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Generalization of strength versus water–cementitious ratio relationship to age

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Abstract

In this paper, an attempt is made to generalize Abrams' law to any given age. It is intended to enhance the applicability of this law for practical applications by covering 3 to 365 days range. The result makes the prediction of concrete strength more convenient. Two novel methodologies, parameter-trend-regression and four-parameter-optimization methodology, have been proposed to extend the Abrams' formula and a power formula to any given age without collecting data at that age. Experimental data from several different sources are used to validate the reliability of these methodologies. As a result of the analysis presented in this study, a set of generalized water–cementitious ratio formulas is proposed for concrete with limited replacement percentage of fly ash. It is shown that the generalized formulas agree with the experimental data better than the original formulas do.

Keywords: Concrete; Compressive strength; Modeling

1. Introduction

Abrams' water–cement ratio pronouncement of 1918 has been described as the most useful and significant advancement in the history of concrete technology. He proposed that the strength of a fully compacted concrete processed with a specific type of aggregates at a given age cured and at a prescribed temperature, is dependent primarily on the water–cement ratio. Abrams suggested a mathematical relationship between concrete strength and water–cement ratio as [1]

$$f_c' = \frac{A}{B^x} = AB^{-x} \quad (1)$$

where f_c' is the compressive strength of the concrete; A and B are experimental parameters for a given age, material, and cure conditions; and x is water–cement ratio by mass

$$x = \frac{W}{C} \quad (2)$$

where W is the water content; C is the cement content.

Because the parameter A controls the maximum strength regardless of the water–cement ratio, it is called the magnitude parameter in this paper. On the other hand, because the parameter B controls the curvature of the strength versus water–cement ratio curve, it is called the curvature parameter in this paper.

For an average Portland cement concrete cured under normal temperature and moisture, Abrams gave the relationship between compressive strength and water–cement ratio as [1]

$$f_{c,7}' = \frac{63.45}{14^x} \quad (3)$$

for the 7-day strength (MPa), and

$$f_{c,28}' = \frac{96.55}{8.2^x} \quad (4)$$

for the 28-day strength (MPa).

When the above relationships were formulated by Abrams, the use of concrete admixtures like fly ash and granulated blast-furnace slag (GBFS) was virtually unheard of. The only cementitious material was cement. These materials are now used in concrete for economy and ecology reasons. They do not add much strength in the short term, but greatly enhance the long term strength gain. Since the use of fly ash and GBFS was not popular in concrete mix design at the time the water–cement ratio law was proposed, it is not clear whether the water–cement ratio law is applicable to concrete mixes with fly ash or GBFS. Many studies have shown that when the water–cementitious ratio is

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Table 1
Ranges of components of data sets

Component content	Minimum weight (kg/m^3)	Maximum weight (kg/m^3)	Average weight (kg/m^3)	S.D. of weight (kg/m^3)
Cement	102	540	281	105
Fly ash	0	200	54	64
Slag	0	359	74	86
Water	122	247	182	21
Superplasticizer	0	32.2	6.2	6.0
Coarse aggregate	801	1145	973	78
Fine aggregate	594	993	774	80

used instead of water–cement ratio as the basis for mix designing, strength prediction becomes more accurate [1,2]. Moreover, Babu and Rao [2] made a comprehensive study of the design of fly ash concretes. They studied the cementitious efficiency of fly ash relative to cement as measured by the effect of the ash on the water–cement ratio. The efficiency factor K was defined as the ratio of the mass of cement to the mass of the fly ash when they had equivalent effect on the water–cement ratio. Their study was an effort directed towards a quantitative understanding of the effect of fly ash in concrete. After a thorough evaluation, a set of value was proposed for K at various ages from 7 to 90 days, and with various percentages of replacement from 15% to 75%. The range of K proposed is rather wide. For example, the proposed K is 0.13 at the age of 7 days with 75% percentage of replacement and the K is 1.40 at the age of 90 days with 15% percentage of replacement. Based on these results, the Eq. (2) can be rewritten in the following form

$$x = \frac{W}{C + KF + S} \tag{5}$$

where x is water–cementitious ratio; W is water content; C is cement content; F is fly ash content; S is GBFS content; K is efficiency factor.

Although Abrams' water–cement ratio law was seriously criticized for according to its inter filler role to the aggregate in a concrete mix, experimental data have shown the practical acceptability of this rule within wide limits. However, a few deviations have also been reported. Much work has been done on the modification or generalization of Abrams' formula to the composition of material [1,3,4]. Nevertheless, little research has been done on the formula with respect to the age of the concrete. Currently, the only ways to predict the concrete strength at any given age are (1) to collect a lot of data at that age, then build a specific formula, and (2) to use a time factor (a function of age) multiplied by specific age strength (usually 28-day strength) to estimate the strength at a given age. In this paper, novel methodologies will be proposed to solve this problem. The central idea is if parameters A and B are functions of age, then it is possible to build regression equations for parameters A and B based on age. Therefore, the Abrams' formula can be generalized to the following form

$$f'_{c,t} = \frac{A_t}{B_t^x} = A_t B_t^{-x} \tag{6}$$

where generalized parameters A_t and B_t are functions of age.

