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Formulation of a new micromechanic model of three phases for ultrasonic characterization of cement-based materials

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Abstract

The objective of multiphasic models applied to cement-based materials is to estimate the global elastic constants as a function of properties of their constituents. However, these models have limited success for porosity characterization.

In this investigation, a new approach that determines the elastic properties of three-phase materials is presented. The proposed model takes into account the microstructural characteristics of the constituent phases, as well as their elastic properties. The micromechanical model of three phases is applied to mortar, considering the material composed by cement paste matrix and two types of inclusions, aggregate and pores. The influence of geometry and elastic properties of both inclusions on the ultrasonic velocity has been evaluated theoretically. The effect of the volume fraction of pores on the ultrasonic velocity is also presented.

Keywords: Ultrasound velocity; Micromechanics; Porosity; Characterization; Modeling

1. Introduction

Although multiphasic theory has been developed and applied, to ceramic composites, fundamentally, the application of these models to cement composite is more recent. The objective of multiphasic models applied to composites is to predict the global elastic properties as a function of the properties of their constituent materials and volume fraction.

A review of several models, which have been proposed for the prediction of elastic modulus of multiphasic materials, has been presented in Ref. [1]. The two simplest models, by Reuss and by Voigt, permit one to establish the upper and lower bounds of global elastic modulus from elastic properties and volume fraction of phases. To get a better estimate of the modulus, some other models have been suggested, such as that by Hirsch, which derived an equation to express the elastic modulus of concrete in terms of an empirical constant. Hirsch's model is actually the geometric mean of the Reuss and Voigt models. Counto [2] considered the case of prismatic aggregate surrounded by a prism of concrete, while Hansen [1] modeled an aggregate surrounded by concrete, but in this case, the surrounding shape was spherical.

Hashin and Shtrikman [3] developed the more stringent upper and lower bounds on the elastic modulus of multiphase quasi-homogeneous and quasi-isotropic materials with arbitrary phase geometry, using the variational principles of the linear theory of elasticity.

Kuster and Toksöz [4] considered a longitudinal (or shear) wave impinging on an assemblage of inclusions, and calculated the sum of the waves which were scattered off each inclusion. This wave was then equated to that which would be scattered off a single equivalent spherical inclusion, leading to an expression for the effective elastic modulus.

Several models have been developed using the theory of micromechanics, including the models of single inclusion [5] and double inclusion [6] formulated by

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Yang. These models contain a complex manipulation of the strain field together with the concepts of eigenstrain and eigenstress [7].

However, when they attempted to characterize porosity, the models had limited success. In a few cases, the models concluded that the composite modulus is equal to the matrix modulus, which implied a considerable error, while in other models, the formulations were not valid.

In two previous papers, we applied the micromechanics model formulated by Jeong et al. [8], for estimating the porosity of mortar [9], and for determining the mechanical properties of this material [10] employing ultrasonic methods.

In this paper, a new approach that determines the elastic properties of three-phase materials as a function of microstructural characteristics of phases is presented. This model allows one to establish a relationship between the ultrasonic velocity and the microstructural characteristics of different phases of the composite. The formulation of the model of three phases is based on average field theory of Mori and Tanaka [11], and Eshelby's [12] equivalent inclusion principle. This model allows one to estimate the porosity in cement composite of three phases, considering the porosity as another phase. The model is analyzed in detail using the concept of orientation-dependent average fields and strain concentration factors. The strain concentration factors are evaluated by the Mori–Tanaka approximation.

The micromechanical model of three phases is applied to mortar, considering the material composed of three phases: cement paste without pores, aggregate and pores. The influence of geometry and elastic properties of both inclusions on the ultrasonic velocity is studied as an example. The effect of the porosity is also evaluated. The inclusions are considered as ellipsoids and the particular cases of oblate spheroids, prolate spheroids and spheres are included.

2. Theoretical formulation of micromechanical model of three phases

Consider a composite material represented by a volume element V and subjected to constant uniform overall strains ε_0 as shown in Fig. 1.

