77	29.1	24.9	31.1	28.3
28.48	26.8	25.2	31.3	28.7
29.39	33.0	25.6	31.6	29.1
29.43	31.5	25.7	31.6	29.1
26.37	30.0	24.3	30.7	27.7
28.53	29.0	25.3	31.4	28.7
32.59	32.4	27.0	32.6	30.6
28.75	29.0	25.4	31.4	28.8
29.94	30.2	25.9	31.8	29.4
29.81	27.5	25.8	31.7	29.3
27.95	30.8	25.0	31.2	28.4
27.28	26.5	24.7	31.0	28.1
27.74	25.6	24.9	31.1	28.3
27.46	25.2	24.8	31.0	28.2
27	27.2	24.6	30.9	28.0
28.52	27.3	25.3	31.4	28.7
26.44	26.5	24.3	30.7	27.7
22.1	21.8	22.2	29.3	25.6
27.11	23.9	24.6	30.9	28.0
26.34	24.0	24.3	30.7	27.6
26.05	24.9	24.1	30.6	27.5
27.84	25.3	25.0	31.1	28.4
28.92	26.8	25.4	31.5	28.9
25.73	25.7	24.0	30.5	27.4
27.8	26.0	24.9	31.1	28.3
28.59	27.5	25.3	31.4	28.7
27.91	26.2	25.0	31.2	28.4
20.59	23.9	21.5	28.7	24.9
18.4	21.9	20.3	27.9	23.8
23.41	26.3	22.9	29.7	26.2
29.93	30.4	25.9	31.8	29.4
22.92	26.5	22.6	29.6	26.0
25.26	28.1	23.8	30.3	27.1
23.65	27.2	23.0	29.8	26.4
27.39	27.09	24.8	31.0	28.1

error between the observations and predictions. However, similar to the number of subclasses for histogram construction in statistics where rather subjectively depending on the expert view, the number is chosen between 5 and 15, the number of fuzzy subsets has been established empirically between 4 and 8 in practical studies.

3. For each data point m (one value for each X_1, X_2 and Y) compute the membership values for $X_1(W_i)$ and $X_2(W_j)$ in each of the fuzzy subsets. Only one or two of each will be nonzero. Set membership values of less than 0.5 to zero.
4. Compute the weight of each rule for data point m by multiplying the membership values of X_1 and X_2 that correspond to that rule and squaring the result

$$W_k = \left[(W_i)(W_j)\right]^2 \tag{2}$$

5. Store the output Y_k along with the complete set of rule weights W_k.
6. Repeat with all the other data points.
7. Compute the weighted average with expression similar to Eq. (2) [20,21].

4. Fuzzy algorithm for prediction of elastic modulus of NSC and HSC

There is no doubt that the modulus of elasticity increases with an increase in the compressive strength of concrete, but there is no agreement on the precise form of the relationship. In normal strength concrete, the modulus of elasticity of concrete is affected by the modulus of elasticity of the aggregate and by the volumetric proportion of aggregate in the concrete, but it has limited influence on strength. In high strength concrete, the aggregate remains important for volume stability, but it also plays an important role on the strength and elastic modulus of concrete. Several attempts and different national building codes propose various formulas for an estimate concrete modulus of elasticity from its compressive strength. In practice, there are always sources of uncertainty of different types such as vagueness and ambiguities and/or errors attached to determining of elastic modulus of NSC and HSC. The advantage of fuzzy models is their ability to describe knowledge in a descriptive human-like manner in the form of simple rules using linguistic variables only. In addition, many parameters effected elastic modulus E_c of NSC and HSC can be taken into account easily by adding related rules to the corresponding fuzzy approach.

The application of fuzzy methodology is performed only for experimental results by Wee et al. [22] for HSC and Özturan [23] for NSC. The following relationship proposed by Lydon and Balendran [24] is used to estimate static elastic modulus E_c from dynamic elastic modulus E_d of NSC given by Özturan [23].

$$E_c = 0.83E_d \tag{3}$$

Again the main purpose of this study is to estimate elastic modulus of NSC and HSC from compression strength values. During the training of fuzzy model, experimental results of Wee [22] (Table 1) for HSC and results of Özturan [23] (Table 3) for NSC were used. The fuzzy model was tested on data given by Gesoglu [25] and Turan [26] for HSC and for NSC, respectively. Models and fuzzy results are presented in Tables 1–4.

