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Cement and Concrete Research 36 (2006) 592 – 598
Cement and Concrete Research

Study of modal parameters and vibration signatures of notched concrete prisms

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Received 1 July 2005; accepted 22 August 2005

Abstract

In the present study, vibration tests have been carried out on notched concrete prisms. The tests have been conducted to study modal parameters and vibration signatures in order to enhance the knowledge of monitoring concrete structures. Notch (artificially introduced crack) of constant widths and varying depths has been introduced in the concrete prisms at different locations. The specimens have been subjected to impact excitations by dropping a specific weight from a fixed height and at a particular location. The natural frequencies of notched and intact prisms have been experimentally measured. The frequency response functions (FRF) as obtained from multichannel pulse analyzer have been synthesized to evaluate the fundamental mode shape of vibration of the prisms by a curve fitting method. These curves have been further post-processed to obtain the modal curvature values. Pattern Recognition Scheme is applied to synthesize vibration signatures for the evaluation of the curvature differential values. The curvature differential values corresponding to different sensor locations in both intact and notched specimens have been obtained. Experimental results with different depth of flaws suggest that natural frequency alone cannot be a reliable parameter for the detection of damage in concrete beams with shallow depth of flaws. The modal curvature values have been found to be a reasonably good indicator of the location of the damage. The curvature differential values show the extent of damage but found to be dependent on the sensor location from the position of notch.

Keywords: Vibration; Concrete; Pattern recognition; Crack detection

1. Introduction

The vibration testing has been recognized as a promising tool for monitoring and testing the integrity of concrete structural members. Crack detection in concrete structures in a more effective and accurate manner is an important area of research to predict the remaining life of structures. Visual inspection by experienced team of engineers may often produce valuable information on the existing state of structures and future assessment of load carrying capacity. However, if the cracks or flaws remain inside the structure, visual inspection cannot reveal the actual state of the structure. The traditional nondestructive tests such as rebound hammer and ultrasonic pulse velocity tests are widely used to evaluate the quality of concrete. Ultrasonic pulse velocity methods (UPV) have been applied by Ben-Zeitun [1] and Popovics et al. [2]. Calibration curves relating pulse velocity and concrete strength are established using regression analysis. The lack of success of UPV method for quantification of damage has been pointed out by Popovics and Popovics [3]. They concluded that as the frequency is reduced, axial resolution of ultrasound waves worsened. Due to this reason, the method becomes insensitive to small voids, micro cracking and matrix porosity, which are of orders of magnitude smaller than the signal wavelength. Moreover, practical difficulties encountered often in placing transmitter and receiver in opposite faces of a structural element and presence of moisture in specimen produce erroneous information about the quality of concrete. The improvement of UPV technique to extract more reliable information has been reported by Wei Due [4]. He has shown that it is possible to obtain quantitative information in regards to the status of the concrete by effective analysis of the frequency spectrum. Combined rebound hammer and UPV tests have been carried out by Qasrawi [5] to predict the concrete strength. Qasrawi and Marie [6] have used UPV tester

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doi:10.1016/j.cemconres.2005.08.005