Although Abrams' formula is good to predict concrete strength, a few other types of formula have been studied. In this paper, the power formula will be examined. The basic form is as follow

$$f'_c = Ax^{-B} \tag{7}$$

and the generalized form is as follows

$$f'_{c,t} = A_t x^{-B_t} \tag{8}$$

where generalized parameters A_t and B_t are functions of age.

To obtain the experimental parameters A and B, test data of concretes containing cement, fly ash and GBFS will be assembled for analysis, and a comprehensive statistical analysis will be done. The experimental data were taken from several publications [5–15]. In all, about 1400 concretes from the above investigations were evaluated. The efficiency factor of fly ash is dependent on age and percentage of replacement and it may have some effects on concrete strength formulas. To focus our study on generalizing the Abrams' law to any given age, some of the concretes were deleted from the data due to high percentages of replacement (over 50%) and a value of 1.0 is adopted for K in this study. Finally, 1030 concretes made with ordinary Portland cement and cured under normal conditions were evaluated. Tables 1 and 2 present the general details of the concretes evaluated in this study. It was ensured that these will form a fairly representative group governing all the major parameters that influence the strength behavior of concrete and present the complete information required for such an evaluation.

To test the accuracy of the prediction formulas, the coefficient of determination, R^2, is adopted. The coefficient is a measure of how well the independent variables considered accounts for the measured dependent variable. The higher the value of R^2 is, the better the prediction relation is.

In this paper, several important problems will be solved, including

(1) Is there a trend of magnitude parameter A and curvature parameter B through the age?
(2) Which one is the more accurate formula, Abrams' or the power formula?

Table 2
Ranges of ratio of data sets

Ratio	Minimum	Maximum	Average	S.D.
Water/Cement	0.28	1.88	0.77	0.32
Water/Binder	0.27	0.90	0.48	0.12
Water/Solid	0.058	0.130	0.087	0.011
SP/Binder	0.000	0.057	0.015	0.013
Fly ash/Binder	0.00	0.50	0.14	0.17
Slag/Binder	0.00	0.61	0.17	0.20
(Fly ash + Slag)/Binder	0.00	0.74	0.31	0.21
Aggregate/Binder	2.37	9.85	4.53	1.24
Fine aggregate/ Coarse aggregate	0.53	1.16	0.80	0.11

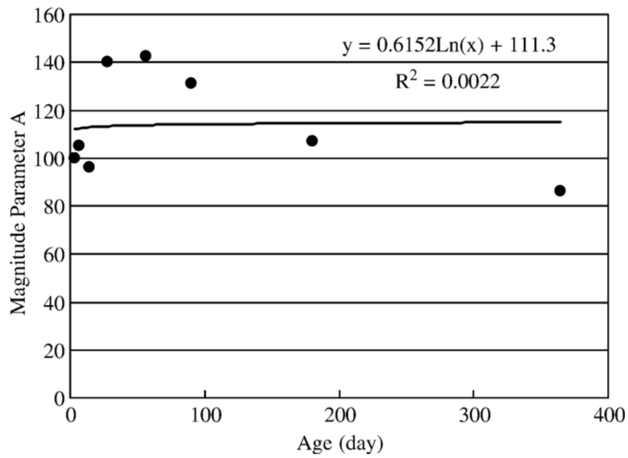

Fig. 1. Parameter *A* vs. age for Abrams' formula.

(3) How to generalize the Abrams' and power formulas to any given age without collecting data at that age?

2. Specific-age-parameter methodology

The specific-age-parameter methodology is to collect a lot of data at that age, then build a specific formula with a set of specific magnitude parameter *A* and curvature parameter *B* to satisfy the demand.

Figs. 1–6 show the results of magnitude parameter *A*, curvature parameter *B*, and compressive strength versus water–cementitious ratio using the methodology. Table 3 shows the evaluation of the methodology. In the table, the RMSE means root of mean square of error, and the MAPE means the mean absolute percentage error. From these results, the following conclusions may be made:

(1) The magnitude parameter *A* and curvature parameter *B* in Abrams' formula and the power formula are correlated with age. Moreover, the trend of parameter *B* is clearer than that of parameter *A*, and regularity of power formula is clearer than that of Abrams' formula.

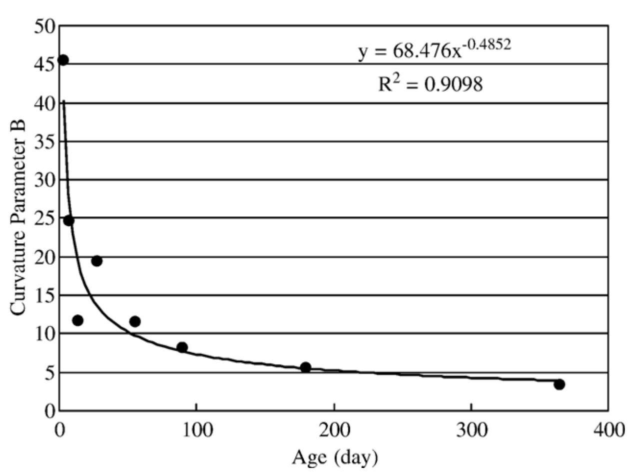

Fig. 2. Parameter *B* vs. age for Abrams' formula.