The volume of the composite is the sum of the volume of matrix V^m, and the volume of the inclusions V^a and V^p; these phases are considered homogeneous, elastic and isotropic.

$$V = V^m + V^a + V^p \tag{1}$$

Similarly, the composite mass is defined as:

$$M = M^m + M^a + M^p \tag{2}$$

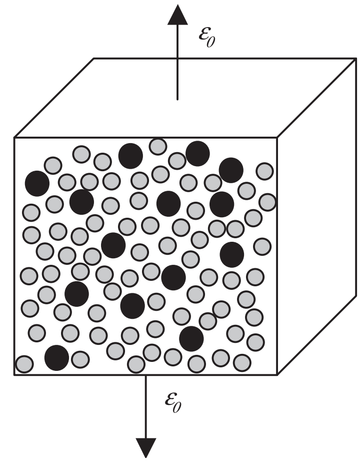


Fig. 1. A representative volume element of a composite medium, with two types of ellipsoidal inclusions in an isotropic matrix, subjected to the overall strain ε_0.

Hence, the triphasic composite density is expressed as follows:

$$\rho = \frac{M}{V} = \frac{\rho^m V^m + \rho^a V^a + \rho^p V^p}{V} = \rho^m v^m + \rho^a v^a + \rho^p v^p \tag{3}$$

In this paper, the superscripts m, a and p refer to quantities related to the matrix, inclusions type 'a' and 'p', respectively, and v represents the volume fraction.

To facilitate the determination of the elastic constant tensor (denoted by $\boldsymbol{C}$), it is convenient to work with averages of overall and local fields. Suppose the strain field in the volume V is ε_0).

The overall average strain in V is defined by the integral:

$$\bar{\varepsilon} = \frac{1}{V}\int \varepsilon \mathrm{d}V \tag{4}$$

where the overbar denotes the volume average.

Because the composite is composed of the matrix and two types of inclusions, the above integral can be decomposed into the following sum of integrals:

$$\bar{\varepsilon} = \frac{1}{V}\int_{V^m} \varepsilon^m \mathrm{d}V + \frac{1}{V}\int_{V^a} \varepsilon^a \mathrm{d}V + \frac{1}{V}\int_{V^p} \varepsilon^p \mathrm{d}V \tag{5}$$

If one introduces the average strain in the matrix $\bar{\varepsilon}^m$ by taking V=V^m in Eq. (4), the first integral in Eq. (5) becomes:

$$\frac{1}{V}\int_{V^m} \varepsilon^m \mathrm{d}V = v^m \bar{\varepsilon}^m \tag{6}$$

where v^m=V^m/V.

However, the second and third integrals should be carried out over all possible orientations because the strain field in each inclusion is orientation dependent. This can be accomplished by doing the integration weighted by orientation distribution function (ODF) in the Euler space. Denoting the ODF by $\Gamma(\phi, \theta, \psi)$, the volume integral over inclusions yields:

$$\frac{1}{V}\int_{V^a} \varepsilon^a \mathrm{d}V = v^a\langle\varepsilon^a\rangle \qquad \text{for inclusions of type } a \tag{7}$$

$$\frac{1}{V}\int_{V^p} \varepsilon^p \mathrm{d}V = v^p\langle\varepsilon^p\rangle \qquad \text{for inclusions of type } p \tag{8}$$

where $\langle\varepsilon^p\rangle$ and $\langle\varepsilon^a\rangle$ are the orientation-dependent average strain in the inclusion p and a, respectively, and the angle bracket $\langle\rangle$ denotes the orientational average, defined by:

$$\langle\varepsilon^p\rangle = \frac{\int_0^{\phi=2\pi}\int_0^{\theta=\pi}\int_0^{\psi=2\pi} \varepsilon^p\,\Gamma(\phi,\theta,\psi)\sin\theta\mathrm{d}\phi\mathrm{d}\theta\mathrm{d}\psi}{\int_0^{\phi=2\pi;}\int_0^{\theta=\pi}\int_0^{\psi=2\pi} \Gamma(\phi,\theta,\psi)\sin\theta\mathrm{d}\phi\mathrm{d}\theta d\psi} \tag{9}$$

$$\langle\varepsilon^a\rangle = \frac{\int_0^{\phi=2\pi}\int_0^{\theta=\pi}\int_0^{\psi=2\pi} \varepsilon^a\,\Gamma(\phi,\theta,\psi)\sin\theta\mathrm{d}\phi\mathrm{d}\theta\mathrm{d}\psi}{\int_0^{\phi=2\pi}\int_0^{\theta=\pi}\int_0^{\psi=2\pi} \Gamma(\phi,\theta,\psi)\sin\theta\mathrm{d}\phi\mathrm{d}\theta d\psi} \tag{10}$$

Fig. 2 shows the coordinate system used for representing the general state of orientation of inclusions. In this model, the representation is the same for both ellipsoidal inclusions, with principal axes a_1, a_2 and a_3.

