

# Application of a micromechanical model of three phases to estimating the porosity of mortar by ultrasound

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## Abstract

Among the factors that affect the durability of cement-based materials, porosity is important due to its role in transporting substances inside the material. Porosity can be determined using destructive testing methods, but the civil construction industry needs a nondestructive method for the estimation of the volume of pores in cement-based materials to evaluate the deterioration process.

In this investigation, a micromechanical model of three phases developed in a previous paper is used to estimate the porosity of a series of mortar samples. The proposed model takes into account the microstructural characteristics and elastic properties of the constituent phases. In a first stage, the model is used to predict the influence of both the volume fraction of sand and pores and the elastic constants of the matrix and the sand on the ultrasonic velocity. Next, experimental measurements are made on a series of mortar samples of varying water/cement ratio, cement type and sand concentration, with the results used to demonstrate that porosity can be estimated using the proposed nondestructive method.

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## 1. Introduction

A common family of structural building materials is, of course, cements, mortars and concretes. The durability of these cementitious composites is affected by factors such as the quality of the raw components from which the material is made, the environment within which the material is used and their porous structure. Today, the porous structure of a cement-based material is characterized by means of destructive testing by qualified workers in the laboratory. Several methods exist for estimating the volume of pores in this type of materials, including mercury porosimetry, ASTM C 642-90 C standard and electronic microscopy. These methods can add substantially to the costs of main-

tenance, and yet, they do not provide a means for total inspection of the structure or for taking measurements in situ. For these reasons, nondestructive testing (NDT) is useful for the characterization of porosity in cement-based materials.

The characterization of porosity by means of NDT is a topic discussed relatively little in the literature. A homogenization theory based on the collision of ultrasonic waves with inclusions has been proposed [1]. This method has been applied to estimate the porosity in rocks and also used to evaluate the behavior of longitudinal ultrasonic waves velocity through rocks saturated with water. The utility of this approach has been recognized and applied to cement-based materials, but the method had limited success for the characterization of the porous structure [2]. Mehta and Monteiro [3] exposed the advantages and disadvantages of the different models as applied to cement composites, identifying the differential method and model from Mori

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and Tanaka [4], as most consistent when studying the influence of porosity and cracks on the elastic modulus. These models take into account two phases—solid (matrix) and a fluid phases (pores or cracks)—and their microstructural information—elastic properties, volume fraction and geometry.

In two previous papers, we have applied the biphasic micromechanical model formulated by Jeong et al. [5], to estimate the porosity [6] and the mechanical properties [7] in mortar by means of ultrasonic methods, considering the material to be composed of two phases: a solid (cement paste plus sand) and a fluid phase (pores).

However, to characterize cement composites (e.g., mortar), it is more convenient to take into consideration the influence of the elastic and microstructural characteristics of three material phases (cement paste without pores, aggregates and pores). However, as it is not possible to manufacture cement samples *without* pores, estimating the elastic characteristics of the nonporous matrix constitutes a specific problem, which has been studied in Ref. [8].

In a previous paper [9], a micromechanical model of three phases has been proposed that considers the microstructural properties, such as the elastic constants, volume fractions, geometry and distribution and orientation of the inclusions. In this paper, we apply the micromechanical model of three phases to mortar with two objectives: (1) to study the influence of the elastic properties and volume fraction of the mortar phases on the ultrasonic wave velocity and (2) to estimate the porosity of mortar samples that have been fabricated with different cement types and water/cement ratios (w/c).

## 2. Micromechanical model of three phases

To study the influence of porosity in the cement composites of three phases, the material is considered to be formed by a solid matrix (cement paste without pores) and two types of inclusions: pores and sand.

In the three-phase model, the volume fraction of the matrix  $v^m$  is given by:

$$v^m = 1 - x - v^a \quad (1)$$

where  $v^a$  represents the volume fraction of sand and  $x$  is the volume fraction of the pores (which is unknown).