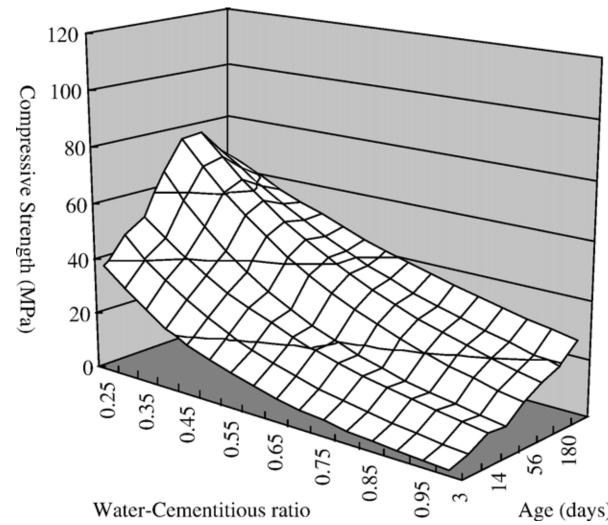

Fig. 3. Compressive strength vs. water–cementitious ratio for Abrams' formula using specific-age-parameter methodology.

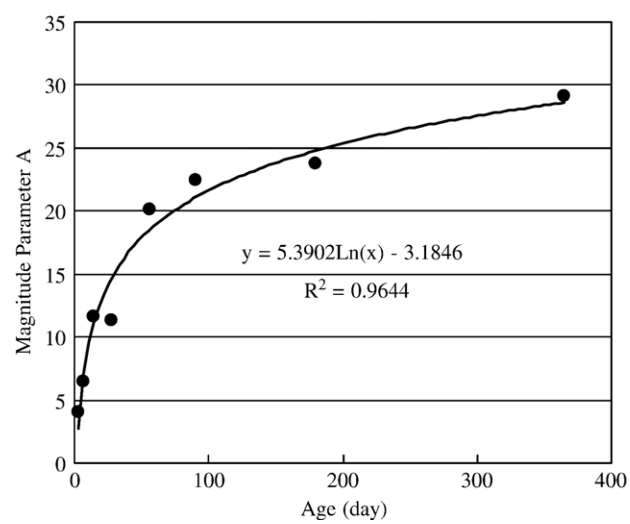

Fig. 4. Parameter *A* vs. age for power formula.

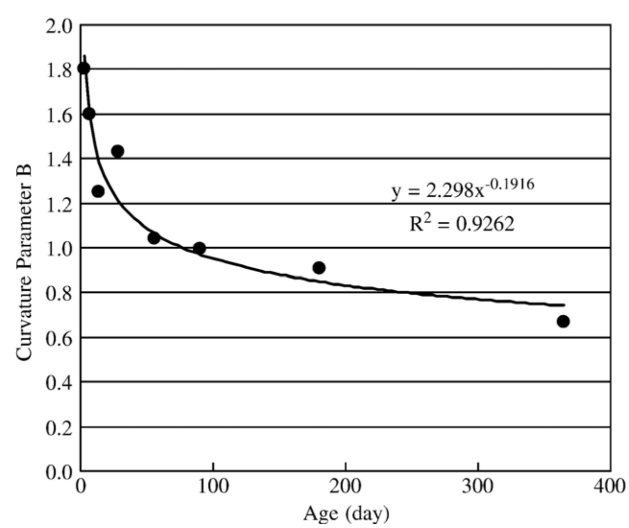

Fig. 5. Parameter *B* vs. age for power formula.

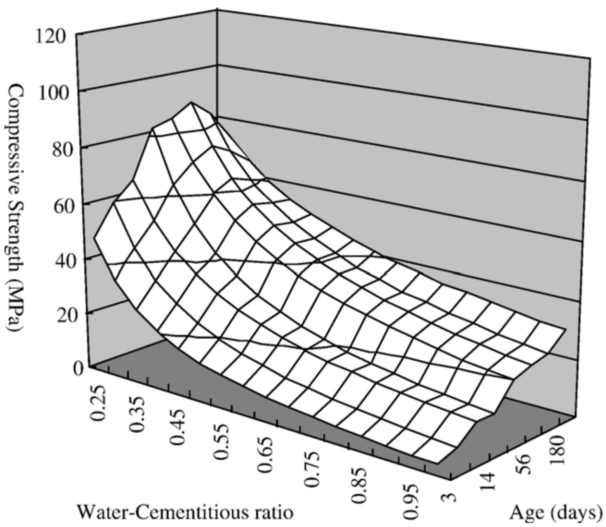

Fig. 6. Compressive strength vs. water–cementitious ratio for power formula using specific-age-parameter methodology.