The local axes of inclusions that represent aggregates are denoted by primed coordinates X_1', X_2' and X_3' whereas pores are represented as X_1'', X_2'' and X_3''. The global axes of the composite are represented by unprimed coordinates X_1, X_2 and X_3.

We assume that both types of inclusions can be represented by an ellipsoid, where the geometry and orientation distribution function will change with the type of inclusion considered.

ε^p and ε^a can be obtained from $\varepsilon^{p''}$ and $\varepsilon^{a'}$ through the tensor transformation

$$\varepsilon_{ij}^p = q_{ki}q_{lj}\varepsilon_{kl}^{p''} \tag{11}$$

$$\varepsilon_{ij}^a = q_{ki}q_{lj}\varepsilon_{kl}^{a'} \tag{12}$$

where q_{ij} denotes the direction cosine between the unprimed i-axis and j-axis, and their components are given in Ref. [14].

Then, the average strain in the composite can be explicitly written in terms of averages over individual phases to give:

$$\bar{\varepsilon} = v^m\bar{\varepsilon}^m + v^p\langle\varepsilon^p\rangle + v^a\langle\varepsilon^a\rangle \tag{13}$$

Similarly to the average strain, the average stress in the composite is obtained as:

$$\bar{\sigma} = v^m\bar{\sigma} + v^p\langle\sigma^p\rangle + v^a\langle\sigma^a\rangle \tag{14}$$

2.1. Elastic constant tensor

If we assume that the matrix and the inclusions are elastic and their constitutive relations are known, the average stresses in the matrix and the inclusions are related to the average strains by Hooke's law:

$$\bar{\sigma}^m = C^m\bar{\varepsilon}^m$$

$$\langle\sigma^p\rangle = C^p\langle\varepsilon^p\rangle$$

$$\langle\sigma^a\rangle = C^a\langle\varepsilon^a\rangle \tag{15}$$

where C^m, C^p and C^a correspond to elastic constants tensor of matrix and the inclusions p and a, respectively.

For relating the global uniform strain ε_0 applied to the composite to the average strain of matrix and the inclusions p and a, we introduce the average strain concentration factor $\bar{A}^m$, $\bar{A}^p$, $\bar{A}^a$.

$$\bar{\varepsilon}^m = \bar{A}^m\varepsilon_0$$

$$\langle\varepsilon^p\rangle = \langle\bar{A}^p\rangle\varepsilon_0$$

$$\langle\varepsilon^a\rangle = \langle\bar{A}^a\rangle\varepsilon_0 \tag{16}$$

The global effective stress tensor of the composite can be defined through the relation:

$$\bar{\sigma} = C\varepsilon_0 \tag{17}$$

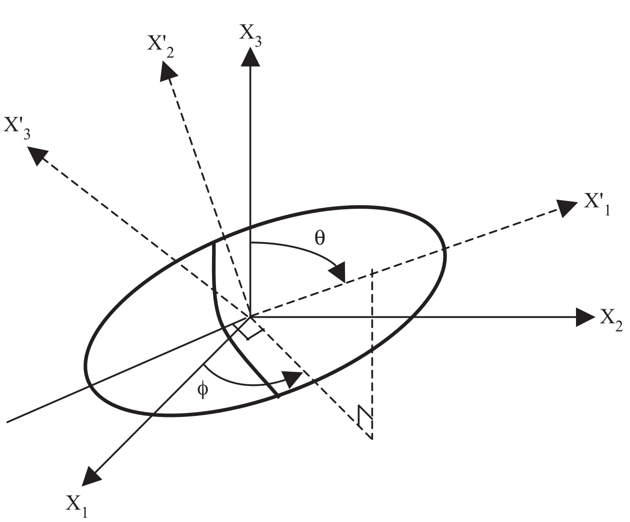


Fig. 2. Coordinate system used for specifying a general state of orientation of an ellipsoidal inclusion.