For application of fuzzy approach, the compression strength f_c and elastic modulus E_c are first fuzzified into fuzzy subsets so as to cover the hole range changes. The compression strength fuzzy subsets for NSC and HSC as $C_1, C_2, C_3, \ldots, C_{13}$ are considered to have triangular membership functions represented in Fig. 2. Once the fuzzy rule base inference machine is set up, it is straightforward to adjust the fuzzy partition on computer until the best fit is obtained. Of course, domain of elastic modulus of NSC and HSC change is observed from experimental studies. The elastic modulus of concrete is considered maximum at 56 GPa and its subdivision to 13 subsets are labelled as $E_{c1}, E_{c2}, E_{c3}, \ldots, E_{c13}$ in increasing magnitude. This subdivision is considered as valid for hole range of experimental results. However, subsets of fuzzy changes in elastic modulus of NSC

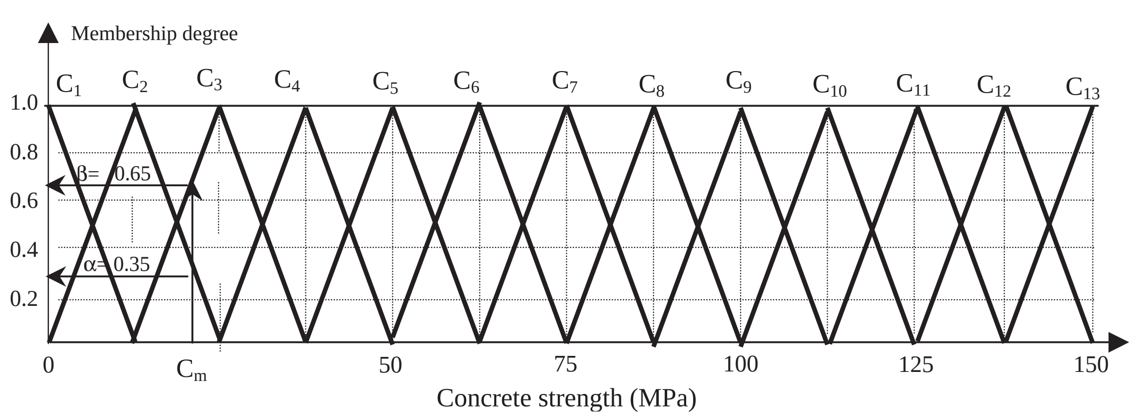

Fig. 2. Fuzzy subset membership functions for compression strength class.

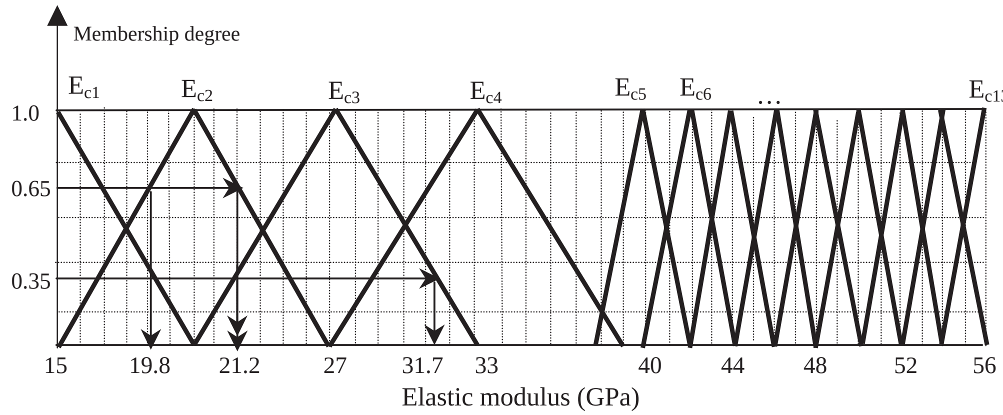

Fig. 3. Fuzzy subset membership functions for elastic modulus.

and HSC depend on many parameters such as aggregate characteristics. Since elastic modulus of concrete data changes slightly and smoothly over very range, the fuzzy portion of elastic modulus of concrete is expected to be slightly different for different test results. The compression strength and elastic modulus of concrete fuzzy subsets are combined according to Eq.(4) through the following fuzzy rule base.

$$R_i: \text{ IF } C \text{ is } C_i + C_{i+1}$$
$$\text{THEN } E_c \text{ is } E_{ci} + E_{ci+1} \ (i = 1, 2, 3, \ldots, 12) \quad (4)$$

Such a rule base for prediction elastic modulus of concrete is not used in any previous study in literature since the consequent part is in terms of two successive fuzzy subsets from the compressive strength of NSC and HSC. For a given compressive strength, C_m, there are two successive compressive strength fuzzy subsets (see Fig. 2). Once fuzzy subsets of elastic modulus of concrete and compressive strength are determined, it is possible to estimate elastic modulus from given concrete strength through the following steps where numerical calculations are given experimental data.