If it is taken that the density and elastic constants of pores are zero and that the three phases are isotropic, the elastic tensor is reduced to two independent elastic constants, given by Ref. [10]:

$$C_{11} = C_{11}^m - \frac{x(C_{11}^m - 4/3C_{44}^m)\langle T^p \rangle}{1 - x - v^a + x\langle T^p \rangle + v^a\langle T^a \rangle} + \frac{v^a(C_{11}^a - 4/3C_{44}^a)\langle T^a \rangle}{1 - x - v^a + x\langle T^p \rangle + v^a\langle T^a \rangle} + \frac{8/3x(-C_{44}^m)\langle T_{1212}^p \rangle + 8/3v^a(C_{44}^a - C_{44}^m)\langle T_{1212}^a \rangle}{1 - x - v^a + 2x\langle T_{1212}^p \rangle + 2v^a\langle T_{1212}^a \rangle} \quad (2)$$

$$C_{44} = C_{44}^m + \frac{x(-C_{44}^m)2\langle T_{1212}^p \rangle + v^a(C_{44}^a - C_{44}^m)2\langle T_{1212}^a \rangle}{1 - x - v^a + 2x\langle T_{1212}^p \rangle + 2v^a\langle T_{1212}^a \rangle} \quad (3)$$

$$\rho = (1 - x - v^a)\rho^m + v^a\rho^a \quad (4)$$

where:

$$C_{11}^a = C_{11}^a - C_{11}^m \quad C_{44}^a = C_{44}^a - C_{44}^m \quad (5)$$

$$\langle T^p \rangle = \langle T_{1111}^p \rangle + 2\langle T_{1122}^p \rangle \quad \langle T^a \rangle = \langle T_{1111}^a \rangle + 2\langle T_{1122}^a \rangle \quad (6)$$

Where  $C_{ij}$  is the independent components of the elastic tensor in reduced notation as it is shown in Ref. [10]. The superscripts m, a and p refer to quantities related to the nonporous matrix and types 'a' and 'p' inclusions, respectively.  $T^p$  and  $T^a$  represent the Wu's tensor [11] in global coordinates for pores and aggregates, respectively, and the angle bracket  $\langle \rangle$  denotes the average over all possible orientations. The tensor  $\langle T \rangle$  is a function of the geometry, the distribution and orientation of the inclusions and can be calculated using the method given in Ref. [11].

The independent elastic constants are related to the longitudinal ( $v_L$ ) and transversal ( $v_T$ ) velocities by:

$$v_L = \sqrt{\frac{C_{11}}{\rho}} \quad (7)$$

$$v_T = \sqrt{\frac{C_{44}}{\rho}} \quad (8)$$

From these expressions, we obtain a nondestructive method for determining the microstructural characteristics of the three phase composites, such as porosity.

Table 1  
Properties of constituent phases of mortar

	$C_{11}$ (GPa)	$C_{44}$ (GPa)	$\rho$ (kg/m <sup>3</sup> )
Matrix (cement paste)	30...60	13	2200
Sand	80.7	32.64	2600

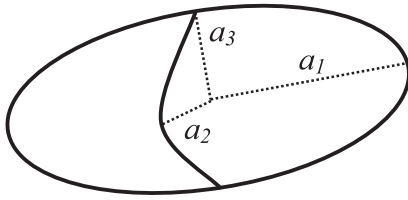


Fig. 1. Ellipsoid that describes a general form of the inclusions. ( $a_1$ ,  $a_2$  and  $a_3$  are ellipsoid radii).

In the following discussion, the study of the influence of the microstructural characteristics on the ultrasonic velocity will be examined.

### 3. Theoretical prediction of the ultrasonic velocity from the three-phase model in cement composites.

The purpose of this theoretical study is to predict the change in the ultrasonic wave velocity as a function of the elastic and microstructural properties of a series of mortars. To this end, mortars with properties similar to the mortar samples used in the experimental phase have been selected (see Table 1).

In the micromechanical model, the geometry of the inclusions is represented by an ellipsoid and is characterized by means of Eshelby's tensor. Formulas for  $\mathbf{S}$  for various shapes of inclusions can be found in Ref. [10]. For an isotropic matrix,  $\mathbf{S}$  is given by the Poisson's ratio of the matrix and by the aspect ratio  $\alpha$  or relative size of the three axes  $a_1$ ,  $a_2$  and  $a_3$  of the inclusion, as shown in Fig. 1. The geometry of inclusion is modeled as a spheroid. The spherical inclusions can be characterized by the aspect ratio  $\alpha$  and by the corresponding orientation distribution function. If the aspect ratio is defined by the relationship between the different axes and the two coincident axes, then  $\alpha = a_3/a_1 < 1$  for an oblate spheroid. Similarly,  $\alpha = a_1/a_3 > 1$  for a prolate spheroid and  $\alpha = 1$  for the spherical inclusions.

