

A methodology to assess robustness of SCC mixtures

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Abstract

The present paper highlights the importance of a robust SCC mixture for the successful introduction of this innovative technology in the concrete industry. A methodology to quantify SCC mixture robustness is also proposed. A central composite design was carried out to mathematically model the influence of five mixture parameters and their coupled effects on deformability, passing and filling abilities and compressive strength of SCC mixtures. The target SCC mix composition, to be applied during full-scale tests in a precast factory, was selected to be the central point in the factorial design. The mixture parameters suggested by the Japanese SCC-designing method were adopted. The derived models were used to estimate SCC properties while mixture parameters variations were simulated, based on daily fluctuations inherent from production process. SCC robustness was assessed by measuring how frequently the properties of simulated mixes fall inside the acceptance intervals.

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1. Introduction

Self-compacting concrete (SCC), initially developed in Japan [1], is an innovative type of concrete which main characteristic is to fill the forms and consolidate without the need of vibration. In the last years, there has been a growing interest on SCC technology among constructors and construction industry in several countries. The principal reasons for this interest concern the ease of placing this type of concrete in heavily reinforced areas difficult to access, the reduced effort in accomplishing some of the casting tasks and the significant reduction of the construction period. Along with these advantages, in terms of environment, this technology will enable a considerable reduction of the acoustic noise levels and the use of secondary raw materials.

Until now, in Portugal, many mixtures have been developed and tested in the laboratory but the experience of producing

SCC on site is limited. Within a national research project, “BACPOR” [2], which covered a wide range of applications and materials, some problems occurred on site during full-scale tests [3]. Variations in cement or mineral additives due to changes in the production process as well as changes in aggregate type, e.g. from one sand pit to another, were observed to cause large variations on properties of fresh SCC. Therefore it is of great importance to have a robust mixture, which is minimally affected by external sources of variability. In “The European Guidelines for Self-Compacting Concrete” the robustness checking is recognized as an important step in the SCC design process [4]. Since variability of most constituent materials can be translated by a change in water requirement, it is suggested in [4] that compositions with plus and minus 5 to 10 L of the target water content be tested and the respective changes in fresh state properties be measured. A robust SCC should tolerate these deviations, i.e. should maintain its fresh properties inside the specified limits [4]. But this approach seems too simplistic because it does not take into account the specific characteristics of the production center like the existing level of quality control, equipment performance, skills and knowledge of the personnel involved.

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SCC is a complex material exhibiting several sensitive interactions between the constituent materials [4] and further work is needed to better understand the effect of mixture parameters governing material performance. Many approaches to design SCC proceed by trial and error using conventional single factor experiments, which can mask true effects on the response of concrete [5]. A better approach to dealing with several factors is to conduct a factorial experiment. This is an experimental strategy in which factors are varied together, instead of one at a time [6]. This strategy was followed by other authors [5,7–10] to derive numerical models relating mixture parameters to the key properties of cementitious materials. These models were used to design and optimize the mixtures, to correlate different workability characteristics, to analyze the effect of changes in mixture parameters and to evaluate trade-offs between different constituent materials (for example, superplasticizer and viscosity agent).

The objective of the present paper is to develop a methodology that can be used to assess the robustness of an SCC mixture. First, the feasibility of using factorial design to establish numerical models describing SCC key properties based on mixture parameters is investigated. The central point in the factorial design corresponds to the target SCC mix to be applied during full-scale tests in a precast concrete factory. Then, the derived numerical models are used to compute a measure of the target SCC mix robustness based on simulations of mixture parameters. The proposed measure represents the probability that a SCC mix verifies the fulfilment of the acceptance criteria.

2. Experimental programme

2.1. Materials characterization

Crushed calcareous aggregate (1–12 mm), a siliceous natural fine sand (sand 1) with a fineness modulus of 2.47 (ASTM) and natural coarse sand (sand 2) with a fineness modulus of 2.87 were used (see Table 1). The specific gravity of the coarse aggregate, sand 1 and sand 2 were 2.61, 2.54 and 2.54, and absorption values were 1.31%, 1.10% and 0.96%, respectively, according to EN 1097-6:2000. In this study SCC mix was prepared with Portland cement (CEM I 52.5 R) and a mineral additive (limestone filler), with a specific gravity of 3.12 and 2.70, respectively. A polycarboxylate type superplasticizer was used having a specific gravity of 1.05 and 20.2% solid content.