From Eq. (14) and taking into account Eqs. (15) and (16), we obtain C:

$$C = v^m C^m \bar{A}^m + v^p C^p \bar{A}^p + v^a C^a \bar{A}^a \tag{18}$$

Because $\bar{\varepsilon}=\varepsilon_0$ when ε_0 is applied at the boundary, Eqs. (13) and (16) give

$$v^m \bar{A}^m + v^p \bar{A}^p + v^a \bar{A}^a = I \tag{19}$$

where I is the fourth-rank unit tensor defined by

$$I_{ijkl} = \frac{(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})}{2} \tag{20}$$

and where δ_{ik} is the Kronecker delta [15].

Finding A^m from Eq. (19) and substituting it in Eq. (18), the elastic constant tensor is expressed as:

$$C = C^m + v^p\left(C^p - C^m\right)\langle \bar{A}^p\rangle + v^a\left(C^a - C^m\right)\langle \bar{A}^a\rangle \tag{21}$$

This procedure enables the evaluation of the overall elastic properties C with the help of orientation-dependent average strain concentration factor. This factor can be evaluated using Mori and Tanaka's [11] average field theory, as described below.

2.2. Evaluation of strain concentration factor by Mori–Tanaka method

Now, we will suppose that the composite is subjected to the average strain ε_0 at its boundary. Because inclusions are embedded in the matrix, these will experience a further perturbation ε^*. The objective here is to find the average strain in the inclusions $\langle\varepsilon^p\rangle$ and $\langle\varepsilon^a\rangle$ in terms of the applied strain.

In Mori and Tanaka's method, each inclusion is regarded as a single heterogeneity embedded in an infinite matrix subjected to the matrix average strain ε^m.

If the strain perturbation caused by the presence of this heterogeneity is denoted by ε^{pt} and ε^{at}, the total strain in the inclusions p and a, respectively, depending on the specific orientation, is:

$$\varepsilon^{p''} = \bar{\varepsilon}^{m''} + \varepsilon^{pt''} \tag{22}$$

$$\varepsilon^{a'} = \bar{\varepsilon}^{m'} + \varepsilon^{at'} \tag{23}$$

where the prime indicates that the variables are referred to local coordinates.

From Eshelby's [12] equivalent inclusion principle, the eigenstrains that correspond to inclusions p and a ($\varepsilon^{*p''}$ and $\varepsilon^{*a'}$), in local coordinates, and the perturbed strain can be related by:

$$\varepsilon^{pt''} = S^p \varepsilon^{*p''} \tag{24}$$

$$\varepsilon^{at'} = S^a \varepsilon^{*a'} \tag{25}$$

where S is Eshelby's tensor [7].

Then, applying Hooke's law, the stresses in the inclusion will follow as:

$$\sigma^{p''} = C^p \varepsilon^{p''}$$

$$\sigma^{p''} = C^m\left(\varepsilon^{p''} - \varepsilon^{*p''}\right) \tag{26}$$

$$\sigma^{a'} = C^a \varepsilon^{a'}$$

$$\sigma^{a'} = C^m\left(\varepsilon^{a'} - \varepsilon^{*a'}\right) \tag{27}$$

From Eq. (26), we obtain the following expression:

$$\varepsilon^{*p''} = \left(C^m\right)^{-1}\varepsilon^{p''}\left(C^m - C^p\right) \tag{28}$$

Accordingly, Eq. (27) gives:

$$\varepsilon^{*a'} = \left(C^m\right)^{-1}\varepsilon^{a'}\left(C^m - C^a\right) \tag{29}$$

where $(.)^{-1}$ denotes the inverse of the enclosed quantity.

Substitution of Eq. (28) into Eq. (22) using Eq. (26) gives the strain for inclusion p in local coordinates as:

$$\varepsilon^{p''} = T_p{}'' \bar{\varepsilon}^{m'} \tag{30}$$

Similarly, for the inclusion a, substitution of Eq. (29) into Eq. (23) and using Eq. (27), we will obtain the strain for this inclusion as:

$$\varepsilon^{a'} = T_a{}' \bar{\varepsilon}^{m'} \tag{31}$$

where

$$T_p'' = \left[I - S_p\left(C^m\right)^{-1}\left(C^m - C^p\right)\right]^{-1} \tag{32}$$

and

$$T_a' = \left[I - S_a\left(C^m\right)^{-1}\left(C^m - C^a\right)\right]^{-1} \tag{33}$$

