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Residual stress–strain relationship for concrete after exposure to high temperatures

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Abstract

An experimental research is performed on the complete compressive stress–strain relationship for concrete after heating to temperatures of 100–800 °C. All concrete specimens are ϕ15 cm × 30 cm standard cylinders, made with siliceous aggregate. The heated specimens are tested at 1 month after they are cooled to room temperature. From the results of 108 specimens with two original unheated strengths, a single equation for the complete stress–strain curves of heated concrete is developed to consider the shape varying with temperature. Through the regression analysis, the relationships of the mechanical properties with temperature are proposed to fit the test results, including the residual compressive strength, peak strain and elastic modulus. Compared with the experimental curves, the proposed equation is shown to be applicable to unheated and heated concrete for different temperatures. In addition, the split-cylinder tests of 54 specimens are also carried out to study the relationship of splitting tensile strength with temperature.

Keywords: Temperature; Stress–strain curve; Compressive strength; Elastic modulus

1. Introduction

Concrete structures generally behave well in fires. Most fire-damaged concrete buildings can be repaired and put back to use even after severe fires. Certainly, the damaged structural members must be repaired to reach again the minimum strength, stiffness and ductility they ought to have had before the fires. When concrete is exposed to heat, chemical and physical reactions occur at elevated temperatures, such as loss of moisture, dehydration of cement paste and decomposition of aggregate. These changes will bring a breakdown in the structure of concrete, affecting its mechanical properties. Therefore, concrete members without visible damage may have reduced strength due to elevated temperatures. To evaluate and repair the fire-damaged concrete members, it is essential to understand the effect of temperature on the mechanical properties of concrete, especially the stress–strain relationships used to predict the entire behavior in a future strong earthquake.

Many studies have been made on the residual mechanical properties of concrete after exposure to elevated temperatures such as compressive strength, splitting tensile strength and elastic modulus [1–5]. The results obtained by different works in different countries are not easy to compare quantitatively with each other, because of the differences in the materials, specimen sizes and test conditions. In addition, very little information is available on the compressive stress–strain curves of concrete after exposure to elevated temperatures. Wu et al. [6] have tested 44 prismatic concrete specimens after heating to temperatures up to 600 °C and proposed a stress–strain equation for unheated and heated concrete with the same shape of the nondimensional curves for different temperatures. The purposes of this paper are to establish a database of the mechanical properties of concrete after heating to temperatures up to 800 °C and to propose a single equation of the complete stress–strain curve applicable to unheated and heated concrete for different temperatures. The compression and split-cylinder tests are carried out to examine the validity of the relationships of temperature with the residual compressive strength, corresponding peak strain, elastic modulus, splitting tensile strength and stress–strain curves.

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2. Experimental work

2.1. Specimens and test temperatures

All tests are carried out on standard concrete cylinders of 15 cm diameter × 30 cm height, which is more representative of the quality of normal concrete at room temperature. The specimens are made with the Portland cement of Type I and the siliceous aggregate commonly used in Taiwan. For compression tests, two kinds of compressive strengths are tested. Eight specimens of 40 MPa and four of 27 MPa are carried out for each of nine temperatures: room temperature, 100, 200, 300, 400, 500, 600, 700 and 800 °C. A total of 108 specimens are tested to obtain the complete stress–strain curves for different temperatures. For split-cylinder tests, two kinds of compressive strength are also tested. Three specimens of 32 MPa and three of 21 MPa are tested for each of the same nine temperatures. A total of 54 specimens are tested to provide the splitting tensile strength for different temperatures. The authors of this paper [7] have tested eight 30 cm × 45 cm × 300 cm RC columns exposed on all sides to the ISO-834 Standard fire for 2 and 4 h. The results showed that the maximum concrete temperatures are 745 °C at depth 3.5 cm from the exposed surface for 4-h exposure and 261 °C at the centre for 2-h exposure. Besides, for fire-damaged concrete members, the cover concrete with higher temperature may encounter spalling damage during fires or be chipped off before repairing. Consequently, the test temperatures ranging from 100 to 800 °C could satisfy the practical assessment.