2.2. Experimental design

A $2^{(5-1)}$ fractional factorial statistical design [6], corresponding to five parameters at two levels, was used to

establish models that describe key SCC properties. A fractional factorial design was selected since it involves fewer runs than the complete set of $2^5=32$ runs while it can still be used to obtain information on the main effects and on the two-factor interactions. In this design three factor and higher order interactions are assumed to be negligible. Since the central point in the factorial design applied in this study corresponds to a SCC mixture optimized in a previous study, the analysed region is relatively close to the optimum. In this situation a second order model is usually required to approximate the response because of the curvature that may be present in the true response surface. For this reason the factorial design ($2^4=16$ runs) was augmented with 10 axial runs plus 4 central runs, resulting in a central composite design that can be used to fit a second-order model [6]. The generic form of a second order model is:

$$y = \beta_0 + \sum_{i=1}^k \beta_i x_i + \sum_{i=1}^k \beta_{ii} x_i^2 + \sum_{i < j} \sum \beta_{ij} x_i x_j + \varepsilon \quad (1)$$

where y is the response; x_i are the independent variables; β_0 is the independent term; β_i , β_{ii} and β_{ij} are the coefficients of independent variables and interactions, representing their contribution to the response; ε is the random residual error term representing the effects of variables or higher order terms not considered in the model.

SCC mix proportions can be established based on the following parameters: water to powder volume ratio (Vw/Vp); filler to cement weight ratio (wf/wc); superplasticizer to powder weight ratio (Sp/p); sand to mortar volume (Vs/Vm); solid volume (Vap), as suggested by Okamura et al. [1]. An additional parameter must be considered when fine aggregate is a combination of two sands. In this work weight ratio (s1/s) sand 1 to total sand was used, resulting in five factors used in the modelling: Vw/Vp; wf/wc; Sp/p; s1/s and Vap. The volumetric ratio Vs/Vm was kept constant and equal to 0.462. The effect of each factor was evaluated at five different levels $-\alpha$, -1 , 0 , $+1$, $+\alpha$ as presented in Table 2. In order to make the design rotatable (i.e. the standard deviation of the predicted response is constant in all points at the same distance from the center of the design) the α value should be taken equal to $F^{1/4}$, where F is the number of points in the factorial part of the design [6]. In the case treated in this paper, this corresponds to taking α equal to 2.0. The parameters β_0 , β_i , β_{ii} and β_{ij} of the models are estimated by regression techniques and are valid for mixtures made within the range of values presented in Table 3.

2.3. Mixing sequence, testing methods and test results

Mixtures from the experimental plan were tested in a random order. The mixes were prepared in the laboratory in 25 L batches

Table 1
Grading of aggregates

Sieve size (mm)	0.074	0.150	0.297	0.59	1.18	2.38	4.75	6.30	9.5	12.5
Sand 1	0.0	5.3	19.2	48.1	81.5	98.9	100	100	100	100
Sand 2	0.9	4.5	14.7	38.7	66.6	88.1	99.9	100	100	100
Coarse aggregate	0.0	0.11	0.5	0.7	1.0	2.3	20.7	46.3	90.5	100

Table 2
Coded values for the factors used in the experimental design

Mix number	Point type	Vw/Vp	wf/wc	Sp/p	s1/s	Vap
0 (*)	Central	0	0	0	0	0
1	Factorial	−1	−1	−1	−1	1
2	Factorial	1	−1	−1	−1	−1
3	Factorial	−1	1	−1	−1	−1
4	Factorial	1	1	−1	−1	1
5	Factorial	−1	−1	1	−1	−1
6	Factorial	1	−1	1	−1	1
7	Factorial	−1	1	1	−1	1
8	Factorial	1	1	1	−1	−1
9	Factorial	−1	−1	−1	1	−1
10	Factorial	1	−1	−1	1	1
11	Factorial	−1	1	−1	1	1
12	Factorial	1	1	−1	1	−1
13	Factorial	−1	−1	1	1	1
14	Factorial	1	−1	1	1	−1
15	Factorial	−1	1	1	1	−1
16	Factorial	1	1	1	1	1
17	Axial	2	0	0	0	0
18	Axial	−2	0	0	0	0
19	Axial	0	2	0	0	0
20	Axial	0	−2	0	0	0
21	Axial	0	0	2	0	0
22	Axial	0	0	−2	0	0
23	Axial	0	0	0	2	0
24	Axial	0	0	0	−2	0
25	Axial	0	0	0	0	2
26	Axial	0	0	0	0	−2

(*) Central point was replicated four times to estimate the degree of experimental error.

and mixed in an open pan mixer. The mixing sequence consisted of mixing both sands and coarse aggregate with 1/4 of the mixing water during 2.5 min, waiting for 2.5 min for absorption, adding of the powder materials, followed by the rest of the water with the superplasticizer and finally mixing concrete during a further 8 min. Slump-flow, V-funnel and Box tests were then carried out to characterize fresh state. Details on equipment used for testing fresh concrete and on test procedures can be found in Ref. [4]. After fresh concrete tests, three standard 150 mm cubes were moulded to evaluate 28 days compressive strength ($f_{c,28}$). Concrete cubes were demoulded one day after casting and kept inside a chamber under controlled environmental conditions (Temp. = 20 °C and HR = 100%) until a compressive strength test was carried out at 28 days concrete age. The Slump-flow test was used to evaluate deformation capacity, viscosity and resistance to segregation of SCC (by visual observation). From this test final slump flow diameter (Dflow) and time necessary for concrete to reach a 50 cm diameter (T50) were recorded. The V-funnel test was used to assess viscosity and passing ability of SCC. Test flow time was recorded (Tfunnel). The Box test was used to assess the ability of concrete to pass through tight openings between reinforcing bars and filling ability; filling height was recorded (H). Dflow, T50, Tfunnel, H and $f_{c,28}$ were the selected concrete properties to be analysed and modelled.