as a tool to monitor basic initial cracking of concrete structures and thereby introduced a threshold limit for possible failure of the structures. Nanni et al. [7] have developed fibre optic sensor, which can be embedded within the material and is capable of measuring the internal displacements due to crack openings. Ohtsu [8] has applied acoustic emission technique for diagnosing damage in concrete. All of these experimental techniques require that the portion of the structure being investigated is readily accessible. The need for quantitative global damage detection methods that can be applied to complex structures has led to the development and continued research of methods that examine changes in the vibration characteristics of the structure. The involvement of physical properties such as mass, damping, and stiffness of structures are the most important key parameters to utilize for determination of modal parameters like frequencies, mode shapes and modal damping. Vibration technique with mode shapes as identifying parameter plays an important role in crack identification. Cawley and Adams [9] and Yuen [10] have investigated the change of natural frequencies of notched beams. Several other theoretical and experimental studies have also confirmed the decrease of natural frequencies of notched beams [11–14]. However, mode shapes have not been found very sensitive to identify the presence of crack. The study has also recommended considering the effect of ambient conditions on the dynamic response of the structures, consistency and reliability of the analytical/experimental procedure. To measure the similarity between two different mode shapes, the modal indices termed as modal assurance criteria (MAC) and coordinate modal assurance criteria (COMAC) have been used by Wolf and Richardson [15] and Lievens and Ewins [16]. However, Pandey et al. [17] have shown for a cantilever and a simply supported beam that MAC and COMAC techniques are insensitive to damage location. For localization of damage, other dynamic parameters such as frequency response function, power spectral density and curvature of the mode shape have also been used by Pandey et al. [17], William and Salawu [18] and Wahab and Roeck [19], which showed more sensitivity to damage than the mode shapes themselves. Gladwell and Morassi [20] have utilized the changes in node positions in principal mode of axial vibration for detection of the crack. Subsequently Dilena and Morassi [21] have extended the study for bending vibration of beams supported by experimental findings on thin walled section steel beam. Methods based on digital signal processing and pattern recognition technique have been applied for the nondestructive evaluation of structures by different researchers [22–24]. The report LA-13070-MS by Doebling et al. [25] contains a review of the technical literature concerning the detection, location and characterization of structural damage using techniques that examine changes in measured structural vibration response. The development of the damage-identification methods and summaries of the current state-of-the-art of the technology has been found in the report.

Literature survey indicates that a single method of nondestructive evaluation of structure is not adequate to extract complete and reliable information of the damaged pattern. Moreover, detailed post processing of the test results and careful interpretation and visual inspection can yield sufficient information about the state of damage. In the present investigation, the vibration test has been carried out on intact and notched specimens of concrete prisms ($150 \times 150 \times 700$ mm). The specimens have been subjected to impact excitations by dropping a specific weight from a fixed height at a particular location. The Frequency Response Functions (FRF) obtained in pulse analyzer is treated as vibration signatures of the specimen. These are further processed to obtain fundamental mode shape and modal curvature. Pattern recognition scheme [24] has been applied to the digitized signatures of both intact and notched specimens to obtain curvature differential values. The test has been conducted with specimens containing different depth of notches at center. The usefulness of vibration test for the detection of anomalies in concrete specimens are demonstrated with modal parameters and vibration signatures, which should be useful in providing information for structural health monitoring.

2. Experimental program

Experimental studies have been carried out using $700 \times 150 \times 150$ mm plain concrete prisms. The concrete prisms have been cast in mild steel moulds and are removed after 24 h. Subsequently, these prisms have been kept for curing by immersing in a water tank for the maximum period of 28 days.

2.1. Material properties

The concrete mix 1 : 1.5 : 3 by weight with water cement ratio 0.42 has been used. The quantities of material per cubic meter of finished concrete are 380 kg of cement, 570 kg of fine aggregates, 1140 kg of coarse aggregates and 160 l of water. Locally available coarse and fine aggregates have been used. The grading of fine aggregates falls under the classification 'M' as per BS 882-1992. The nominal maximum size of the coarse aggregate is 20 mm. Ordinary Portland Cement (OPC) of 53 grades (Grade 53 indicates that 28-day compressive strength of 70.7 mm cement mortar cube is 53 N/mm^2) has been used for the required mix of concrete. The slump value of the concrete was observed in the range of 25–35 mm. The average 7-day and 28-day compressive strengths of the cubes cast from the concrete have been obtained as 16 and 24.89 N/mm^2, respectively. Further, flexural strength of the prisms at 7 and 28 days are obtained as 2.95 and 3.74 N/mm^2, respectively. The mix chosen is a nominal mix as per Indian Standard: 456, 2000 [26], which is expected to produce 28 days cube strength of not lower than 20 N/mm^2. With normal weight aggregate of 20 mm nominal maximum size, maximum free water–cement ratio of 0.5 has been suggested in the Code for such a nominal mix. Lower water–cement ratio used in the present investigation yielded strength as 24.9 N/mm^2 higher than the strength expected from such nominal mix. However, a controlled mix with water–cement 0.42 is expected to yield marginally higher strength than the obtained result as indicated in the guidelines of Indian Standard: 10262, 1982 [27] for the design of concrete