2.2. Heating rates and furnace

Previous tests for heat treatments often employed the heating rates of 1–10 °C/min. The rate remains the same for different maximum temperatures [1,6]. However, the results of RC column fire tests [7] showed the higher the maximum temperature, the larger the rising rate of concrete temperature. The measured rates were in the range of 2–4 °C/min for the maximum temperatures of 251–745 °C. Therefore, the heating rates are decided to be from 1 to 4.5 °C/min with an increment of 0.5 °C/min respectively corresponding to the test temperatures from 100 to 800 °C with an increment of 100 °C.

An electrical furnace with 70 × 40 cm cross section × 40 cm height is used. The temperatures of the specimens are monitored by five K-type thermocouples: four attached to the specimen surface at mid-height with equal spacing and one embedded at the centre during casting. The rising rates of average surface temperatures are followed as closely as the assigned heating rates by adjusting the heat output of the electrical furnace.

2.3. Test setup

Fig. 1 shows the test setup. The tests are performed with a universal testing machine capable of 2000 kN. In the uniaxial compression test, a compressometer is attached to the specimen surface to measure the average strains over a central 20 cm length. The tests are performed at a constant displacement rate to gain the complete stress–strain curves of unheated and heated concrete. The tensile strength of heated concrete for different temperatures is obtained by the split-cylinder tests according to the ASTM C496 for unheated concrete.

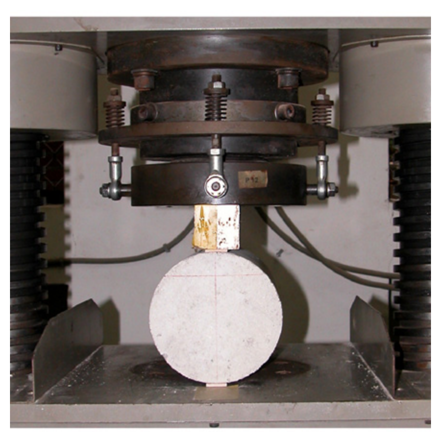
2.4. Test procedure

The specimens are heated at about 18 months after casting when they have the low moisture content to avoid spalling. Three specimens are heated in the furnace at the same time. When the average surface temperature of the specimens reaches 20 °C more than the specified test temperature, the surface temperature is kept constant for about 1.5–2.5 h by adjusting the heat output of the furnace until the centre temperature reaches the same specified temperature. This is followed by natural cooling down to room temperature in the furnace. Tests are carried out at about 1 month thereafter to ensure that the residual strengths of concrete after heating would be reduced to minima at the time of compression tests [3,4]. The maximum mean of the surface and centre temperatures during heating and cooling is taken as the maximum temperature of the specimen.

3. Experimental results

Figs. 2, 3, 4 and 5 show the effect of temperature on the residual compressive strength f'_{cr}, peak strain ε_{or} (corresponding to f'_{cr}), elastic modulus E_{cr} and splitting tensile strength f'_{tr}. The

(a) Compression test



(b) Split-cylinder test

Fig. 1. Test setup.

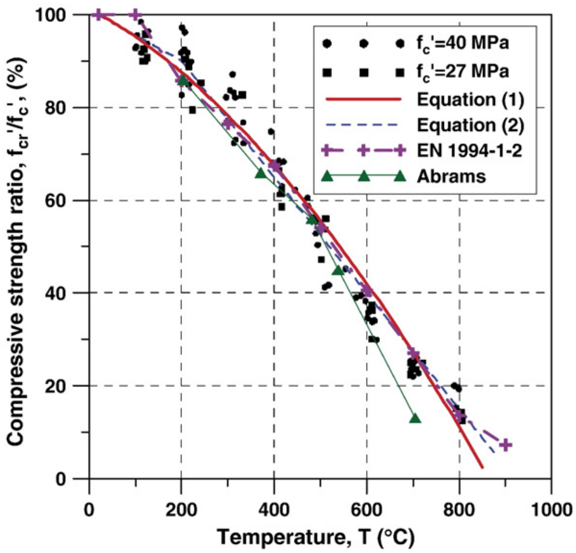


Fig. 2. Compressive strength after heating for different temperatures.

experimental results are compared with those obtained from other studies [2,3,5], EN 1994-1-2 [8] and EN 1992-1-2 [9] for siliceous concrete. All the data in these figures are normalized to average original unheated values: f_c', ε_o, E_c and f_t' respectively. For unheated specimens of this paper, the average peak strains are 1.73×10^{-3} and 2.11×10^{-3} corresponding to the average compressive strengths of 27 MPa and 40 MPa respectively. It should be mentioned that the curves given by EN 1994-1-2 in Figs. 2, 3 and 4 are for heated concrete having cooled down to the room temperature, and the curve given by EN 1992-1-2 in Fig. 5 is for concrete at elevated temperatures under heating.