The mix proportions and test results of the 30 mixes prepared as described above are summarized in Table 4. In this table W_c , W_f , W_{sp} , W_w , W_{s1} , W_{s2} and W_g represent the weight (by unit of

concrete volume) of cement, limestone filler, superplasticizer, free water, sand 1, sand 2 and coarse aggregate, respectively. From these results it may be observed that with this experimental plan a wide range of SCCs was covered with Dflow ranging from 505 to 750 mm, T50 ranging from 1.47 to 6.31 s, Tfunnel ranging from 6.59 to 28.47 s and $f_{c,28}$ ranging from 52.20 to 75.31 MPa. All mixes exhibited a filling height in the Box-test higher than 300 mm, meaning that blocking is not a critical aspect in the analysed region; this can be attributed to low maximum aggregate size and low coarse aggregate content. None of the mixes exhibited severe segregation.

3. Response models

In this work commercial software (Design-Expert) [11] was used to analyse the results for each response variable by examining summary plots of the data, fitting a model using regression analysis and ANOVA, validating the model by examining the residuals for trends and outliers and interpreting the model graphically.

3.1. Fitted models

For each response variable a quadratic model can be estimated from the central composite design data (see Eq. (1)). The model parameters are estimated by means of a multilinear regression analysis. It may happen however that, for some response variables, some of the terms may not be significant. Since model parameters are assumed to be normally distributed the significance of each factor on a given response can be evaluated using Student's t -test. A Backward elimination was used in this work to eliminate non-significant terms in the regression model [11], i.e. those terms that had a p -value greater than the chosen significance level (in this study, $\alpha=0.05$). The results of the estimated models, including the residual error term, along with the correlation coefficients, are given in Table 5. An analysis of variance showed that these models are significant when describing the effect of Vw/Vp; wf/wc; Sp/p; s1/s and Vap on the modelled responses.

Notice that the observed value of 14.44 in the variable T50 (marked with * in Table 4) is not typical of the rest of the data. This value was identified as an outlier in the statistical analysis and for this reason it has been excluded from the data when fitting the model. Residual analysis did not reveal any obvious model inadequacies or indicate serious violations of the normality assumptions, except in the case of Tfunnel. This problem was overcome after a variable transformation of the type $1/\sqrt{y}$, as indicated in Table 5.

Table 3
Correspondence between coded values and actual parameter values

Mixture parameter	−2	−1	0	+1	+2
Vw/Vp	0.727	0.791	0.855	0.919	0.983
wf/wc	0.432	0.470	0.508	0.546	0.584
Sp/p	0.020	0.021	0.023	0.025	0.026
s1/s	0.598	0.673	0.748	0.823	0.897
Vap	0.530	0.544	0.557	0.571	0.585

Table 4
Mix proportions and properties of fresh and hardened concrete specimens used in the experimental design