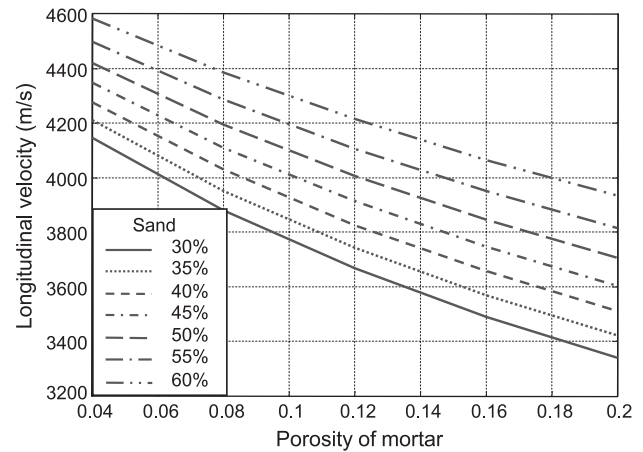


Fig. 3. Influence of porosity and volume fraction of sand on the longitudinal velocity.

The change of the elastic properties of the nonporous matrix, as a function of the volume fraction of pores and sand, was evaluated. Two pore geometry cases were considered: spheres ( $\alpha=1$ ; representing the air voids) and prolate spheroids ( $\alpha \gg 100$ ) (representing capillary pores). Previous studies have shown that the shape of the sand inclusions has little influence on the ultrasonic velocity [8], and hence, sand was modeled as a sphere.

The influence on the ultrasonic velocity of the porosity, type of sand and pore geometry were studied. The elastic properties of sand were kept constant, while the properties of the matrix,  $C_{11}^m$  (Table 1) and the porosity were varied.

Fig. 2 shows the longitudinal velocity behavior with changing porosity and changing elastic constant  $C_{11}^m$  of the matrix while  $C_{11}^m$  is fixed to 13 GPa. An increase in the stiffness of the matrix and in the level of porosity causes an increase and decrease, respectively, of the longitudinal velocity, with the change being of approximately the same magnitude (250 m/s, approximately). On the other hand, prolate pores exhibit the lowest velocities.

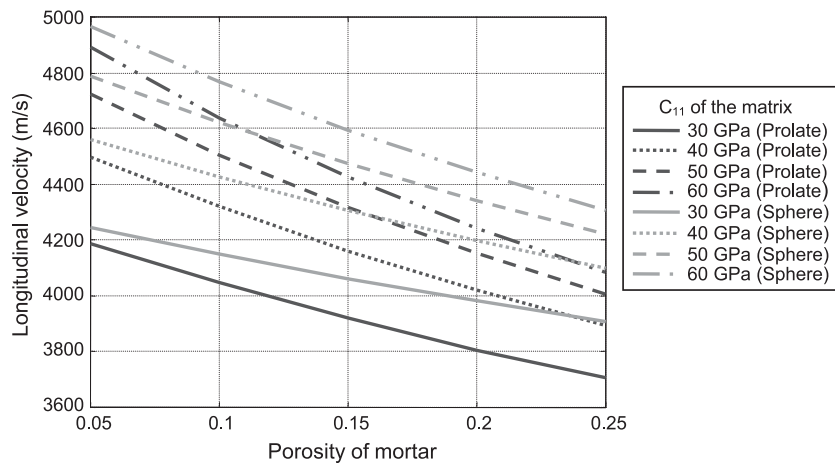


Fig. 2. Influence of the porosity, geometry of pores and elastic constants of the matrix on the longitudinal velocity. --Prolate pore ( $\alpha=1000$ ); -- spherical pore ( $\alpha=1$ ).