3.1. Residual compressive strength

Fig. 2 shows that the compressive strength decreases continuously with an increase in temperature, and the decrease rate is lower for the temperature below 200 °C than above it. Besides, the original strength f_c' does not appear to have a significant effect on the normalized compressive strength after heating to various temperatures. The residual strength at 200 °C still retains about 90% of the original unheated value; however, the values at 400, 600 and 800 °C reduce to about 65%, 40% and 15%, respectively. The increase in decrease rate above 200 °C is mainly due to the fact that dehydration of hydrated cement paste occurs continuously from 105 to 850 °C, and crystalline transformation from α-quartz to β-quartz occurs between 500 and 650 °C [10]. Through the regression analysis, the relationship of the residual normalized compressive strength f_{cr}'/f_c' with temperature T can be expressed as Eq. (1) and a simplified bi-linear equation by Eq. (2). As shown in Fig. 2, the proposed Eqs. (1) and (2) both fit the test data well.

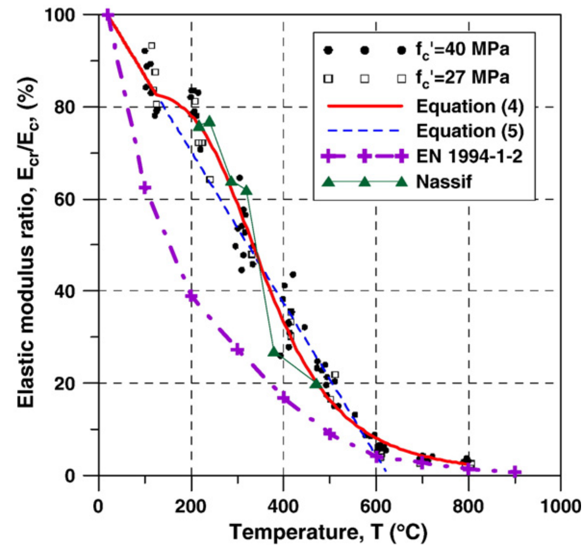


Fig. 4. Elastic modulus after heating for different temperatures.

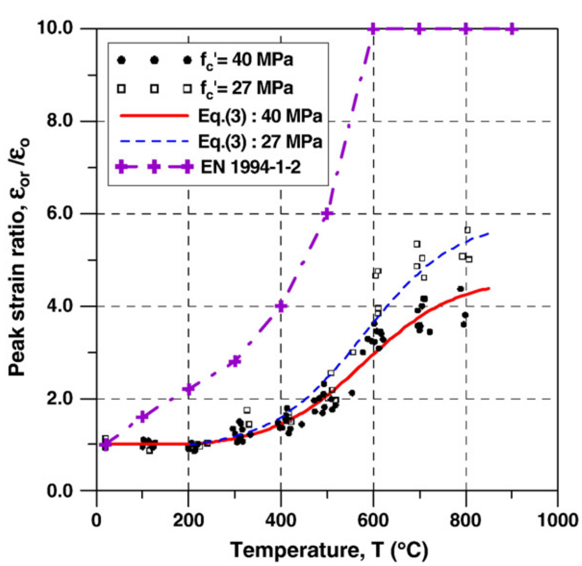


Fig. 3. Peak strain after heating for different temperatures.