1. Locate the measured concrete strength C_m, say, as 23 MPa, on the horizontal axis of Fig. 2. It is possible to find two successive fuzzy subsets, for instance, C_2 and C_3, for which the membership does not vanish.
2. Find the membership degrees, namely, α=0.35 and β=0.65, corresponding to these two successive concrete strength fuzzy subsets C_3 and C_2, respectively. It is important to notice at this stage that, by definition, $\alpha+\beta$=1.0.
3. Enter the compressive strength fuzzy subset domain from fuzzy rule base by considering α and β membership degrees as shown, for instance, in Fig. 2.
4. In Fig. 3 for α=0.35, compressive strength should yield two values, namely, α_1=21,200 MPa and α_2=31,700 MPa from the corresponding E_{c3} fuzzy subset in the elastic modulus of concrete domain. Likewise, for β=0.65 MPa, β_1=19,800 MPa and β_2=21,200 MPa are obtained from E_{c2} as shown in the same figure. In fact, E_{c2} and E_{c3}

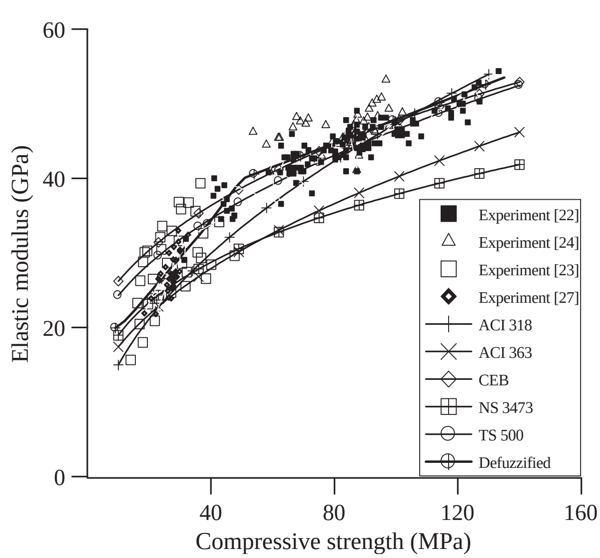

Fig. 4. Elastic modulus as a function of the compressive strength.

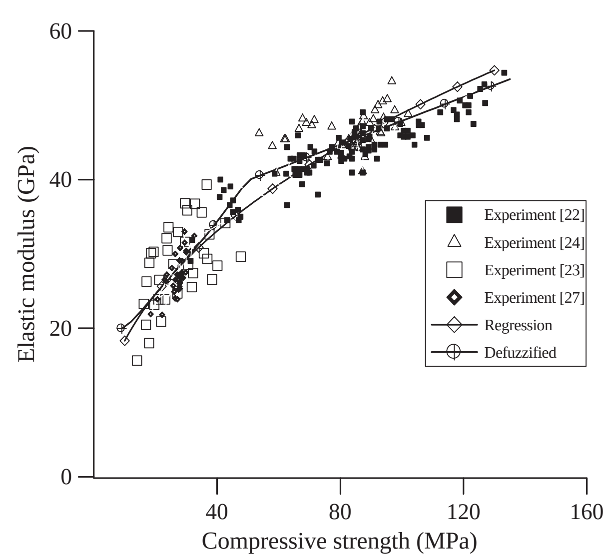

Fig. 5. Fuzzy prediction elastic modulus.

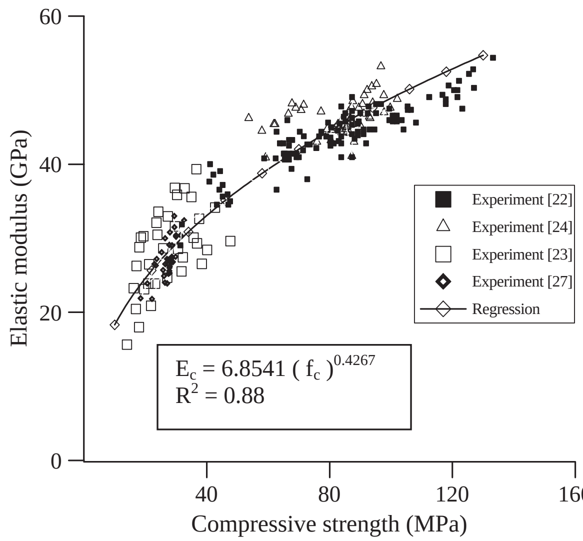

Fig. 6. Scatter diagram and regression curve.

together present the fuzzy consequent, i.e. answer to the elastic modulus estimation in the form of fuzzy subset.