$T_p{}''$ and T_a' are fourth-rank Wu's tensors in local coordinates for the inclusion p and a, respectively. T' possesses symmetry properties such that:

$$T'_{ijkl} = T'_{jikl} = T'_{ijlk} \tag{34}$$

The average strain in the inclusions p and a, can be found by taking the orientational average on both sides of Eqs. (30) and (31), respectively, and can be written as:

$$\langle\varepsilon^p\rangle = \langle T_p\rangle \bar{\varepsilon}^m \tag{35}$$

$$\langle\varepsilon^a\rangle = \langle T_a\rangle \bar{\varepsilon}^m \tag{36}$$

where T_p and T_a represent Wu's tensor in global coordinates for both inclusions.

The tensor T in global coordinates has the same properties as T' in local coordinates. The components of T in global coordinates can be found through tensor transformation:

$$T_{ijkl} = q_{mi} q_{nj} q_{ok} q_{pl} T'_{mnop} \tag{37}$$

Because $\bar{\varepsilon}=\varepsilon_0$, we have

$$\varepsilon_0 = v^m \bar{\varepsilon}^m + v^p \langle \varepsilon^p \rangle + v^a \langle \varepsilon^a \rangle \tag{38}$$

and

$$\bar{\varepsilon}^m = \left[v^m I + v^p \langle T_p \rangle + v^a \langle T_a \rangle \right]^{-1} \varepsilon_0 \tag{39}$$

Substituting Eq. (39) in Eqs. (35) and (36), we obtain

$$\langle \varepsilon^p \rangle = \langle A^p \rangle \varepsilon_0 \tag{40}$$

and

$$\langle \varepsilon^a \rangle = \langle A^a \rangle \varepsilon_0 \tag{41}$$

So that the average strain concentration tensor for the inclusion p is

$$\langle A^p \rangle = \langle T_p \rangle \left[v^m I + v^p \langle T_p \rangle + v^a \langle T_a \rangle \right]^{-1} \tag{42}$$

Similarly, the average strain concentration tensor for the inclusion a is expressed as:

$$\langle A^a \rangle = \langle T_a \rangle \left[v^m I + v^p \langle T_p \rangle + v^a \langle T_a \rangle \right]^{-1} \tag{43}$$

2.3. Micromechanical model of three-phase composite

From Eqs. (42) and (43), we obtain the elastic constant tensor for composites of three phases:

$$C = C^m + v^p \left(C^{pm} \right) \langle T_p \rangle \left[v^m I + v^p \langle T_p \rangle + v_a \langle T_a \rangle \right]^{-1} + v^a \left(C^{am} \right) \langle T_a \rangle \left[v^m I + v^p \langle T_p \rangle + v_a \langle T_a \rangle \right]^{-1} \tag{44}$$

where

$$C^{pm} = C^p - C^m \quad \text{and} \quad C^{am} = C^a - C^m \tag{45}$$

Here C^m, C^p and C^a are the elastic constant tensor for the matrix and the inclusions p and a, respectively. T is Wu's tensor [16] in the global coordinates and the angle bracket denotes the orientational average over all possible orientations. The tensor $\langle T \rangle$, of fourth range, evaluates the geometry, distribution and orientation of the inclusions and their calculus will be shown in the next section.

2.4. Relationship between the elastic constant tensor and the ultrasonic velocity

If we consider that the matrix and both types of inclusions are isotropic, the elastic constants tensor is reduced to two independent elastic constants, C_{11} and C_{44}:

$$C_{11} = C_{11}^m + \frac{v^p \left(C_{11}^{pm} - 4/3 C_{44}^{pm} \right) \langle T^p \rangle + v^a \left(C_{11}^{am} - 4/3 C_{44}^{am} \right) \langle T^a \rangle}{v^m + v^p \langle T^p \rangle + v^a \langle T^a \rangle} + \frac{8/3 v^p C_{44}^{pm} \langle T_{1212}^p \rangle + 8/3 v^a C_{44}^{am} \langle T_{1212}^a \rangle}{v^m + 2v^p \langle T_{1212}^p \rangle + 2v^a \langle T_{1212}^a \rangle} \tag{46}$$