Mix	W_c (kg/m ³)	W_f (kg/m ³)	W_{sp} (kg/m ³)	W_w (kg/m ³)	W_{s1} (kg/m ³)	W_{s2} (kg/m ³)	W_g (kg/m ³)	Date	Dflow (mm)	T50 (s)	Tfunnel (s)	H (mm)	fc,28 (MPa)
0	380	193	13.23	165	600	202	810	08-03-2005	645.0	3.00	10.94	320	64.79
0	380	193	13.23	165	600	202	810	09-03-2005	665.0	2.84	10.53	335	62.16
0	380	193	13.23	165	600	202	810	10-03-2005	657.5	2.40	9.28	340	62.01
0	380	193	13.23	165	600	202	810	11-03-2005	650.0	2.66	10.03	337	62.16
1	400	188	12.56	157	534	260	830	08-03-2005	525.0	6.21	21.16	305	62.39
2	382	180	11.99	174	547	266	789	10-03-2005	692.5	1.75	8.16	340	59.70
3	388	212	12.79	160	547	266	789	11-03-2005	585.0	3.60	13.53	325	57.48
4	354	193	11.66	170	534	260	830	11-03-2005	650.0	2.22	8.41	327	57.46
5	410	193	14.94	160	547	266	789	08-03-2005	588.0	5.25	19.94	323	66.77
6	374	176	13.62	170	534	260	830	10-03-2005	693.0	2.06	9.03	338	60.98
7	379	207	14.52	157	534	260	830	09-03-2005	590.0	4.25	16.75	318	62.39
8	362	198	13.87	174	547	266	789	10-03-2005	743.0	1.47	6.94	330	58.86
9	410	193	12.85	160	667	144	789	09-03-2005	600.0	3.53	14.00	325	62.54
10	374	176	11.72	170	652	141	830	14-03-2005	660.0	2.57	7.66	338	58.66
11	379	207	12.49	157	652	141	830	09-03-2005	552.5	6.31	19.10	305	65.80
12	362	198	11.93	174	667	144	789	11-03-2005	695.0	1.78	8.44	340	57.56
13	400	188	14.59	157	652	141	830	14-03-2005	610.0	3.50	15.19	327	66.35
14	382	180	13.94	174	667	144	789	11-03-2005	707.5	1.68	7.37	340	67.34
15	388	212	14.86	160	667	144	789	11-03-2005	620.0	3.47	14.13	330	67.10
16	354	193	13.55	170	652	141	830	14-03-2005	747.5	1.50	6.59	340	57.47
17	356	181	12.37	178	600	202	810	11-04-2005	750.0	1.84	6.72	335	52.20
18	409	207	14.21	151	600	202	810	11-04-2005	505.0	14.44 (*)	28.47	305	75.31
19	360	210	13.16	165	600	202	810	12-04-2005	685.0	2.38	10.15	340	62.01
20	403	174	13.30	165	600	202	810	12-04-2005	640.0	2.72	11.84	337	65.03
21	380	193	15.21	165	600	202	810	12-04-2005	707.5	2.53	9.41	340	61.91
22	380	193	11.24	165	600	202	810	12-04-2005	595.0	4.40	11.78	327	59.81
23	380	193	13.23	165	719	82	810	11-04-2005	640.0	3.87	10.50	337	66.17
24	380	193	13.23	165	481	323	810	11-04-2005	650.0	3.06	9.91	337	59.42
25	371	189	12.92	162	586	198	850	11-04-2005	650.0	2.81	10.94	325	54.09
26	389	198	13.53	169	614	207	769	11-04-2005	650.0	3.22	10.56	340	59.25

3.2. Accuracy of the proposed models

Even though the majority of the fitted models presented relatively high correlation coefficients (see R^2 in Table 5) their

Table 5
Fitted numerical models

Response variable	Dflow (mm)	T50 (s)	[Tfunnel (s)] ^{-0.5}	H (mm)	fc,28 (MPa)
Model terms	Estimate	Estimate	Estimate	Estimate	Estimate
Independent	650.53	2.936	0.304	333.36	63.00
Vw/Vp	58.67	-1.291	0.054	8.12	-3.29
wf/wc	8.21	-0.110	0.006	NS	-1.11
Sp/p	23.5	-0.355	NS	2.79	1.25
s1/s	NS	-0.035	0.004	NS	1.26
Vap	-8.46	0.220	NS	-3.54	-0.67
(wf/wc) × (s1/s)	NS	0.344	-0.009	NS	NS
(Sp/p) × (Vap)	10.44	-0.451	NS	3.44	-1.24
(Vw/Vp) ²	-6.95	0.316	NS	-3.95	NS
(Vap) ²	NS	NS	NS	NS	-1.46
Residual error, ε					
Mean	0	0	0	0	0
Standard deviation	13.188	0.468	0.015	5.023	2.248
R^2	0.95	0.86	0.92	0.79	0.78

(NS) non-significant terms; (*) error term is a random and normally distributed variable.

accuracy must be verified. The results of four central points included in the experimental design, together with four additional central runs from a previous study (see data in Table 6) were analysed in order to estimate the experimental error. The corresponding mean value, standard deviation, coefficient of variation and estimated error (95% confidence interval) are presented in Table 7. The lowest coefficient of variation (2.6%) was associated to Dflow and the highest one (8.1%) was associated to T50. As can be observed in Tables 4 and 6 replicate runs of central points were spread out in time to get a rough check on the stability of the process during the experimental programme. The accuracy of the derived models can be assessed by comparing the residual standard deviation (see Table 5) and the standard deviation calculated from the

Table 6
Coded values and test results from a previous study

Vw/Vp	wf/wc	Sp/p	s1/s	Vap	Date	Dflow (mm)	T50 (s)	Tfunnel (s)	H (mm)	fc,28 (MPa)
0	0	0	0	0	14-02-2005	645.0	2.78	10.07	335	69.03
0	0	0	0	0	14-02-2005	630.0	3.10	10.69	320	69.17
0	0	0	0	0	14-02-2005	683.0	2.59	9.44	322	—
0	0	0	0	0	15-02-2005	670.0	2.72	10.09	318	62.82
1	0	0	0	0	15-02-2005	705.0	2.57	7.85	340	65.20
-1	0	0	0	0	15-02-2005	610.0	3.97	13.47	330	68.90
0	0	-1	0	0	15-02-2005	645.0	2.47	10.43	332	61.85
0	0	1	0	0	15-02-2005	697.5	2.66	10.72	325	66.03