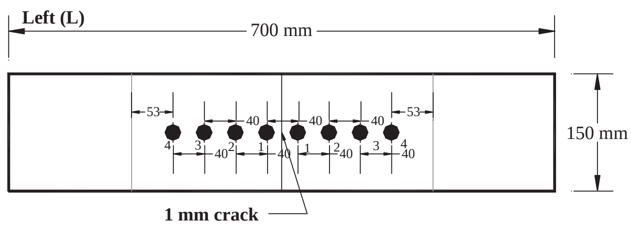

Fig. 1. Plan view of the positions of inserts on the concrete prisms.

mix. The possible reason for this shortfall can be attributed to the fact that for the same cement content, water cement ratio and slump as used in nominal mix concrete, the controlled design has been observed to have little lesser amount of fine aggregate and slightly increased amount of coarse aggregate. Thus, it is quite likely that such a controlled proportioning of concrete may lead to a slight increase in the strength.

2.2. Introduction of notch

Notch or artificial cracks are introduced during casting with the help of 1 mm thick oiled aluminum sheet. The notch has been introduced at the center of the prism along the width and on the top surface by putting the aluminum sheet while casting. After one day, the oiled aluminum sheet pieces are very carefully removed. The notch or artificial cracks have been introduced in the prism at one location only. Different specimens have been prepared with the central notch having depths 10, 20, 30, 40, 50 and 70 mm.

2.3. Sensor locations and fittings

The locations of sensors for picking up responses play important role in identifying the quality of the specimen under study. 30 mm diameter and 6 mm thick mild steel inserts are fabricated and placed during concreting of the specimen. The inserts are introduced to provide mounting for the accelerometer on the specimen. Eight inserts are placed at distances of 20, 60, 100 and 140 mm on each side of the notch as shown in Fig. 1.

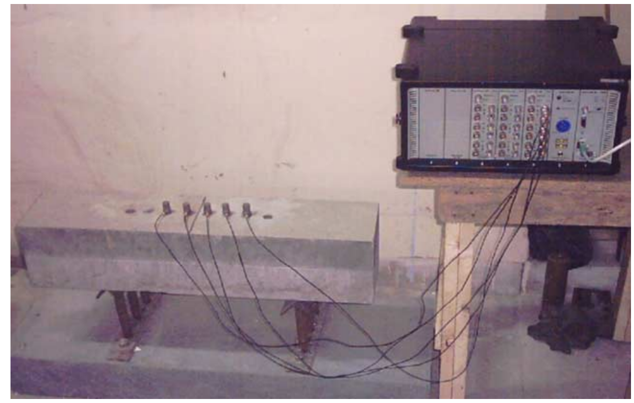
Fig. 3. Vibration testing setup for the concrete prism.

2.4. Fabrication of the support

As shown in Fig. 2, the support for the concrete prism has been fabricated by using angle section, $50 \times 50 \times 6$ mm thick and 300 mm long, which is welded to four numbers of 20 mm mild steel rods. The mild steel rods are again welded to 10×5 mm thick mild steel flat section, which is fixed to a concrete block of size $900 \times 150 \times 300$ mm width by means of bolts. Two such supports are placed at 193 mm apart from the center of the block.

2.5. Vibration test procedure

The specimens are subjected to impact load on the top surface at a particular location. This is carried out by allowing a load of 456 g to fall freely from a height of 250 mm on the top of the prisms and 50 mm away from the center towards the left. The complete experimental set up is shown in Fig. 3. Vibration testing has been conducted to obtain the response of specimens with different notch depths at center using piezoelectric accelerometers. These accelerometers are having a sensitivity of 10 mV/ms^{-2} with the working frequency range of 1 Hz to 6 kHz. The accelerometers are fed to input channels in the front end of FFT analyzer. The FFT analyzer is consisting of acquisition front end with AC/DC power