$$C_{44} = C_{44}^m + \frac{v^p C_{44}^m 2 \langle T_{1212}^p \rangle + v + v^a C_{44}^{am} 2 \langle T_{1212}^a \rangle}{v^m + 2v^p \langle T_{1212}^p \rangle + 2v^a \langle T_{1212}^a \rangle} \tag{47}$$

where

$$C_{11}^{pm} = C_{11}^p - C_{11}^m \quad C_{44}^{pm} = C_{44}^p - C_{11}^m \tag{48}$$

$$C_{11}^{am} = C_{11}^a - C_{11}^m \quad C_{44}^{am} = C_{44}^a - C_{11}^m \tag{49}$$

$$\langle T^p \rangle = \langle T_{1111}^p \rangle + 2 \langle T_{1122}^p \rangle$$
$$\langle T^a \rangle = \langle T_{1111}^a \rangle + 2 \langle T_{1122}^a \rangle \tag{50}$$

C_{ij} are the independent components of the elastic tensor in reduced notation. The manipulation of isotropic tensors contained in Eq. (44) can be performed according to the symbolic notation given by Hill [13].

The components of elastic constant tensor are related to the longitudinal (v_L) and transversal (v_T) velocities by

$$v_\mathrm{L} = \sqrt{\frac{C_{11}}{\rho}} \tag{51}$$

$$v_\mathrm{T} = \sqrt{\frac{C_{44}}{\rho}} \tag{52}$$

Thus, we obtain expressions that relate the microstructural characteristics of the phases of the composite to the ultrasonic velocity.

3. Prediction of behavior of ultrasonic velocity in mortar from the micromechanical model of three phases

In this section, the influence of the microstructural characteristics on the ultrasonic velocity is theoretically analyzed, applying the three-phase model to mortar. The calculus of Wu's tensor is made taking into account the particularities of cement-based materials.

The effect of the geometry of inclusions (pores and sand) on the ultrasonic velocity is presented.

3.1. Application of the micromechanical model of three phases to mortar

In the micromechanical model, the geometry of the inclusions is represented by an ellipsoid and is characterized by means of Eshelby's tensor. Formulas for S for various shapes of inclusions can be found in Ref. [7]. For an isotropic matrix, S is given by Poisson's ratio of the matrix and by the aspect ratio α or relative size of the three axes a_1, a_2 and a_3 of the inclusion as shown in Fig. 3.

In this section, the geometry of inclusion is modeled as a spheroid. The spherical inclusions can be characterized by the aspect ratio α, and by the corresponding

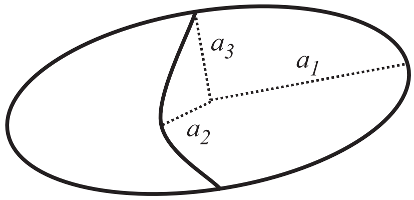


Fig. 3. Ellipsoid that describes a general form of the inclusions. (a_1, a_2, a_3 are ellipsoid radii).

orientation distribution function. If the aspect ratio is defined by the relationship between the different axes and the two coincident axes, then $\alpha = a_3/a_1 < 1$ for an oblate spheroid. Similarly, $\alpha = a_1/a_3 > 1$ for a prolate spheroid and $\alpha = 1$ for the spherical inclusions, as shown in Fig. 4.

The elastic properties of composite materials depend on the distribution and the orientation of the inclusions in the matrix. In these materials, the proportion of the inclusions in the different directions is specified by the orientation distribution function (ODF) $\Gamma(\phi, \theta, \psi)$, where the arguments ϕ, θ, ψ are the Euler angles which describe the orientation of inclusions with reference to the global axes of the composite. The Euler angles permit one to transform a system of the Cartesian coordinates in other coordinates by means of three consecutive rotations.

If we consider that the inclusions are spheroids, only two angles ϕ and θ are needed to describe the distribution of orientation. When the distribution of the orientation is uniform, as happens with the pores in cementitious materials, the distribution function $\Gamma(\phi,\theta)$ is constant [14].

3.2. Calculation of the orientational average of fourth range tensor T

If we assume a spherical inclusion in the homogeneous matrix, the local and global axes are X_1', X_2', X_3' and X_1, X_2, X_3 respectively, as in Fig. 2. The lengths of the major axes of the spheroid in coordinates X_1', X_2' and X_3' are denoted by a_1, a_2 and a_3, respectively.