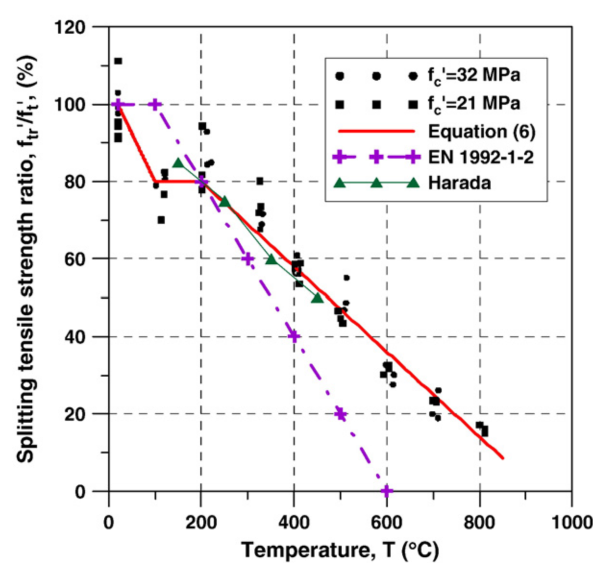


Fig. 5. Tensile strength after heating for different temperatures.

Besides, they are very close to the experimental results of Abrams [2] and the values given by EN 1994-1-2.

$$f'_{\mathrm{cr}}/f'_{\mathrm{c}} = 1.008 + \frac{T}{450\ln(T/5800)} \geq 0.0,\ 20\,^{\circ}\mathrm{C} < T \leq 800\,^{\circ}\mathrm{C} \tag{1}$$

$$f'_{\mathrm{cr}}/f'_{\mathrm{c}} = \begin{cases} 1.01-0.00055T & ,20^{\circ}\mathrm{C} < T \leq 200^{\circ}\mathrm{C} \\ 1.15-0.00125T & ,200^{\circ}\mathrm{C} \leq T < 800^{\circ}\mathrm{C} \end{cases} \tag{2}$$

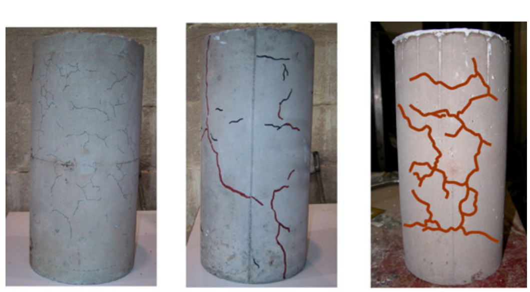
3.2. Peak strain

Fig. 3 shows that, for the temperature below 200 °C, the peak strain is approximately equal to the original unheated value. Above 200 °C, however, the peak strain increases rapidly with an increase in temperature, especially in the range of 500–600 °C. Besides, the original strength f'_c has a significant effect on the normalized peak strain for the temperature above 200 °C, and the lower the original strength, the greater the increase in the normalized peak strain. The effect is more evident for higher temperature. For example, the peak strain ratios at 400, 600 and 800 °C are respectively about 1.5, 3.2 and 4.0 for $f'_c = 40$ MPa, and 1.7, 4.0 and 5.2 for $f'_c = 27$ MPa. By the regression analysis, the relationship between the normalized peak strain $\varepsilon_{\mathrm{or}}/\varepsilon_{\mathrm{o}}$ and temperature T for different original strength f'_c can be expressed as Eq. (3).

$$\varepsilon_{\mathrm{or}}/\varepsilon_{\mathrm{o}} = \begin{cases} 1.00 & ,20^{\circ}\mathrm{C} < T \leq 200^{\circ}\mathrm{C} \\ (-0.1f'_{\mathrm{c}} + 7.7)\left[\frac{\exp(-5.8+0.01T)}{1+\exp(-5.8+0.01T)} - 0.0219\right] + 1.0, & 200^{\circ}\mathrm{C} < T \leq 800^{\circ}\mathrm{C} \end{cases} \tag{3}$$

The increase in the peak strain can be attributed to the cracks caused by thermal incompatibility of aggregate and cement paste during heating and cooling. Dougill [11] has indicated that the differential strains between the aggregate and cement paste can initially induce a small compressive stress in the paste. As the temperature increases, the compressive stress decreases and changes to a larger tensile stress. Roux [12] has concluded that the tensile stress may be large enough to cause cracks in concrete. Fig. 6 shows the surface cracks after cooling. A visible network of cracks is formed after heating to 300 °C. As the temperature increases, larger cracks occur at 500 °C and become numerous at 700 °C. As a result, the peak strain does not increase for the temperature below 200 °C, but, rather, increases rapidly above 200 °C.