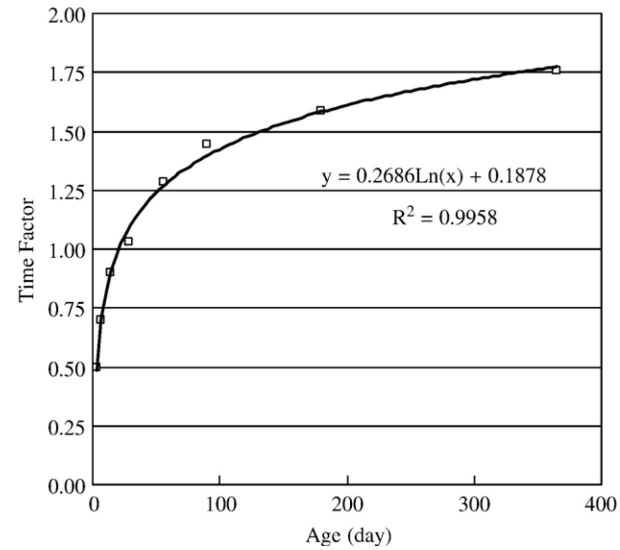

Fig. 7. Average of time factor vs. age for Abrams' formula.

(2) In Abrams' formula, the magnitude parameter A increases as the age increase from 3 days to 56 days, then decreases as the age increases from 56 days to 365 days. On the other hand, the curvature parameter B monotonically decreases through age.
(3) In the power formula, the magnitude parameter A monotonically increases through age. On the other hand, the curvature parameter B monotonically decreases through age.
(4) According to Figs. 3 and 6, the surfaces of compressive strength versus water–cementitious ratio and age are somewhat unreasonable in that the surfaces are not regular and smooth. However, both of the two surfaces show the phenomenon of strength retrogression in low water–cementitious ratio concrete at later ages.
(5) According to Table 3, the power formula is slightly more accurate than Abrams' formula.

3. Time-factor-decomposition methodology

An alternative way to predict the concrete strength at any given age is to use a time factor (a function of age) multiplied by specific age strength (usually 28-day strength) to estimate the strength at a given age; i.e.,

$$f'_{c,t} = C_t f'_{c,28} \quad (9)$$

Figs. 7–10 show the results of time factor curve and compressive strength versus water–cementitious ratio using the methodology. Table 4 shows the evaluation of the methodology. From these results, the following conclusions may be made:

(1) The time factor curve for Abrams' formula and that for power formula are very similar.
(2) Comparing Figs. 8 and 10 with Figs. 3 and 6, the surfaces of compressive strength versus water–cementitious ratio and age using time-factor-decomposition methodology are more reasonable than those using specific-age-parameter methodology because the surfaces are regular and smooth. However, both surfaces do not show the phenomenon of strength retrogression in low water–cementitious ratio concrete at later ages.
(3) According to Table 4, the power formula is as accurate as Abrams' formula.

Table 3
Evaluation of specific-age-parameter methodology

Evaluation	Abrams' formula	Power formula
R^2	0.798	0.805
RMSE (MPa)	7.50	7.36
MAPE (%)	15.7	15.4

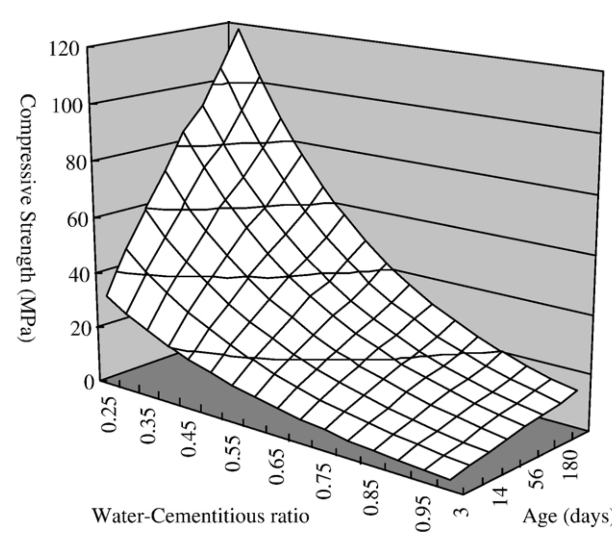

Fig. 8. Compressive strength vs. water–cementitious ratio for Abrams' formula using time-factor-decomposition methodology.

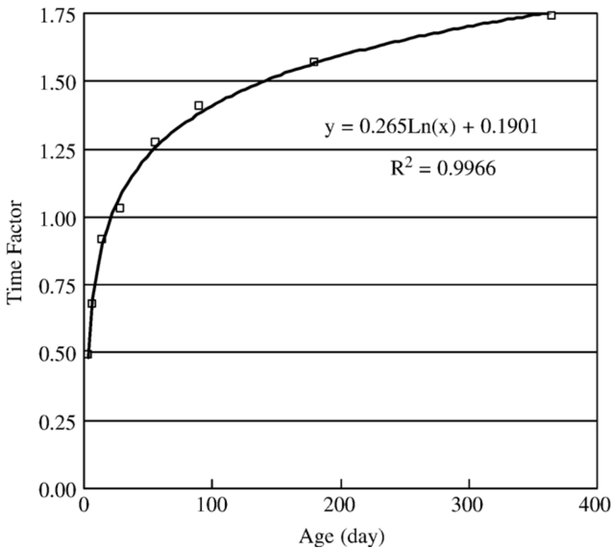

Fig. 9. Average of time factor vs. age for power formula.