If we average Wu's tensor T according to the orientation of inclusions using the orientation distribution function, the orientational average of tensor $\langle T \rangle$ is given by:

$$\langle T \rangle = \frac{\int_0^{\phi=2\pi} \int_0^{\theta=\pi} T\Gamma(\phi,\theta)\mathrm{sen}\theta \mathrm{d}\theta \mathrm{d}\phi}{\int_0^{\phi=2\pi} \int_0^{\theta=\pi} \Gamma(\phi,\theta) sen\theta \mathrm{d}\theta \mathrm{d}\phi} \quad (53)$$

When spherical inclusions are randomly oriented, i.e., $\Gamma(\phi,\theta)$=constant, Eq. (53) becomes:

$$\langle T \rangle = \frac{1}{4\pi} \int_0^{\phi=2\pi} \int_0^{\theta=\pi} T(\phi,\theta) sen\theta \mathrm{d}\theta \mathrm{d}\phi \quad (54)$$

Components of T in global coordinates can be found from T', Eq. (37)

The tensor T' in local coordinates can be conveniently expressed as:

$$T' = \left[I + S(C^m)^{-1}\left(C^i - C^m\right)\right]^{-1} \quad (55)$$

Components of Wu's tensor can be calculated from their matrix formulation [16].

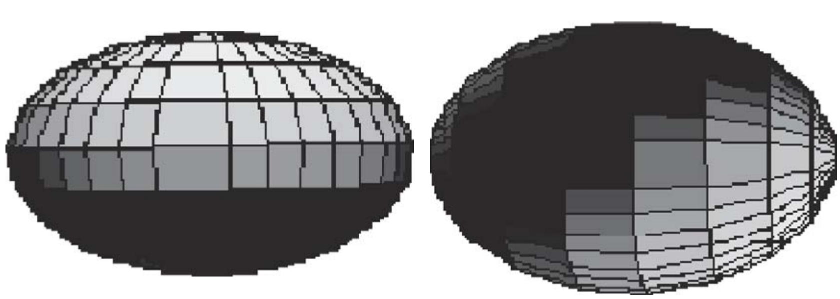
When the inclusions are the pores, the tensor T' is reduced to

$$T' = (I - S)^{-1} \quad (56)$$

3.3. Influence of the geometry of the inclusions on the ultrasonic velocity

In this investigation, the mortar is considered as a material of three phases: matrix (cement paste without pores) and two types of inclusions, namely, pores and sand. From the application of previously discussed micromechanical model, the influence of the geometry of the inclusions, pores and sand, on the ultrasonic velocity is evaluated.

To evaluate the behavior of longitudinal velocity with the aspect ratio of pores, we assume that the elastic properties and density of the matrix are known, the pores and sand are randomly oriented in the matrix and the sand is modeled as spheres.



a) b)

Fig. 4. Particular cases of the spheroid: (a) oblate spheroid, (b) prolate spheroid.

Table 1
Properties of cement paste matrix and sand

	C_{11}^m (GPa)	C_{44}^m (GPa)	ρ (kg/m^3)
Cement paste	40	9	2200
Sand	80.7	32.64	2600

Table 1 shows the elastic properties of cement paste and sand. The matrix corresponds to a cement paste with similar characteristics to a mix of Portland cement (type II, 32.5 N) with water, and water/cement ratio of 0.5, while the properties of sand correspond to natural sand with 98% of silica and a sand volume fraction of 55%.

In this case, we assume that elastic properties and density of matrix and inclusions are kept constant and the porosity can vary between 0.1% and 40% of the volume of the matrix. Evidently, in practice, it is not possible to fabricate a mortar with very low porosity (0.1%) but this represents an interesting case for the present study.

Fig. 5 shows the behavior of longitudinal velocity as a function of the aspect ratio of pores when we change the porosity. It is found that the velocity strongly depends on the volume fraction and shape of the pores. The highest velocities are for prolate (quasi-spherical) pores, while the oblate pores affect sound velocities more. This aspect is relevant because this shape of pores is similar to the geometry of cracks or fissures [17] and can represent an interesting case study.

The influence of the properties of sand on the ultrasonic velocity, based on the three-phase model, is analyzed below. In this case, the elastic property of sand C_{11}^a is varied, between 50 and 100 GPa while C_{44}^a (32.64 GPa), the density (2600 kg/m^3) and volume fraction of sand (55%) are kept constant. The properties of the matrix of cement paste are shown in Table 1 and the porosity of the cement paste is kept constant (30%).