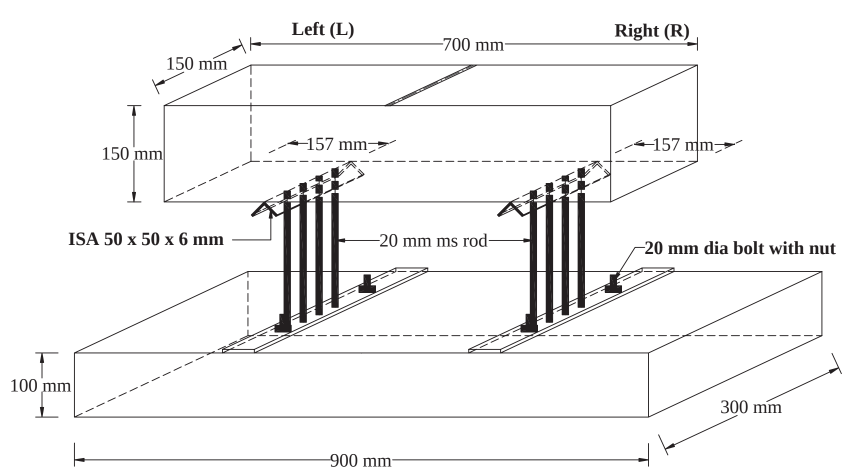

Fig. 2. Support condition of the prism.

supply, 100 kHz input modules, generator modules, signal analyzer input modules and output modules. The signal analyzer interface module handles all digital data communication between the Digital Signal Processing unit (s) and modules in acquisition front end. All the input modules have a dynamic range greater than 80 dB. The output channels from FFT are connected to a PC loaded with PULSE software, which enables the processing of the response signals. The PULSE has 50 beats limit of analysis with 16 active channels with frequency span of 25 kHz.

3. Pattern recognition of vibration signature

Pattern recognition (PR) is the art of simulating human identification of images, graphs, and waveforms using computers. An attempt has been made to analyze and study the characteristics of the different acceleration frequency response functions (FRF) obtained from sensors attached to both notched and intact concrete prisms. The FRF from sensors have been used as signature in the current study.

Recognition of signatures, which is categorized as waveform is done in three stages:

(i) Preprocessing: Signatures are “cleaned up” and read, before recognition tools are applied. This includes noise elimination or signal smoothening.
(ii) Processing: Recognition tools are used to highlight certain features in the signatures.
(iii) Interpretation: The features obtained in the processing stage are used to decide whether two sets of signatures are “different.”

In order to evaluate the performance of signature recognition methods, the following terms are used.

Feature of recognition is a measure of the difference between two signatures. Each feature value is obtained from the result of a waveform recognition technique operating on a pair of signatures.

Feature plot is a figure that illustrates how a certain feature of recognition varies with changes in the condition of the structure. In such a plot, the abscissa represents data point (frequency) and ordinate is the value of the corresponding feature (curvature differential values).

Trend analysis is the process of studying a feature plot to establish a possible relationship between the length of the defect and the corresponding feature of recognition.

The features can be obtained through processing the digital signals as follows:

For a digital signal u, slopes are computed by forward difference as

$$S_n = u_{n+1} - u_n \quad (n = 1, 2, \ldots, M - 1) \tag{1}$$

where u_n is the magnitude of the digitized signal at index n, S_n is the slope value at n. Since lines are equidistant in digital-signal analyzers, the distance between each two lines is taken as unity. To establish the relative nature of the code, the values of S are then normalized with respect to the maximum value of slope as

$$S_n^{\mathrm{r}} = \frac{S_n}{S_{n\ \max}} \tag{2}$$

where S_n^{r} is relative slope at n and $S_{n\max}$ is the maximum slope in the signal.

The relative slope is then used to find the relative curvature.

$$C_n^{\mathrm{r}} = \frac{S_n^{\mathrm{r}} - S_{n-1}^{\mathrm{r}}}{2} \qquad (n = 1, 2, \ldots, M - 2) \tag{3}$$

To compare two signals, u and w, their curvature differential values are given by

$$C_n^{dv} = |C_n^{r:u} - C_n^{r:w}| \tag{4}$$

where. $C_n^{r:u}$ and $C_n^{r:w}$are the values of the relative curvature at point n for signatures u and w, respectively. A large value of C_n^{dv} at a point indicates a significant difference between the two signatures in the neighborhood of that point. To implement the recognition process, the area enclosed by the C^{dv} curve and the frequency axis or the summation of the values in C^{dv} is used as the feature of recognition. The signatures have been obtained from both intact and notched specimen during the impact test and summation of curvature differential values is evaluated.