As shown in Fig. 3, the curves proposed by Eq. (3) fit the test data reasonably. However, they are significantly different from the curve given by EN 1994-1-2, in which the peak strain during



(a) 300 °C (b) 500 °C (c) 700 °C

Fig. 6. Cracks on the specimen surface after cooling.

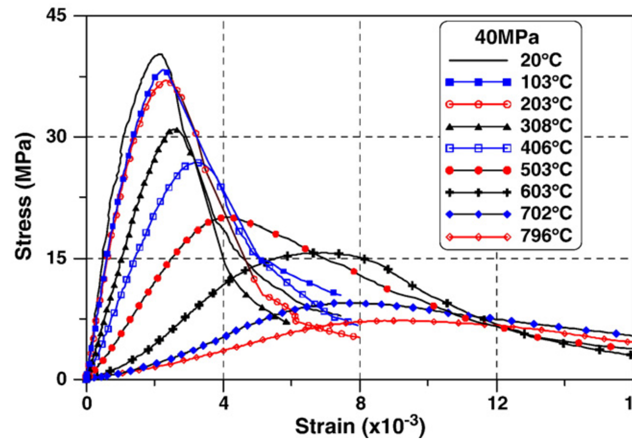


Fig. 7. Experimental stress–strain curves after heating for different temperatures.

cooling down to room temperature is maintained equal to the corresponding value during heating for the same maximum temperature. This is mainly due to the fact that the peak strain at elevated temperature during heating is including the transient strain.

3.3. Elastic modulus in compression

Generally speaking, the compressive strength decreases and peak strain increases with an increase in temperature as mentioned above. In other word, concrete is softening with increasing temperature. Therefore, in evaluating the deformation of fire-damaged concrete structure, it is important to consider the effect of temperature on the elastic modulus of the concrete. For comparative purposes, the elastic modulus of heated concrete in compression is taking the secant modulus at 40% of the peak stress from the experimental compressive stress–strain curve as for the unheated concrete. As shown in Fig. 4, the elastic modulus decreases with increasing temperature. For the same heat treatment, the reduction in elastic modulus is greater than that in the compressive strength. Besides, the original strength f'_c has no significant effect on the normalized elastic modulus after heating to various temperatures. The elastic moduli at 200, 400 and 600 °C are respectively about 80%, 40% and 6% of the original unheated value. Through the regression analysis, the relationship of the normalized elastic modulus $E_{\mathrm{cr}}/E_{\mathrm{c}}$ in compression with temperature T can be expressed as Eq. (4). Alternatively, the relationship can be expressed in an approximately linear form as Eq. (5), neglecting the values at higher temperature above about 700 °C. As shown in Fig. 4, the proposed Eqs. (4) and (5) both fit the test data well. Besides, they are also close to the experimental results of Nassif [5].

$$E_{\mathrm{cr}}/E_{\mathrm{c}} = \begin{cases} -0.00165T + 1.033 & ,20^{\circ}\mathrm{C} < T \leq 125^{\circ}\mathrm{C} \\ \dfrac{1}{1.2 + 18(0.0015T)^{4.5}} & ,125^{\circ}\mathrm{C} < T \leq 800^{\circ}\mathrm{C} \end{cases} \tag{4}$$

$$E_{\mathrm{cr}}/E_{\mathrm{c}} = -0.00165T + 1.033 \qquad ,20^{\circ}\mathrm{C} < T \leq 600^{\circ}\mathrm{C} \tag{5}$$

In EN 1994-1-2, the compressive stress–strain relationship for concrete during heating and cooling have the same shape of the nondimensional curves for different temperatures. Therefore, the reduction in elastic modulus is proposed the same as that in the