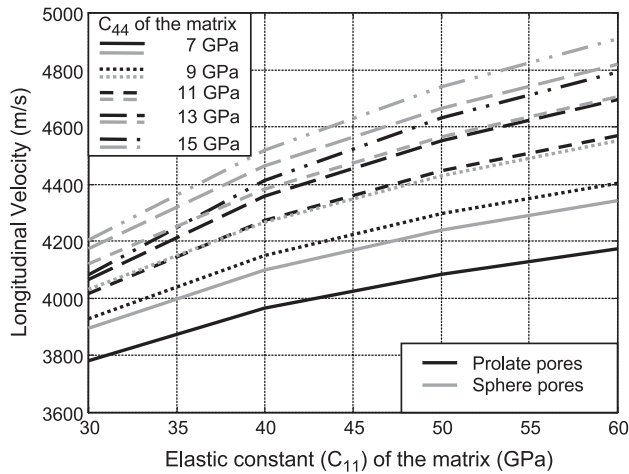


Fig. 4. Influence of the elastic properties of the matrix on the longitudinal velocity in mortar. --Prolate pore ( $\alpha=1000$ ); -- spherical pore ( $\alpha=1$ ).

The influence of the volume fraction of sand and the level of cement paste porosity on the ultrasonic velocity was also analyzed. For this, the volume fraction of both sand and pores were varied by changing the volume of sand for a given level of porosity of cement paste. The elastic properties of the cement paste were kept constant to  $C_{11}^m=40$  GPa and  $C_{44}^m=10$  GPa, respectively. It is assumed that these properties remain constant despite changing the volume fraction of sand in the samples from 30% to 60%. Fig. 3 shows that an increase in sample porosity causes a decrease in the longitudinal velocity, while an increase in the volume fraction of sand induces the opposite effect (an increase in the velocity). From Fig. 3, it can be seen that an increase in the volume fraction of sand results in a small decrease in the slope of the velocity–porosity curve. When comparing, for example, the curves corresponding to a volume fraction of 35% and 60%, in the former, the velocity decreases by approximately 800 m/s as mortar porosity is increased from 0.04 to 0.2, while in the latter, the decrease is limited to approximately 650 m/s over the same porosity

Table 2

Description of groups of samples of mortar

Group of samples	Cement type	Water/cement ratio	Sand/cement ratio
CI-04 1/1	Cement 32.5 MPa	0.4	1/1
CI-05 1/1	Cement 32.5 MPa	0.5	1/1
CII-04 1/1	Cement 42.5 MPa	0.4	1/1
CII-05 1/1	Cement 42.5 MPa	0.5	1/1
CI-04 2/1	Cement 32.5 MPa	0.4	2/1
CI-05 2/1	Cement 32.5 MPa	0.5	2/1
CII-04 2/1	Cement 42.5 MPa	0.4	2/1
CII-05 2/1	Cement 42.5 MPa	0.5	2/1

range. To amplify, a 5% increment in the volume fraction of sand produces a less significant decrease in the longitudinal velocity in the sample with lower levels of porosity.

To evaluate the influence of the elastic properties of the matrix on the ultrasonic velocity, it is assumed that the density of the matrix is known and constant (Table 1) and that pores are randomly oriented in the matrix at a fixed volume fraction (25%). In this case, the elastic constants of the matrix,  $C_{11}^m$  and  $C_{44}^m$ , are varied while the elastic properties of the sand are kept constant, as shown in Table 1. The volume fraction of sand is fixed at 45%. Fig. 4 shows that an increase in the longitudinal velocity occurs with increasing the elastic constants of the nonporous matrix. The samples with spherical pores exhibit the highest velocity.

Fig. 5 shows the influence of  $C_{11}^m$  and  $C_{44}^m$  on the transverse velocity. The increase in elastic constants of the matrix results in an increase in the transverse velocity, with similar slopes exhibited regardless of pore geometry. Once again, the spherical pores display the highest velocities.

#### 4. Experimental validation

The objective of the experimental validation is to verify the previously obtained theoretical results and estimate the