4. Results and discussions

The results of the vibration tests are presented and interpreted in three ways. First the fundamental natural frequencies of the intact and notched prisms are compared. The FRF corresponding to each sensor location has also been obtained through an in-built software in the pulse analyzer used in the experiment. The test results of the specimens at different notch depths have been presented. From FRF obtained in the impact test, the fundamental mode shapes of the prisms are determined using a curve fitting method. Further, absolute differences in curvature mode shapes of the intact and notched specimens are evaluated. Lastly, curvature differential values of the vibration signatures for both intact and notched specimens obtained in pulse analyzer are presented.

4.1. Fundamental natural frequency

Table 1 shows the natural frequency of flexural vibration of notched and intact prisms obtained by impact-induced excita-

Table 1
Fundamental natural frequency of intact and notched beam

Notch depth (mm)	Frequency (Hz)	% decrease
Nil	1152	–
10	1088	5.56
20	1056	8.33
30	1024	11.11
40	1010	12.33
50	960	16.67
70	704	38.89

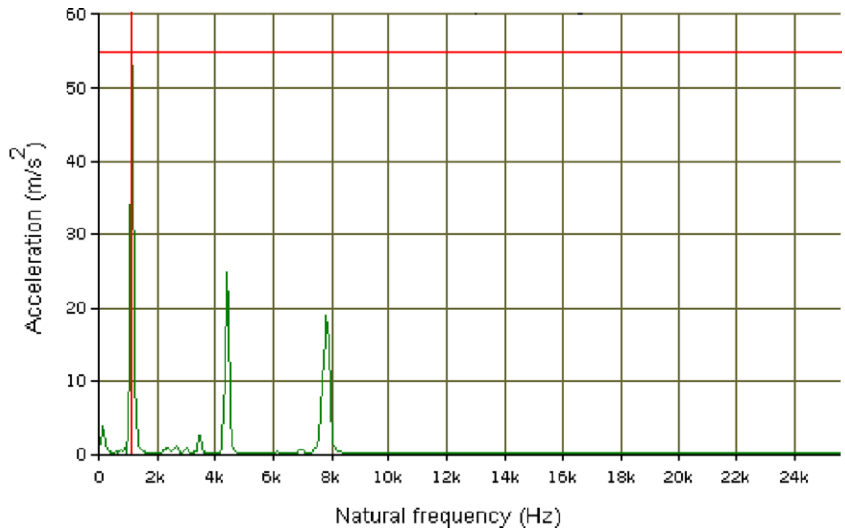

Fig. 4. Natural frequency response function for intact concrete prism at sensor location 1L.

tion. The notch is located at the center of the specimens and depths have been varied from 10 to a maximum of 70 mm. The obtained results show that the fundamental frequency corresponding to flexural mode decreases with the increase in notch depth. The decrease has been observed to be quite significant if the notch depth is more than 30% of the depth of the specimen.

4.2. Mode shape and modal curvature

The fundamental flexural mode of vibration has been obtained from frequency response function (FRF) for the sensors at different locations of the prism specimen, which has been excited by impact loading. Figs. 4 and 5 show two such typical FRFs obtained during the impact excitation on intact and notched specimen (notch at the center) corresponding to sensor location 1R. It may be seen that FRFs obtained during such event on notched and intact specimen does not reveal much information by direct comparison, except the absolute magnitude of the peak acceleration in notched prisms being slightly larger than that of intact specimen. Hence effort has been made to utilize this basic output from the actual test by evaluating mode shapes and modal curvature values. A simple methodology has been adopted based on the fact the fundamental frequency and amplitude at each sensor location is easily read from the FRF curves obtained during impact test. Further, assuming constant input forcing harmonic function, the displacement amplitudes are obtained by dividing the magnitude of acceleration by the square of corresponding natural frequency. The method will not work in the other higher modes of vibration. However, fundamental mode is practically the most significant mode of vibration, which governs the engineering design. Hence, in the present work fundamental mode shapes between intact and notched specimens are compared. The magnitude of acceleration corresponding to fundamental frequency as obtained from FRF curves at different sensor locations are presented in Table 2. It may be observed that the acceleration values corresponding to different