(4) Comparing Table 4 with Table 3, the time-factor-decomposition methodology is slightly less accurate than the specific-age-parameter methodology. However, the former can be used to predict concrete strength at any given age.

4. Parameter-trend-regression methodology

In this section, a new methodology is proposed to overcome the above mentioned shortcomings. The central idea is that there are trends of parameters A and B through age; therefore, it is possible to build regression equations for parameters A and B based on age. The methodology can be listed as follows

$$\text{Min} \sum_t \left(A_t - \hat{A}_t(t)\right)^2 \quad t = 3, 7, 14, 28, 56, 90, 180, 365 \text{ days} \tag{10}$$

Table 4
Evaluation of time-factor-decomposition methodology

Evaluation	Abrams' formula	Power formula
R^2	0.765	0.761
RMSE (MPa)	8.07	8.16
MAPE (%)	16.9	17.1

where

A_t = the parameter gotten from the specific−age − parameter methodology

$$\hat{A}_t(t) = a\ln(t) + b \tag{11}$$

and

$$\text{Min} \sum_t \left(B_t - \hat{B}_t(t)\right)^2 \quad t = 3, 7, 14, 28, 56, 90, 180, 365 \text{ days} \tag{12}$$

where

B_t = the parameter gotten from the specific −age − parameter methodology

$$\hat{B}_t(t) = ct^{-d} \tag{13}$$

Eqs. (11) and (13) are two-parameter (a and b; c and d) optimization problems, and can be solved with numerical methods. Figs. 1, 2, 4, and 5 show the regression curves and regression coefficients (a and b; c and d). Figs. 11 and 12 show the surfaces of compressive strength versus water–cementitious ratio and age using the methodology. Table 5 shows the evaluation of the methodology. From these results, the following conclusions may be made:

(1) Comparing Figs. 11 and 12 with Figs. 3 and 6, the surfaces of compressive strength versus water–cementitious ratio and

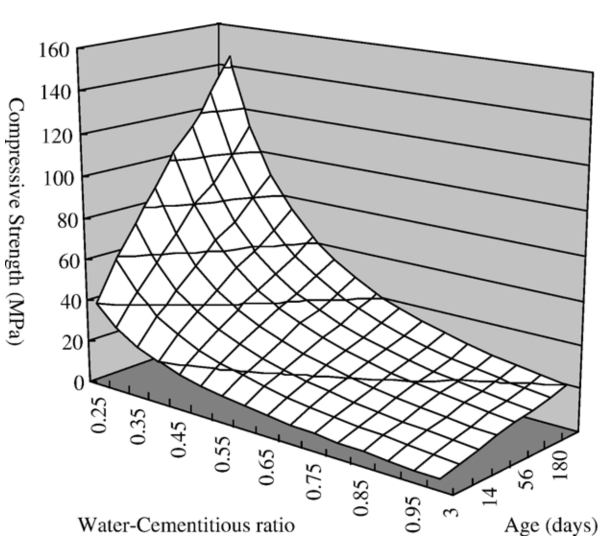

Fig. 10. Compressive strength vs. water–cementitious ratio for power formula using time-factor-decomposition methodology.

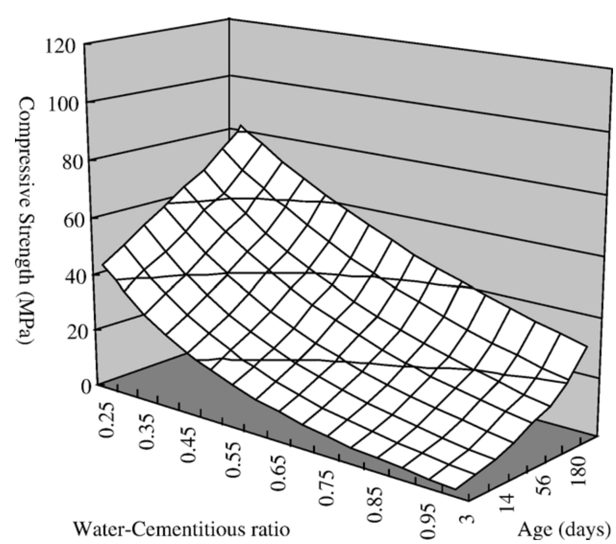

Fig. 11. Compressive strength vs. water–cementitious ratio for Abrams' formula using parameter-trend-regression methodology.