Table 7
Statistics of the results for the central points

Central points ($n=8$)	Dflow (mm)	T50 (s)	$[T_{\text{funnel}} \text{ (s)}]^{-0.5}$	H (mm)	fc,28 (MPa)
Mean	655.7	2.76	0.314	328	64.6
Standard deviation	16.7	0.22	0.009	9.2	3.2
Coefficient of variation	2.6%	8.1%	2.9%	2.8%	5.0%
Estimated error*	± 11.6	± 0.15	± 0.006	± 6.3	± 2.4

(*) Corresponding to a 95% confidence level.

central points (see Table 7) [6]. A good fitting can be expected when residual standard deviation does not exceed the experimental error by far. In this study, the standard deviation measured on the central points was always higher or close to the residual standard deviation, except in the case of T50.

The accuracy of the proposed models was also analysed by comparing predicted-to-measured values obtained with the eight mixtures presented in Table 6. The ratio between predicted-to-measured values for Dflow, T50, Tfunnel, H and fc,28 ranged between 0.95 and 1.03, 0.76 and 1.33, 0.99 and 1.18, 0.97 and 1.05, 0.91 and 1.02, respectively. Again, these values indicate good accuracy for the established models except in the case of T50. The predicted-to-measured values of Dflow, T50, Tfunnel, H and fc,28 are shown in Fig. 1, respectively, with the prediction intervals corresponding to a 95% confidence level. In this figure the black dots represent the experimental design results and the white dots represent results obtained in a previous study (not used to derive the numerical models). One can observe that all points fall within or very close to the limits of the prediction intervals. Thus, one can expect the established models to be sufficiently accurate to predict the analysed fresh and hardened properties.

3.3. Individual and interaction effects

The estimates of the model coefficients presented in Table 5 give an indication of the relative significance of the various mixture parameters on each response. Naturally a negative coefficient means that the response variable will decrease if the given mixture parameter increases. The results in Table 5 clearly show that V_w/V_p exhibit the greatest effect on all five measured responses. The variables Sp/p and V_{ap} also influence SCC properties. Significant interaction effects were found between wf/wc and $s1/s$ on both T50 and $1/\sqrt{T_{\text{funnel}}}$ responses and between Sp/p and V_{ap} on all the analyzed responses except $1/\sqrt{T_{\text{funnel}}}$. The quadratic term in V_w/V_p was significant for Dflow, T50 and H responses. The quadratic term in V_{ap} was significant for the fc,28 response. The effects of selected mixture parameters on concrete responses were analyzed more in detail but such a discussion is out of the scope of the present work.

4. Robustness measure

4.1. Introduction

Based on Ref. [4] the definition of robustness can be generalized as the capacity of concrete to retain its performance

requirements (fresh and hardened properties, including durability) when small variations in the properties or quantities of the constituent materials occur. Fig. 2 illustrates this concept for the simplest case, when one is interested only in a single concrete property (Y) which depends on a single variable (X), representing the constituent materials effect. R_{sup} and R_{inf} denote the acceptance limits for Y and X can assume any value between X_{min} and X_{max} . In this figure the behaviour of two different concrete mixes *Mix A* and *Mix B* is described by two functions f and f' , respectively. Observing Fig. 2 one can easily conclude that *Mix A* is more robust than *Mix B*, i.e. *Mix A* allows for larger variations of X while maintaining Y inside the acceptance interval $[R_{\text{inf}}, R_{\text{sup}}]$. To increase the robustness of *Mix B* it would be necessary to reduce the range of the interval $[X_{\text{min}}, X_{\text{max}}]$ and this would only be accomplished by reducing constituent materials variations through more quality control, modernization of existing equipments, etc. In general, these improvements are difficult to implement and very expensive so a more robust mixture is preferred.

In reality, a large number of independent variables (X_i , $i = 1, \dots, n$) influence concrete properties (Y_i , $i = 1, \dots, n$) and a considerable amount of concrete properties must be assessed to verify the fulfilment of the performance criteria. In the present study, as mentioned before, five independent variables were considered, namely, $X_1 = V_w/V_p$; $X_2 = wf/wc$; $X_3 = Sp/p$; $X_4 = s1/s$ and $X_5 = V_{ap}$. Based on these variables numerical models were derived to describe $Y_1 = \text{Dflow}$, $Y_2 = \text{T50}$, $Y_3 = T_{\text{funnel}}$, $Y_4 = H$ and $Y_5 = \text{fc,28}$, which are the dependent variables. Once the acceptance limits for each dependent variable are established, if the typical fluctuations of V_w/V_p ; wf/wc ; Sp/p ; $s1/s$ and V_{ap} associated to production process have a known distribution one can consider the probability that the performance criteria is fulfilled, given by:

$$p = P\left(\bigcap_{i=1}^5 R_{\text{inf}}^i < Y_i < R_{\text{sup}}^i\right) \quad (2)$$

as a measure of the robustness of the SCC mix. Since this probability cannot be computed exactly, one can use the frequency of accepted intervals computed on a large sample:

$$f = \frac{\text{number of occurrences of the event } \left(\bigcap_{i=1}^5 R_{\text{inf}}^i < Y_i < R_{\text{sup}}^i\right)}{\text{sample size}} \quad (3)$$

to estimate Eq. (2), that is, to estimate the robustness of the SCC mix under study. As we will explain in Section 4.3, applying bootstrap re-sampling techniques enables improvement of this estimate and evaluating how accurately a statistic calculated from the observed data estimates the corresponding quantity for the whole population [12].