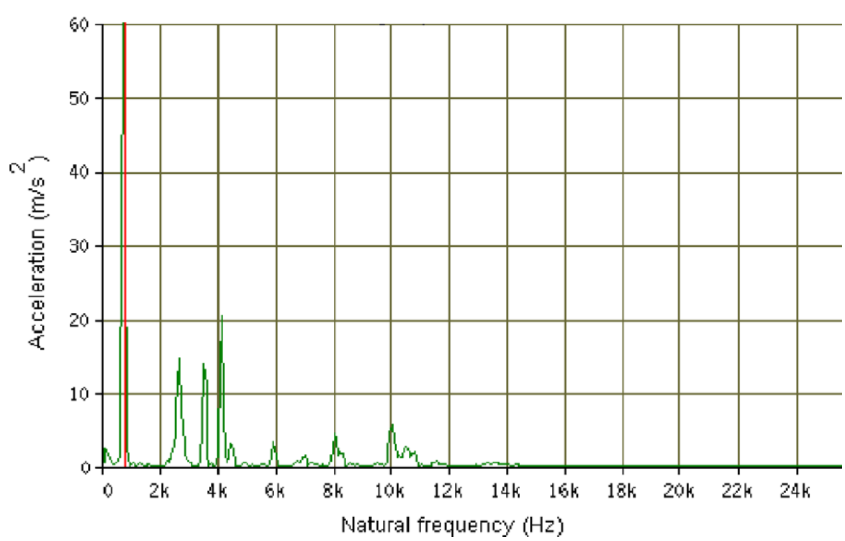

Fig. 5. Frequency response function of notched (70 mm deep) concrete prism at sensor location 1L.

Table 2
Acceleration magnitude at different sensor locations

Sl. no.	Type of specimen	Sensor no.	Location of sensors from the center of the prism (mm)	Fundamental frequency (Hz)	Acceleration in (m/s^2)
1	Intact	1L	20	1088	52.50
		2L	60	1088	48.00
		3L	100	1088	35.20
		4L	140	1088	21.00
		1R	20	1088	53.10
		2R	60	1088	47.90
		3R	100	1088	38.40
		4R	140	1088	23.90
2	Notched at center (70 mm deep)	1L	20	704	79.60
		2L	60	704	58.00
		3L	100	704	40.40
		4L	140	704	22.30
		1R	20	704	78.60
		2R	60	704	60.00
		3R	100	704	40.90
		4R	140	704	23.70

sensor locations are different. However, the fundamental frequency of vibration as obtained corresponding to all the vibration signatures at all sensors locations for a particular specimen are exactly same, since all the points are vibrating in the same frequency.

The displacement magnitudes are calculated at each sensor location and have been used to fit a fifth degree polynomial using the *polyfit* in MATLAB program. The fundamental mode shapes of intact and notched specimen are shown in Fig. 6. Modal displacement of notched specimen increases with the increase in notch depth. For higher notch depth, the discontinuity in the displacement at the location of notch is apparent from the plot.

From the displacement mode shapes obtained as above, curvature mode shapes can be obtained numerically by using a central finite difference approximation as

$$v_n'' = (v_{n+1} - 2v_n + v_{n-1})/h^2 \qquad (5)$$

where v_{n-1}, v_n and v_{n+1} denote the displacement at $n-1$, n and $n+1$ th nodes, respectively and h is the distance between two adjacent nodes. The absolute difference in the modal curvature values between intact and notched specimens are obtained and

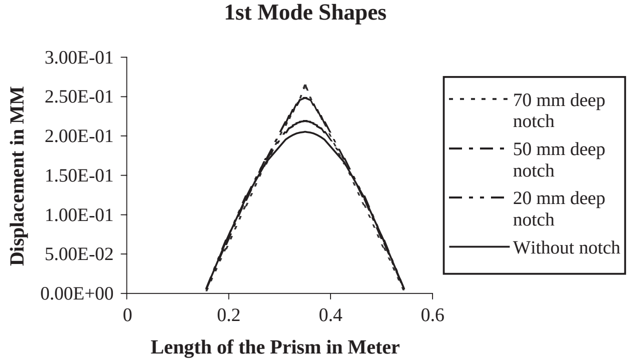

Fig. 6. Fundamental mode shape of intact and notched specimens.