Fig. 6 shows the influence of geometry and elastic constants of sand on the longitudinal velocity. From Fig. 6,

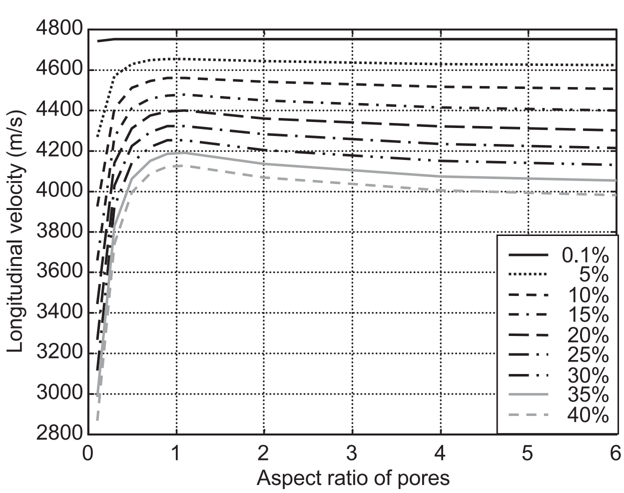


Fig. 5. Influence of the geometry and volume fraction of pores on the velocity.

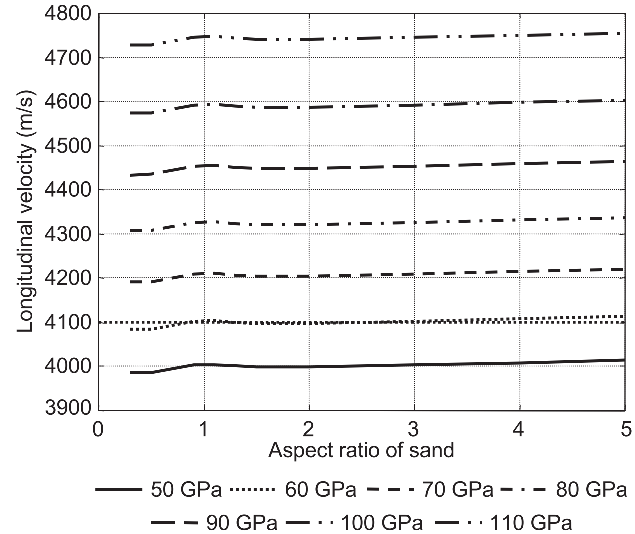


Fig. 6. Influence of the aspect ratio and elastic properties of sand on the velocity.

we can see that the geometry of sand has less influence on the velocity in comparison with the geometry of pores. If this result is confirmed later on, it may induce one to think that if the geometry of aggregates has little influence on the velocity, it may be possible to apply the micromechanical model of three phases to concrete considering a medium aspect ratio. However, if we analyze the influence of elastic properties of fine aggregate in Fig. 6, it can be seen that an increase of C_{11}^a of 10 GPa produces an increase of 100 m/s in the velocity. For this reason, the application of micromechanical model of three phases to concrete must be treated with care.

4. Conclusions

In this paper, a micromechanical model of three phases has been formulated and applied to cement composites of three phases. The model has been applied to mortar considering the material composed of a matrix of cement paste, fine aggregates (sand) and pores. From this formulation, an expression for the global elastic constant tensor of the composite has been obtained, which is more complicated than the corresponding expressions for the biphasic model, as implied by the resolution of two tensors of fourth range which contains the information relevant to the geometry, distribution and orientation of two different phases.

The formulation of the model of three phases allows one to characterize cement composites of three phases, considering the porous structure as an additional phase. From the study of the influence of microstructural characteristics of the inclusions on the ultrasonic velocity, the following conclusions have been obtained:

– An increment in the volume of pores causes considerable reduction in velocity.

– The geometry of pores influences the ultrasonic velocity; the oblate pores affect the velocity more than the prolate pores.
– The geometry of sand has little influence on the ultrasonic velocity.
– An increment in the elastic constants of sand induces an increase in velocity.

The study presented in this work will be complemented with a second part in which the estimation of porosity of mortar, from micromechanical model and ultrasonic velocity, will be performed.

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