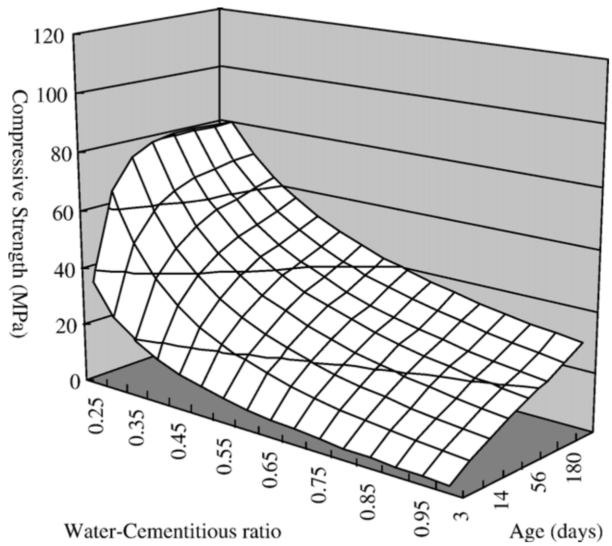

Fig. 12. Compressive strength vs. water–cementitious ratio for power formula using parameter-trend-regression methodology.

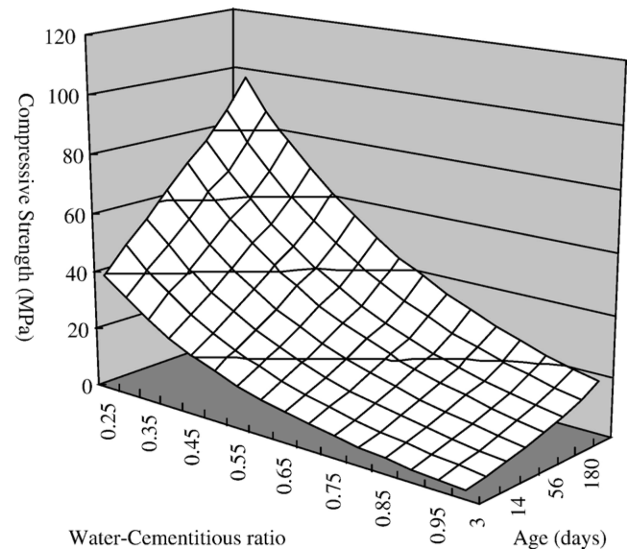

Fig. 13. Compressive strength vs. water–cementitious ratio for Abrams' formula using four-parameter-optimization methodology.

age using parameter-trend-regression methodology are more reasonable than that of specific-age-parameter methodology because the surfaces are regular and smooth. However, only the surface produced by the power formula shows the phenomenon of strength retrogression in low water–cementitious ratio concrete at later ages.

(2) According to Table 5, the power formula is more accurate than Abrams' formula.

(3) Comparing Table 5 with Table 3, the parameter-trend-regression methodology is slightly less accurate than the specific-age-parameter methodology. However, the former can be used to predict concrete strength at any given age.

5. Four-parameter-optimization methodology

In the parameter-trend-regression methodology, both parameter A and parameter B are governed by two regression coefficients, and are solved separately. However, there may be interactions between parameters A and B. Therefore, solving the regression coefficients for parameters A and B simultaneously may be a better approach. The methodology can be listed as follows

$$\text{Min} \sum_{t} \sum_{i=1_{i \in D_t}}^{nt} \left(f'_{c,i} - \hat{f}'_{c,i}(x,t) \right)^2 \tag{14}$$

Table 5
Evaluation of parameter-trend-regression methodology

Evaluation	Abrams' formula	Power formula
R^2	0.732	0.771
RMSE (MPa)	8.63	7.97
MAPE (%)	18.1	16.7

where

$\hat{f}'_{c,i}(x,t) = A_t B_t^{-x}$ for Abrams′ formula;

$\hat{f}'_{c,i}(x,t) = A_t x^{-B_t}$ for power formula;

$$A_t = a\ln(t) + b \tag{17}$$

$$B_t = ct^{-d} \tag{18}$$

The Eq. (14) is a four-parameter (a, b, c, and d) optimization problem, and can be solved with numerical methods. Figs. 13 and 14 show the surfaces of compressive strength versus water–cementitious ratio and age using the methodology. Table 6

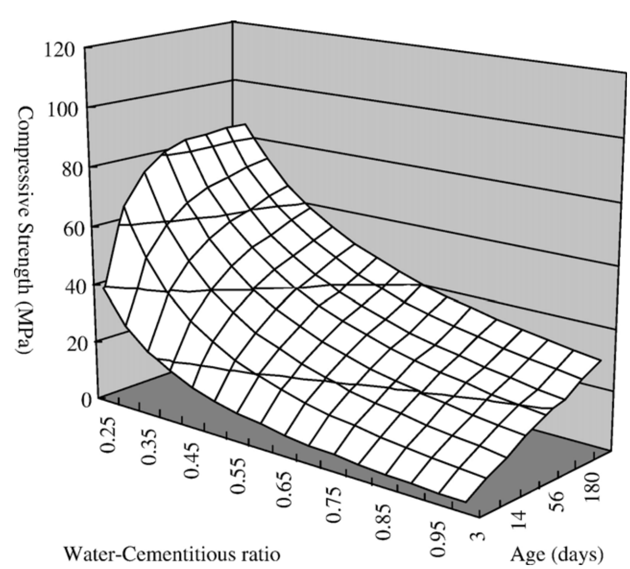

Fig. 14. Compressive strength vs. water–cementitious ratio for power formula using four-parameter-optimization methodology.