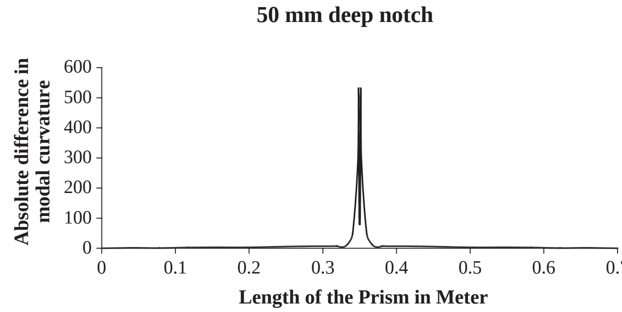

Fig. 7. Absolute difference in modal curvatures between intact and notched (50 mm deep) specimens.

plotted along the length of the beam. Fig. 7 shows the absolute difference in the modal curvature between intact and a notched specimen with 50 mm deep notch at the center of the prism. The results show that the curvature difference between intact and notched prisms show minor changes except at the location of the notch, where a sudden increase in the value of the absolute difference of modal curvature is observed. This is indicative of the usefulness of obtaining curvature difference to locate the damage. The absolute differences of curvature for specimens with other notch depths showed similar patterns and therefore have not been presented in the paper. The absolute difference also increases with the increase in notch depth.

4.3. Curvature differential values

Signatures have been obtained corresponding to different sensor locations for both intact as well as notched concrete specimens with the help of experiments using FFT analyzer. The digital data sheets for different signatures have been further analyzed with Eq. (4) for obtaining curvature differentials values. Table 3 shows the summation of curvature differential values of the signatures obtained at sensor location 1R of both intact and the notched specimens corresponding to 20, 50 and 70 mm notch depths at center. It is observed that the summation of curvature differential values increases with the increase in notch depths. Further, comparison has also been carried out among signatures obtained from other sensor locations for a specified notch depth. Table 4 shows the summation of curvature differential values obtained from signatures at different sensor locations for 70 mm depth of notch. The average of the values of the same location to the left and right is presented in the table. It can be observed from the results of Table 4 that the summation of curvature differential values obtained from different signatures is sensitive to the location of the sensors. The nearer the sensor to the damaged

Table 3
Summation of curvature differential values for signatures of intact and notched specimen with varying depth of notch at center

Depth of notch (mm)	Summation of curvature differential values (C^{dv})
20	20.429
50	65.907
70	171.580

Table 4
Summation of curvature differential values for signatures at different sensor locations of intact and notched specimen

Sl. no.	Particulars of the prism	Sensor locations away from the notch (mm)	Summation of curvature differential values
1	Intact and centrally	20	171.580
2	notched prisms	60	160.750
3	(70 mm deep)	100	149.235
4		140	137.450

location, the higher is the summation of curvature differential values obtained from the signature at that location.

5. Conclusion

Vibration test has been conducted on notched concrete specimens, which have yielded useful information about the extent of the notch and its location. The study has shown that for very shallow notch, the natural frequency of the beam alone could not be a reliable parameter for the detection of notch. Natural frequency can be successfully utilized in the detection of notch if the depth of notch is large. The absolute difference of modal curvature between notched and intact specimen can be a good indicator of the localization of notch. The summation of curvature differential values, which have been obtained using pattern recognition scheme is found to increase with the increase in notch depth and can be a measure of the extent of the notch. However, data obtained from sensor located near the notch is more useful for the identification of the notch than those located further away from the notch. The study on notched concrete specimens by vibration testing demonstrated that modal parameters and vibration signatures would provide useful information for health monitoring of the concrete structures.

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