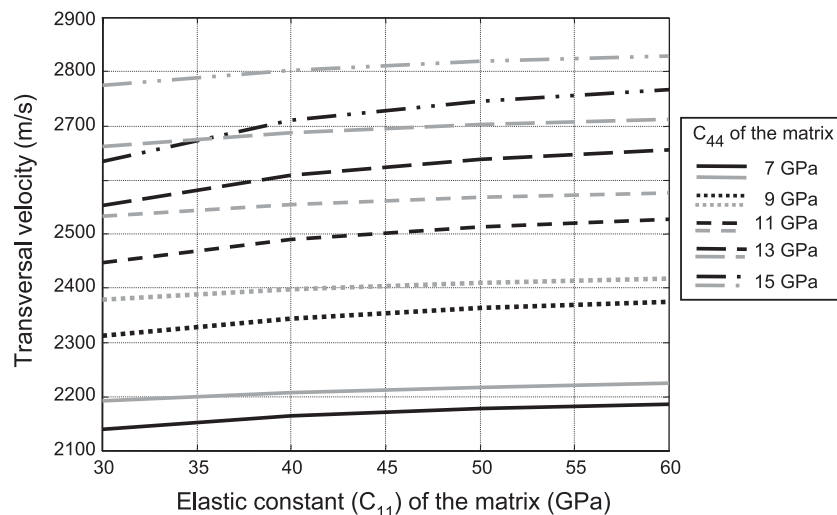


Fig. 5. Influence of the elastic properties of the matrix on transverse velocity in mortar. --Prolate pore ( $\alpha=1000$ ); -- spherical pore ( $\alpha=1$ ).

Table 3

Porosity destructive measurements and ultrasonic longitudinal velocity (mean value and standard deviation) of the mortar samples

Groups of samples	Porosity (%)	Velocity (m/s)
CI-04 1/1	12.6	4281 (24)
CI-05 1/1	18.2	4035 (19)
CII-04 1/1	10.7	4384 (18)
CII-05 1/1	14.8	4136 (27)
CI-04 2/1	10.3	4544 (51)
CI-05 2/1	13.6	4334 (13)
CII-04 2/1	8.6	4625 (22)
CII-05 2/1	12.5	4347 (12)

porosity in cement-based materials from developed ultrasonic techniques using the micromechanical model of three phases. First, destructive techniques were used to measure the porosity of the samples. Second, measurements of ultrasonic velocity were made, the results used in the micromechanical model to theoretically estimate porosity. Finally, the theoretical porosity measurements were compared with those determined through destructive measurement to establish the utility of the modeling approach.

Destructive and nondestructive testing was made on mortar samples that were considered to consist of three phases: a matrix (nonporous cement paste) and two types of inclusions (sand and pores).

#### 4.1. Description of the samples

Two equal collections of 12 mortar samples of  $40 \times 40 \times 160 \text{ mm}^3$  were fabricated, with each collection consisting of three groups (three samples per group). To obtain samples with different degrees of porosity, the water/cement ratio (w/c) and compressive strength of cement were varied. Water to cement ratios of 0.45 and 0.50 were used. The compressive strength was varied by controlling the fineness of the cement powder during the fabrication

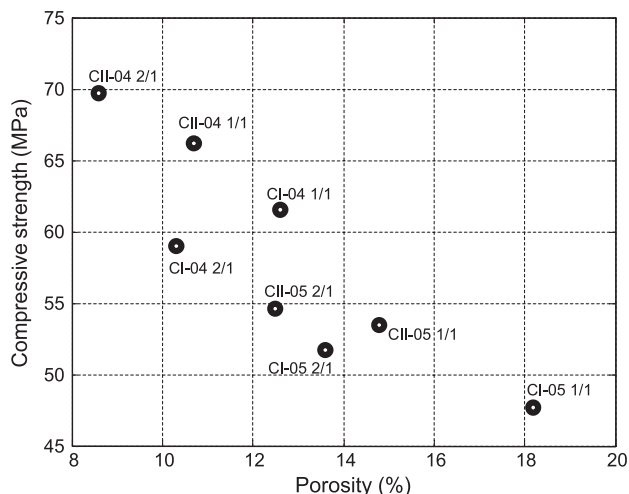


Fig. 6. Mortar compressive strength vs. porosity.