Table 6
Evaluation of four-parameters-optimization methodology

Evaluation	Abrams' formula	Power formula
R^2	0.755	0.779
RMSE (MPa)	8.26	7.84
MAPE (%)	17.3	16.4

shows the evaluation of the methodology. From these results, the following conclusions may be made:

(1) Comparing Figs. 13 and 14 with Figs. 3 and 6, the surfaces of compressive strength versus water–cementitious ratio and age using four-parameter-optimization methodology are more reasonable than that using specific-age-parameter methodology because the surfaces are regular and smooth. However, only the surface produced by the power formula shows the phenomenon of strength retrogression in low water–cementitious ratio concrete at later ages.
(2) According to Table 6, power formula is slightly more accurate than Abrams' formula.
(3) Comparing Table 6 with Table 3, the four-parameter-optimization methodology is slightly less accurate than the specific-age methodology. However, the former can be used to predict concrete strength at any given age.

6. Discussions

The comparisons of the four approaches are listed in Table 7. The parameter-trend-regression methodology and four-parameter-optimization methodology offer better insights into the concrete strength-versus-composition relationship through the presented implications of these formulas. For instance,

(1) strength effect of water–cementitious ratio

In Abrams' formula, the ratios of strength between two water–cementitious ratios at short-term and at long-term are as follows

$$r_s = \frac{A_s B_s^{-x_1}}{A_s B_s^{-x_2}} = B_s^{x_2 - x_1} \quad (19)$$

$$r_l = \frac{A_l B_l^{-x_1}}{A_l B_l^{-x_2}} = B_l^{x_2 - x_1} \quad (20)$$

where x_1, x_2 are the water–cementitious ratios; $x_1 < x_2$

Because the curvature parameter B in Abrams' formula decreases monotonically through age, i.e.

$$B_s > B_l \quad (21)$$

hence,

$$r_s > r_l, \quad (22)$$

it implies that the strength effect of water–cementitious ratio in short-term (with greater B) is more intensive than that in long-term (with smaller B). Similarly, in the power formula, the ratios are as follows

$$r_s = \frac{A_s x_1^{-B_s}}{A_s x_2^{-B_s}} = \left(\frac{x_2}{x_1}\right)^{B_s} \quad (23)$$

Table 7
Comparisons of the four methodologies

Methodology	Model
Specific-age-parameter	Find A_t, B_t to $\text{Min} \sum_{i=1, i \in D_t}^{n_t} \left(f'_{c,i} - \hat{f}'_{c,i}(x)\right)^2$ $t = 3, 7, 14, 28, 56, 90, 180, 365$ days where $\hat{f}'_{c,i}(x) = A_t B_t^{-x}$ for Abrams' formula $\hat{f}'_{c,i}(x) = A_t x_t^{-B_t}$ for power formula D_t = data set of the data at age t n_t = the number of data in D_t
Time-factor-decomposition	Find a,b to $\text{Min} \sum_t \left(\bar{C}_t - \hat{C}_t(t)\right)^2$ $t = 3, 7, 14, 28, 56, 90, 180, 365$ days where $\bar{C}_t = \sum_{i=1, i \in D_t}^{n_t} \frac{f'_{c,i}}{\hat{f}'_{c,28}(x)} / n_t$ $\hat{f}'_{c,28}(x) = A_{28} B_{28}^{-x}$ for Abrams' formula $\hat{f}'_{c,28}(x) = A_{28} x_{28}^{-B_{28}}$ for power formula $\hat{C}_t(t) = a\ln(t) + b$ A_{28}, B_{28}=the parameters gotten from the Specific–Age – Parameter Methology
Parameter-trend-regression	Find a,b to $\text{Min} \sum_t \left(A_t - \hat{A}_t(t)\right)^2$ $t = 3, 7, 14, 28, 56, 90, 180, 365$ days where A_t=the parameter gotten from the specific-age-parameter methodology; $\hat{A}_t$=$a\ln(t)$+b Find c,d to $\text{Min} \sum_t \left(B_t - \hat{B}_t(t)\right)^2$ $t = 3, 7, 14, 28, 56, 90, 180, 365$ days where B_t=the parameter gotten from the specific-age-parameter methodology; $\hat{B}_t(t)=ct^{-d}$
Four-parameter-optimization	Find a,b,c,d to $\text{Min} \sum_t \sum_{i=1, i \in D_t}^{n_t} \left(f'_{c,i} - \hat{f}'_{c,i}(x,t)\right)^2$ where $\hat{f}'_{c,i}(x,t) = A_t B_t^{-x}$ for Abrams' formula $\hat{f}'_{c,i}(x,t) = A_t x_t^{-B_t}$ for power formula $A_t = a\ln(t) + b$ $B_t = ct^{-d}$

$$r_l = \frac{A_l x_1^{-B_l}}{A_l x_2^{-B_l}} = \left(\frac{x_2}{x_1}\right)^{B_l} \tag{24}$$

Because the curvature parameter B in power formula decreases monotonically through age, i.e.

$$B_s > B_l \tag{25}$$

hence,

$$r_s > r_l, \tag{26}$$

it implies that the strength effect of water–cementitious ratio in short-term (with greater B) is more intensive than that in long-term (with smaller B).