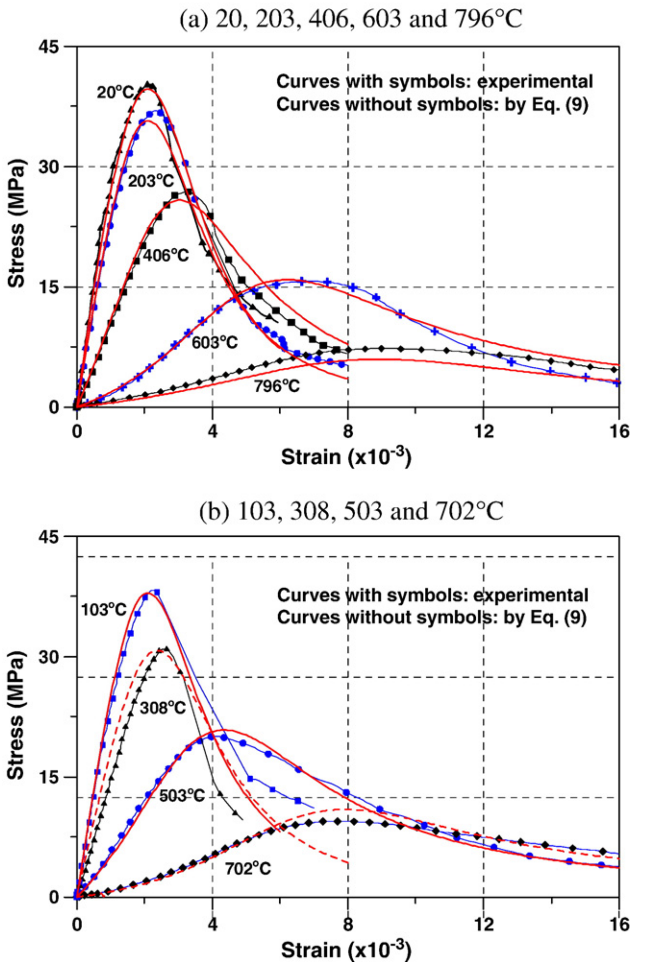


Fig. 8. Comparison of proposed stress–strain curves with experimental results for $f_c'=40$ MPa.

secant modulus at the peak point. Base on the peak stress and strain given by EN 1994-1-2, the elastic modulus can be calculated. As shown in Fig. 4, the values given by EN 1994-1-2 are less than those given by Eq. (5) due to the difference of the peak strain.

3.4. Residual tensile strength

Fig. 5 shows that except for 200 °C the normalized tensile strength decreases with an increase in temperature. For the same heat treatment, the reduction in tensile strength is greater than that in the compressive strength, especially for temperatures less than 400 °C. The original strength f_c' does not appear to have a significant effect on the normalized tensile strength after heating at various temperatures. Through the regression analysis, the relationship of the normalized tensile strength f_{tr}'/f_t' with temperature T can be expressed as Eq. (6).

$$f'_{tr}/f'_t = \begin{cases} 1.05-0.0025T & ,\ 20°C < T \le 100°C \\ 0.80 & ,100°C < T \le 200°C \\ 1.02-0.0011T \ge 0.0 & ,200°C < T \le 800°C \end{cases} \quad (6)$$

As shown in Fig. 5, the proposed Eq. (6) fits the test data well. Besides, it is also close to the experimental results of Harada [3]. The values given by EN 1992-1-2 are larger than those give by Eq. (6) for the temperature below 200 °C, but smaller above 200 °C.

3.5. Compressive stress–strain curves

The complete compressive stress–strain curves for different temperatures are shown in Fig. 7. As the temperature increases, the difference between initial tangent elastic modulus E_o and the secant modulus at peak stress E_p decreases. The ascending curves become more linear. When the temperature increases to above 600 °C, E_o may be less than E_p. There could be a pronounced concave-up curve at the beginning of loading due to the closing of pre-existing cracks caused by heating and cooling. Therefore, the shape of ascending curves for heated concrete is different from that for unheated concrete, and the shape varies with the temperature. Besides, as the temperature increases, the descending curves become flatter.