4.2. Constituent materials variations

Since this study was carried out prior to the industrial application of SCC at Maprel precast factory there was no available data of daily fluctuations concerning SCC production.

Therefore, records of target and measured weight values of each constituent material corresponding to 1.5 m³ batches of conventional concrete (C45/55) were used. These batches were all produced at Maprel precast factory, during one week, using the same materials that were used for the experimental factorial plan. A total of 132 observations were collected. Fig. 3 presents the observed deviations of the constituent materials. The deviations were calculated dividing the difference between the target and the measured values by the target value. Absolute deviations for aggregates were added up and presented as “total aggregates”. For the superplasticizer, no deviations were recorded. It may be observed, from Fig. 3, that deviations respect the limits imposed in the European Standard EN 206-1:2000. Some deviations remained of constant sign (positive in the cases

of water, sand 1 and coarse aggregate and negative in the case of sand 2) along the production process when both positive and negative deviations would be expected to occur randomly.

4.3. Bootstrap re-sampling

A bootstrap sample is obtained by randomly sampling n times, with replacement, from the original N data points. The sampling process consists of randomly generating j_1, j_2, \dots, j_n integers, each of which equals any value between 1 and N with probability $1/N$. These integers determine which members of the original data set are selected to be in the random sample. This process allows a simple member to appear once, more than once or never. For each bootstrap sample (of size n) the statistic