(2) strength effect of age
In Abrams' formula, the ratios of strength between two ages with low and high water–cementitious ratio are as follows

$$r_{low} = \frac{A_l B_l^{-x_1}}{A_s B_s^{-x_1}} = \left(\frac{A_l}{A_s}\right)\left(\frac{B_s}{B_l}\right)^{x_1} \tag{27}$$

$$r_{high} = \frac{A_l B_l^{-x_2}}{A_s B_s^{-x_2}} = \left(\frac{A_l}{A_s}\right)\left(\frac{B_s}{B_l}\right)^{x_2} \tag{28}$$

Because in Abrams' formula the parameter B decreases monotonically through age, i.e.

$$\left(\frac{B_s}{B_l}\right) > 1 \tag{29}$$

and $x_2 > x_1 > 0$, therefore

$$r_{high} > r_{low}, \tag{30}$$

it implies that the strength effect of age in high water–cementitious ratio is more intensive than that in low water–cementitious ratio. Similarly, in power formula, the ratios are as follows

$$r_{low} = \frac{A_l x_1^{-B_l}}{A_s x_1^{-B_s}} = \left(\frac{A_l}{A_s}\right) x_1^{B_s - B_l} \tag{31}$$

$$r_{high} = \frac{A_l x_2^{-B_l}}{A_s x_2^{-B_s}} = \left(\frac{A_l}{A_s}\right) x_2^{B_s - B_l} \tag{32}$$

Because in power formula the parameter B decreases monotonically through age, i.e.

$$B_s - B_l > 0 \tag{33}$$

hence,

$$r_{high} > r_{low}, \tag{34}$$

it implies that the strength effect of age in high water–cementitious ratio is more intensive than that in low water–cementitious ratio.

7. Conclusions

From the results of the study reported herein, the following conclusions are reasonable:

(1) In Abrams' formula, the curvature parameter B monotonically decreases through age. In the power formula, the magnitude parameter A monotonically increases through age. On the other hand, the curvature parameter B monotonically decreases through age.
(2) In general, the power formula is slightly more accurate than the Abrams' formula. Moreover, the power formula can show the phenomenon of strength retrogression. Therefore, the power formula is the more accurate formula.
(3) Two novel methodologies, parameter-trend-regression and four-parameter-optimization methodology, have been proposed to generalize the Abrams' and power formula to any given age without collecting data at that age. As a result of the analysis presented in this study, a set of generalized water–cementitious ratio formulas is proposed for concrete with limited replacement percentage fly ash.
(a) Parameter-trend-regression methodology
Abrams' formula

$$f'_{c,t} = \frac{A_t}{B_t^x} = A_t B_t^{-x} \tag{35}$$

$$A_t = 0.615\ln(t) + 111.3 \quad \text{(MPa)} \tag{36}$$

$$B_t = 64.48t^{-0.4852} \tag{37}$$

Power formula

$$f'_{c,t} = A_t x^{-B_t} \tag{38}$$

$$A_t = 5.390\ln(t) - 3.185 \quad \text{(MPa)} \tag{39}$$

$$B_t = 2.298t^{-0.1916} \tag{40}$$

(b) Four-parameter-optimization methodology
Abrams' formula

$$f'_{c,t} = \frac{A_t}{B_t^x} = A_t B_t^{-x} \tag{41}$$

$$A_t = 16.34\ln(t) + 74.18 \quad \text{(MPa)} \tag{42}$$

$$B_t = 36.919t^{-0.2244} \tag{43}$$

Power formula

$$f'_{c,t} = A_t x^{-B_t} \tag{44}$$

$$A_t = 5.301\ln(t) - 2.417 \quad (\text{MPa}) \tag{45}$$

$$B_t = 2.1288t^{-0.1711} \tag{46}$$

The two novel methodologies provide improved tools to build formulas for the prediction of concrete strength at any given age, and the use of the formulas is as simple as that of the basic formulas.

(4) Although the evaluations of the two novel methodologies show that they are slightly less accurate than that of the specific-age-parameter methodology, because the former uses only four parameters (*a*, *b*, *c*, and *d*) and the latter uses 16 parameters (A_3, A_7, A_{14}, A_{28}, A_{56}, A_{90}, A_{180}, A_{365}, B_3, B_7, B_{14}, B_{28}, B_{56}, B_{90}, B_{180}, and B_{365}) to build the prediction model, the two novel methodologies may be more robust and reliable than the specific-age-parameter methodology. Besides, the surfaces of compressive strength versus water–cementitious ratio and age using these methodologies are more reasonable than that using specific-age-parameter methodology because the surfaces are regular and smooth.

These methodologies can be adjusted for concretes with large replacement percentage fly ash. Because the efficiency factor of fly ash might have some effects on concrete strength formulas and is dependent on age and percentages of replacement, the effects of the efficiency factor of fly ash should be considered in the future development.

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