4. Development of the complete stress–strain relationship

4.1. Selection of the basic model equation

Numerous mathematical equations for the complete stress–strain relationship of unheated concrete have been developed [13–

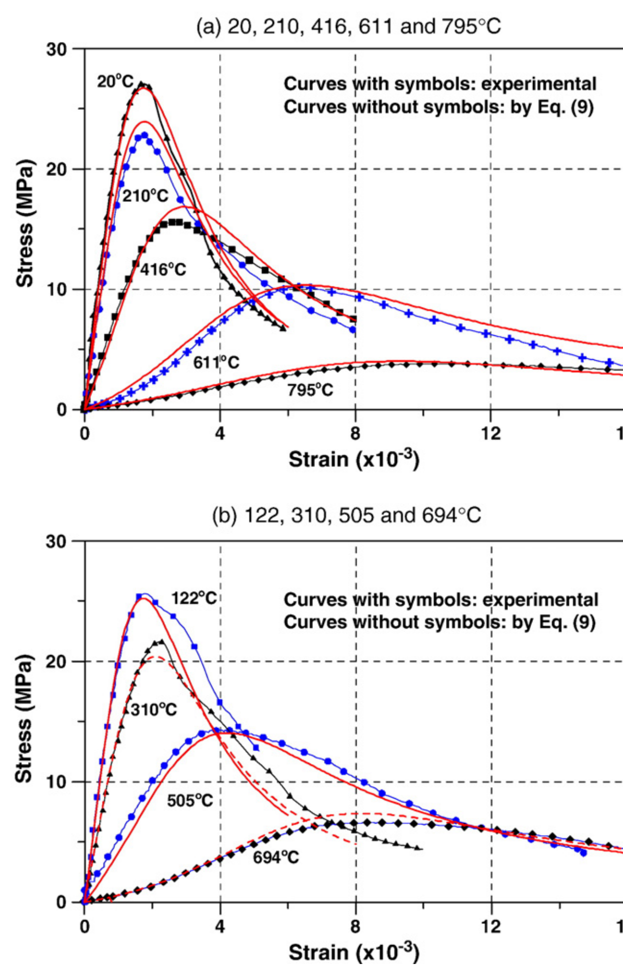


Fig. 9. Comparison of proposed stress–strain curves with experimental results for $f_c'=27$ MPa.

15]. In this paper, the shape suggested by Tsai [15] is introduced as the basic model equation. Tsai's model [15], Eq. (7), is a single equation which has the advantage of not needing another descending branch equation. In Eq. (7), M and n are the independent factors to control the shapes of the ascending and descending curves respectively. The value of M is given by the ratio E_o/E_p. When $M=n/(n-1)$, Eq. (7) reverts to the Eq. (8) suggested by Popovics [14].

$$f_c = \frac{M\left(\frac{\varepsilon_c}{\varepsilon_o}\right)}{1+\left(M-\frac{n}{n-1}\right)\left(\frac{\varepsilon_c}{\varepsilon_o}\right)+\left(\frac{1}{n-1}\right)\left(\frac{\varepsilon_c}{\varepsilon_o}\right)^n} f'_c \tag{7}$$

where f_c is the compressive stress; ε_c is the compressive strain; ε_o is the peak strain corresponding to the compressive strength f'_c.

$$f_c = \frac{n\left(\frac{\varepsilon_c}{\varepsilon_o}\right)}{n-1+\left(\frac{\varepsilon_c}{\varepsilon_o}\right)^n} f'_c \tag{8}$$

Although the Eq. (8) is simpler than the Eq. (7), Tsai's model equation [15] is more suitable to be extended to the heated concrete for the following reasons:

1. For unheated concrete, the ascending and descending curves are steeper with an increase in strength. The Popovics' model equation [14] only uses one factor n to control the shape of complete stress–strain curves for different compressive strength and has good correction with the experimental results. For heated concrete, as the residual strength decreases with increasing temperature, the ascending curve is more linear and the descending curve is flatter. The change in the shape of the stress–strain curves for heated concrete is different from those for the unheated concrete as the strength decreases with increasing temperature. Therefore, Tsai's model equation using two independent factors to control the shapes of the ascending and descending curves respectively is suitable to be extended to the heated concrete.
2. The value n of Eq. (8) is given by $E_o/(E_o-E_p)$, E_o must be larger than E_p. However, in Tsai's model equation, the value M to control the shape of ascending curves is given by E_o/E_p in which E_o could be less than E_p. This is suitable for concrete after heating to higher temperatures. When the value M is less than 1, the Eq. (7) can express the concave-up part of curve at the beginning of loading for heated concrete.