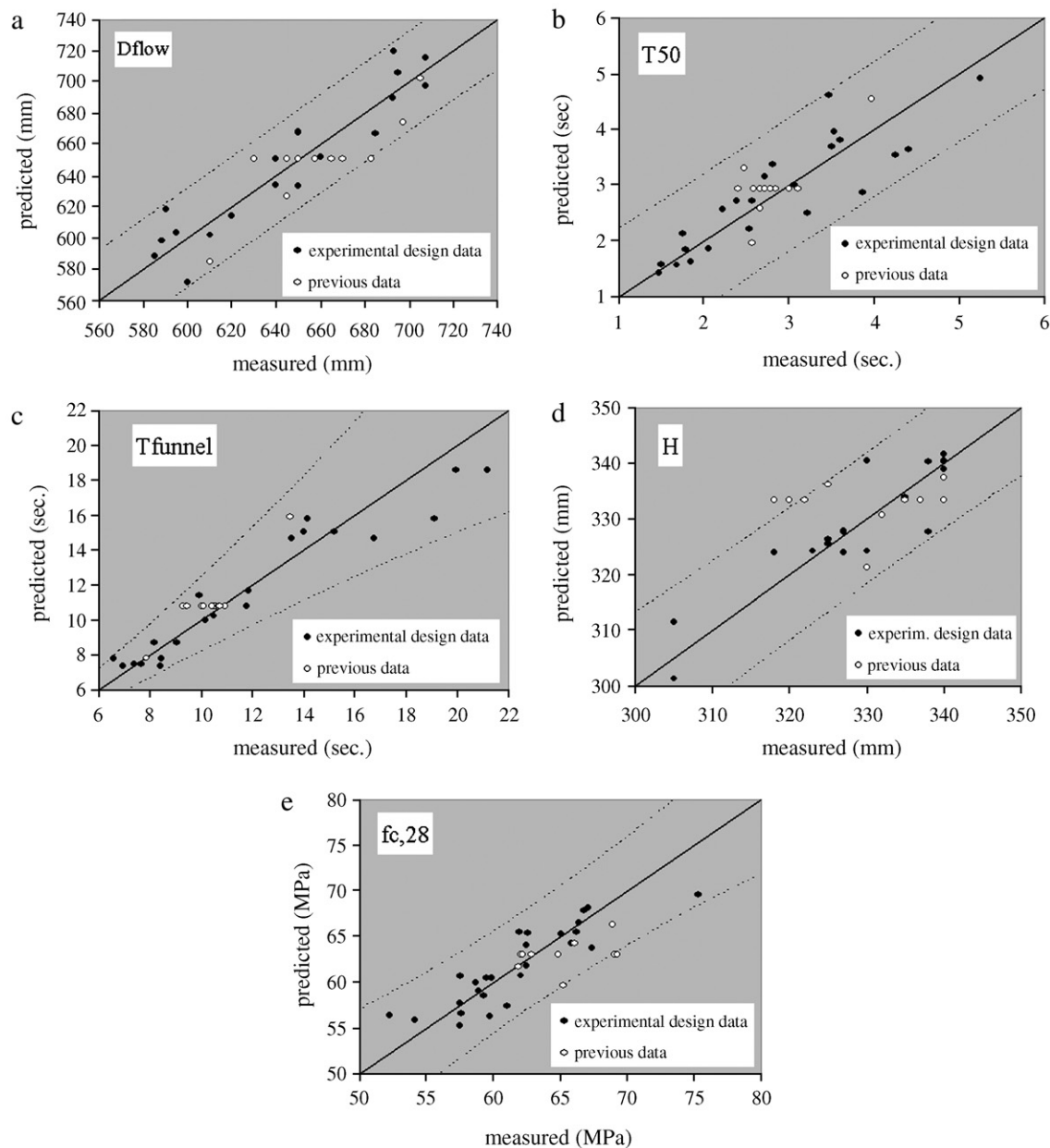


Fig. 1. Comparison of measured versus predicted values of Dflow, T50, Tfunnel, H , and $f_{c,28}$ for all mixes included in the experimental design and additional results from a previous study.

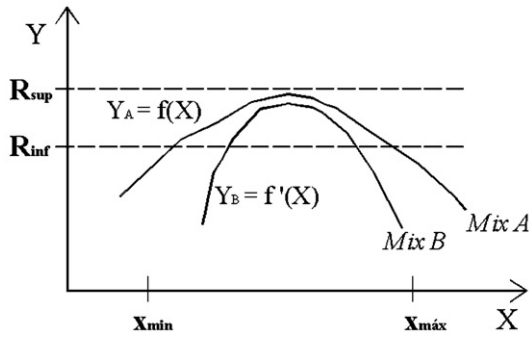


Fig. 2. Comparison between two concrete mixes in terms of robustness.

of interest can be evaluated and it is called a bootstrap replication. This process is repeated many times to generate B bootstrap samples and respective bootstrap replications. Then the sample of the bootstrap replications (size B) is used to assess the accuracy of the computed statistic, for example, to construct confidence intervals [12]. Applying the first percentile method [12] the $100(1-\alpha)\%$ confidence interval for the true value of the unknown parameter of the population is then given by the two values that include the central $100(1-\alpha)\%$ of this distribution. Thus a 95% confidence interval is given by the 2.5% and 97.5% percentiles of the generated distribution. Details on this statistical method are reported in Ref. [12].

In the present study the quantity to be estimated is the probability p given in Eq. (2) and the frequency of accepted intervals given in Eq. (3) is computed for each bootstrap sample. An improved estimate of p , i.e. the robustness measure, is given by the mean of the frequencies obtained in the B replications.

4.4. Implementation, results and discussion

In this work, as previously mentioned, $N=132$ independent data points were observed, consisting of target and measured weight values of each constituent material. From this data, a large number of independent bootstrap samples ($B=2000$) were generated, each one of size $n=100$. For each re-sampled point in the bootstrap sample, the target and measured values of independent variables ($X_1=Vw/Vp$; $X_2=wf/wc$; $X_3=Sp/p$; $X_4=s1/s$ and $X_5=Vap$) were calculated from the weight values of each constituent material. Then, the corresponding deviation

ΔX_i from the target value in each independent variable was calculated:

$$\Delta X_i = X_i^{\text{measured}} - X_i^{\text{target}} \quad (4)$$

These deviations were added to the values of the independent variables of the target SCC mix (corresponding to the central point in the factorial design, X_i^0) to simulate the variations associated to the production process:

$$X_i = X_i^0 + \Delta X_i \quad (5)$$

Finally, concrete properties ($Y_1=D_{\text{flow}}$, $Y_2=T_{50}$, $Y_3=T_{\text{funnel}}$, $Y_4=H$ and $Y_5=fc_{28}$) could be estimated using the derived numerical models presented in Table 5 and detailed in Section 4. The statistic presented in Eq. (3) (a frequency) was computed for each bootstrap sample, as an estimate of the target SCC mixture robustness. Besides this frequency, individual frequencies were also computed as estimates of

$$p_i = P(R_{\text{inf}}^i < Y_i < R_{\text{sup}}^i), \quad i = 1 \text{ to } 5 \quad (6)$$

with the purpose of evaluating the contribution of each concrete property to robustness. The acceptance intervals considered for Y_1 , Y_2 , Y_3 , Y_4 and Y_5 were [600,680] (mm), [2,5] (s), [8,12] (s), [300, ∞] (mm) and [60, 70] (MPa), respectively.