5. For the defuzzification of fuzzy set, the arithmetical averages of the lower and upper values from each fuzzy subsets are calculated as

$$\bar{\alpha} = \frac{\alpha_1 + \alpha_2}{2} = \frac{21,200 + 31,700}{2} = 26,450 \text{ MPa and}$$
$$\bar{\beta} = \frac{\beta_1 + \beta_2}{2} = \frac{19,800 + 21,200}{2} = 205,000 \text{ MPa} \quad (5)$$

6. The elastic modulus of high strength concrete value, E_e, is calculated as the weighted average of these two simple arithmetic averages as

$$E_e = \alpha\bar{\alpha} + \beta\bar{\beta} = 0.35 \times 26,450 + 0.65 \times 20,500$$
$$= 22,583 \text{ MPa} \quad (6)$$

It is possible to execute these steps for each concrete strength which leads to either fuzzy subset estimation in a vague form or after its defuzzification by Eq. (6) to a single elastic modulus estimation value. The final results in the form of defuzzified elastic modulus estimations are indicated in Figs. 4, 5 and 6 respectively by comparing experimental data. It is obvious that these fuzzy results lie within the central parts of the scatter diagram for any given concrete strength. Hence, the proposed method of fuzzy estimation leads to elastic modulus estimations either in a vague form as fuzzy subset or as a defuzzified value, E_e. On the other hand, good agreement was found between computed the fuzzy logic model and measured values for the modulus of elasticity of NSC and HSC (see Fig. 4). Some experimental studies showed that characteristics of aggregate influence elastic modulus and strength of HSC [5–8]. In this study, because these experimental results were used to be compared, several fuzzy results are different from test results. It is also possible to take into account these effects by adding several rules obtained from detailed test results to the rule base given Eq. (4) for future studies. Furthermore, the fuzzy logic model has flexible ranges; the model may be according to the test results but this is not the case for the other models.

Table 6
Error measurement for NSC

	Experiment	ACI 318	TS 500	Regression	Fuzzy
Variance	18.4	8.0	3.8	7.6	9.4
RMSE		4.89	4.62	3.5	3.63

The error measures of the root mean square error (RMSE) and the variances were computed for each model and for experiment, and these are summarized in Tables 5 and 6. To be able to classify any model as a good model by comparing the error measures, it can be said that the model having the smallest RMSE and having the closest variance to the variance of experimental data is a good model. According to the comparison of RMSE calculations, as can be seen from Table 5, RMSE of fuzzy results is the smallest for HSC. RMSE of fuzzy and regression model are very close to each other for NSC and both are smaller than RMSE of ACI 318 and TS 500 codes.

5. Conclusions

Elastic modulus of NSC and HSC estimations from compression strength has been obtained so far in the literature either through the linear equation or via original power function. The herein developed fuzzy algorithm does not provide an equation but can adjust itself to any type of linear or non-linear form through fuzzy subsets of linguistic elastic modulus and compressive strength variables. It is also possible to augment the conditional statements in the fuzzy implications used in this paper to include additional relevant characteristics of aggregate variables that might increase the precision of elastic modulus estimation. The work in this paper may be extended by modelling the

Table 5
Error measurement for HSC

	Experiment	ACI 363	CEB	NS 3473	Wee [22]	Gesoglu [25]	Regression	Fuzzy
Variance	8.0	7.2	6.0	3.7	7.1	20.1	11.8	6.0
RMSE		8.58	2.27	9.95	2.48	3.41	2.67	2.18

participants which affect the characteristics of aggregate with fuzzy logic, the fractional volumes of aggregate and mortar, water–cement ratio, etc.

According to the comparison of RMSE calculations, as can be seen from Table 5, RMSE of fuzzy results is the smallest for HSC. RMSE of fuzzy and regression model are very close to each other for NSC and both are smaller than RMSE of ACI 318 and TS 500 codes. The variance of the fuzzy model is relatively close to the variance of the experimental data. Therefore the fuzzy model is said to be the good model. This shows the benefit of using fuzzy logic in this application.

The necessary fuzzy rule bases of the elastic modulus of NSC and HSC estimation from available experimental results are given and applied to some test data. The application of the proposed fuzzy subsets and rule bases is straightforward for any elastic modulus and compressive strength of NSC and HSC.

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