4.2. Suggestion of the model equation

From the characteristics of experimental stress–strain curves, this paper employs Tsai's model equation to both unheated and heated concrete for different temperatures by redefining some parameters. The following equation is proposed:

$$f_c = \frac{M\left(\frac{\varepsilon_c}{\varepsilon_{or}}\right)}{1+\left(M-\frac{n}{n-1}\right)\left(\frac{\varepsilon_c}{\varepsilon_{or}}\right)+\left(\frac{1}{n-1}\right)\left(\frac{\varepsilon_c}{\varepsilon_{or}}\right)^n} f'_{cr} \tag{9}$$

with

M	E_{or}/E_{pr}
n	$n_o(M/M_o)^{1.014-0.0007T}$
n_o	$[f'_c(\mathrm{MPa})/12]+0.77>1.0$
M_o	E_o/E_p
E_o	$5000\sqrt{f'_c(\mathrm{MPa})}$ [16]

where f'_{cr} is the residual strength after heating, given by Eq. (1); ε_{or} is the peak strain after heating, given by Eq. (3); E_{pr} is the secant elastic modulus at peak stress after heating, determined by Eq. (1) and Eq. (3); E_{or} is initial tangent elastic modulus after heating, shown in detail later. For unheated concrete, f'_{cr}, ε_{or}, E_{pr}, E_{or}, M, n are equal to f'_c, ε_o, E_p, E_o, M_o, n_o respectively.

Although the initial tangent modulus can be obtained by using an experiment technique, this is not practical due to the difficulty of the procedure to obtain it. Since the relationship of the secant elastic modulus with temperature is obtained and expressed as Eq. (4), the initial tangent modulus after heating can be given by the relationship between E_{cr} and E_{or}. For unheated concrete, E_{or} is almost identical with E_{cr}, and this relationship still exits for heated concrete after heating to temperatures below 500 °C. Once the exposure temperature is above 500 °C, the stress–strain curve becomes concave-up at the beginning of loading. E_{or} is less than E_{cr}. Therefore, the relationship of E_{or}/E_o with temperature is proposed the same as Eq. (4) for the temperatures below 500 °C. Above 500 °C, Eq. (4) needs to be multiplied by the following reduction coefficients: 1.0 for 500 °C; 0.6 for 700 °C; 1.0 for 800 °C. A linear interpolation may be used for 500 °C $<T<$ 700 °C and 700 °C $<T<$ 800 °C.

4.3. Verification of the proposed model equation

By redefining the parameters of M and n in Eq. (9), the change in the shape of the complete stress–strain curves for heated concrete could be considered with temperature. To verify the validity of the proposed model equation, the theoretical curves are compared with the experimental curves of unheated and heated concrete for different temperatures. The results for original unheated strengths of 40 MPa and 27 MPa are shown in Figs. 8 and 9, respectively. They show that the proposed model provides a good simulation to the experimental stress–strain curves.

5. Conclusions

The following conclusions can be drawn from the experimental results on the concrete specimens made with siliceous aggregate after heating to temperatures up to 800 °C:

1. The test data indicate that the original compressive strength of concrete f'_c has no significant effect on residual percentage reductions in compressive strength f'_{cr}/f'_c, elastic modulus E_{cr}/E_c and tensile strength f'_{tr}/f'_t. However, it has to be mentioned that variation in the strength of the tested material was only limited.
2. The reductions after exposure to temperatures decrease one by one in this order: elastic modulus, tensile strength and compressive strength.

3. The relationships of the compressive strength, peak strain, elastic modulus and tensile strength with maximum exposed temperature are proposed in this paper and in good agreement with test results.
4. From experimental results of the complete stress–strain curves, as temperature increases, the ascending curve is getting more linear while the descending curve is getting flatter. At 600 and 700 °C, there are pronounced concave-up curves at the beginning of loading, due to the closing of pre-existing cracks caused by heating and cooling.
5. The single equation for the complete stress–strain curves of heated concrete is developed to consider the shape of stress–strain curves varying with temperature. Compared with the experimental curves, the proposed model equation is shown to be applicable to unheated and heated concrete for different temperatures.

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