A statistical analysis using commercial software (SPSS) was carried out on the samples of p_i and p that resulted from running the 2000 bootstrap replications (Table 8). As observed before with the experimental plan results, the values of the filling height (Y_4) were always higher than 300 mm so estimates of p_4 were always equal to 1.0. The obtained estimate of the SCC mixture robustness was 0.74 with a 95% confidence interval of [0.650, 0.820] (see Table 8). In other words, a robustness value of 0.74 means that the target SCC mix, when produced at Maprel precast factory, is expected to fail to satisfy all the acceptance criteria about once in every 4 batches. From Table 8 we can also notice that SCC compressive strength is the property that exhibited more sensitivity with respect to the introduced variations in the constituent materials and it is the one that contributed the most to reducing the robustness estimate.

The variations considered in this work have by no means exhausted the range of factors that affect SCC mixtures

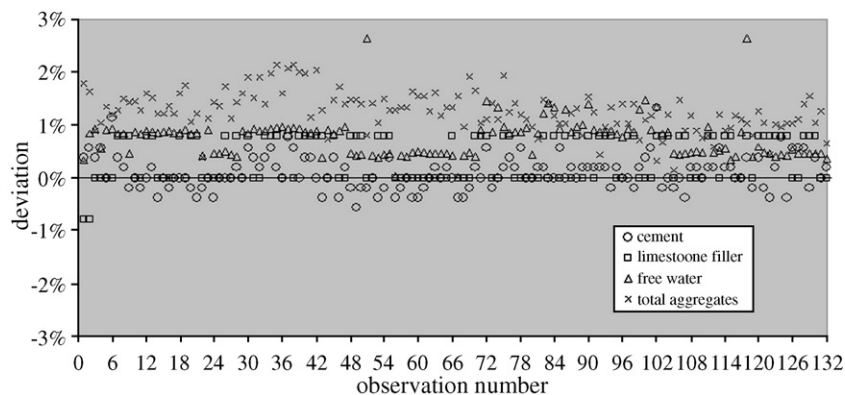


Fig. 3. Observed deviations of constituent materials during one week of concrete production in Maprel precast factory.

Table 8
Descriptive statistics of bootstrap samples

	P_1	P_2	P_3	P_4	P_5	P
No. bootstrap replications (B)	2000	2000	2000	2000	2000	2000
Mean	0.965	0.965	0.918	1	0.864	0.739
Median	0.970	0.970	0.920	1	0.870	0.740
Mode	0.970	0.970	0.920	1	0.860	0.740
Standard error of mean	3.99E-4	4.06E-4	6.21E-4	0	7.63E-4	9.76E-4
Minimum	0.900	0.880	0.800	1	0.730	0.590
Maximum	1.000	1.000	0.990	1	0.960	0.890
Standard deviation	0.018	0.018	0.028	0	0.034	0.044
Percentile 2.5%	0.930	0.930	0.860	1	0.790	0.650
Percentile 97.5%	0.990	0.990	0.970	1	0.930	0.820

robustness. For example, changes in ambient conditions like temperature and humidity, the evolution of concrete properties with time, the variations in material characteristics from different supplies can significantly change SCC fresh and hardened properties. When starting the study an important step is an adequate definition of independent variables, of relevant concrete properties to be analysed and of the respective acceptance limits, since these decisions will affect the final value of robustness. It is also important to collect relevant data about constituent material variations, inherent to the production process, which will then be used to simulate variations of SCC mix proportions.

5. Conclusions

Based on presented results, the following conclusions can be drawn:

1. An experimental plan conducted according to a factorial design is useful to evaluate the effects of mixture parameters and their interactions on SCC properties.
2. The mixture parameters considered in the Japanese-method [1] can be used as factors in the factorial design to characterize the behaviour of SCC mixtures.
3. Data collected during the experimental plan can be used to establish numerical models relating mixture parameters with the SCC properties of interest.
4. V_w/V_p exhibited the greatest effect on all five measured responses; Sp/p and Vap also influenced significantly SCC properties. A significant influence from V_w/V_p , Sp/p and Vap can generally be expected in all SCC mixtures.
5. Significant interaction effects were found between wf/wc and sl/s and between Sp/p and Vap .
6. For all the analysed responses with the exception of $1/\sqrt{T_{funnel}}$, a significant quadratic term was found in V_w/V_p or Vap .
7. Using data of typical material weight deviations inherent to the production process, variations of SCC target mix composition can be simulated, the derived numerical models can be applied to estimate SCC concrete properties and a measure of SCC robustness can be obtained.

Applying the bootstrap technique to the original data sample the robustness value can be accurately estimated and the accuracy of this estimate can be assessed.

8. The target SCC mix, when produced at Maprel precast factory, is expected to fail to satisfy all the acceptance criteria about once in every four batches, corresponding to a robustness value of 0.74. This level of robustness is expected to change if the same mixture is produced in another factory.
9. The number and type of selected independent variables and relevant concrete properties to be analyzed, as well as the respective acceptance limits affect the final value of robustness.
10. The methodology presented in this work is particularly useful to evaluate and compare the robustness of different SCC mixtures and to assist the concrete producer in selecting mixes.

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