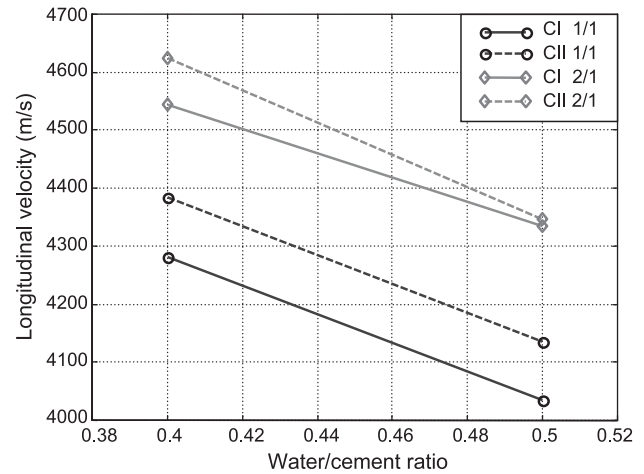


Fig. 7. Longitudinal velocity of mortar samples.

process. Two different strengths were obtained: 32.5 and 42.5 MPa. To study the influence of volume fraction of sand, two sand/cement ratios were used: 1/1 and 2/1. Table 2 describes the groups of samples of mortar.

In addition to the mortar samples, in previous experiments [12], groups of cement paste samples with different degrees of porosity (as given by the water/cement ratio) were used to evaluate the elastic constants of the nonporous matrix. The results obtained are used in this work.

#### 4.2. Results of destructive testing over mortar samples

Destructive testing measurements of the fraction of open porosity were obtained by the standard ASTM C 642-90 of the American Society for testing and materials. The mean value for each group is shown in Table 3. Fig. 6 shows the relationship between compressive strength and porosity in the mortar samples. From the measurements, it can be seen that an increase in w/c ratio results in an increase in porosity and a reduction of the compressive strength. The addition of sand and the use of cement with higher compressive strength produce a reduction in the volume of pores.

#### 4.3. Measurement of ultrasonic velocity in mortar samples

To characterize the mortar samples by means of non-destructive testing, it was necessary to make measurements

Table 4

Elastic constants of the nonporous matrix of the cement paste

Groups of samples	Elastic constants of matrix		
	$C_{11}$ (GPa)	$C_{44}$ (GPa)	$E$ (GPa)
CI-04	46.00	12.90	36.35
CI-05	39.62	11.54	29.87
CII-04	45.70	12.75	33.31
CII-05	40.43	11.37	29.66



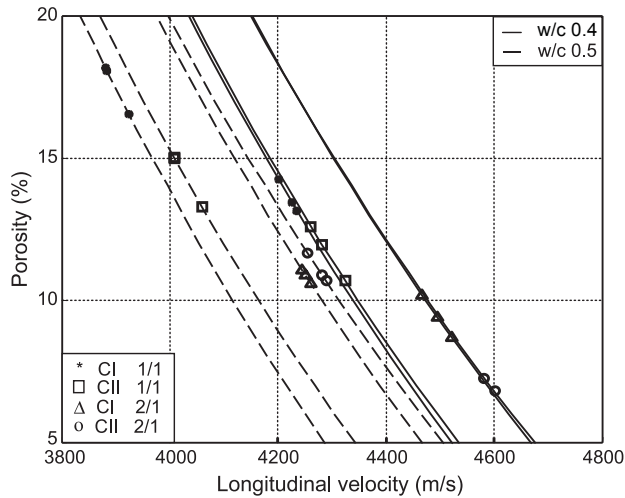


Fig. 8. Curves representing the theoretical function that relates velocity to porosity in mortar. The estimated porosity of the samples, which is obtained from the measured velocity, is also shown.

of ultrasonic velocity. These measurements were made once the sample had suitably matured (28 days).

The ultrasonic velocity was measured using a wide-band ultrasonic transducer with a central frequency of 2 MHz. The transmission technique was employed with medical gel used as the coupling material. The ultrasonic signals were acquired at a sampling frequency of 66 MHz using the SENDAS system [13]. The longitudinal velocity measurement is based on the propagation time through the material.

An increase in the w/c ratio results in a decrease in longitudinal velocity, while an increase in the compressive strength has the opposite effect (Fig. 7). In this case, it can be seen that an increase in the volume fraction of sand induces an increase in the longitudinal velocity by approximately 250 m/s. Table 3 shows the mean velocity and standard deviation for the eight groups of samples.

#### 4.4. Estimation of porosity in mortar samples from micro-mechanical model

To apply the model of three phases [9] to the mortar samples, the following assumptions were made:

- Mortar is a three-phase material consisting of a matrix of cement paste and two types of inclusions—sand and pores.
- The pores are modeled like cylinders of infinite length distributed randomly into the matrix.
- The elastic properties of sand are known with grains of spherical geometry distributed randomly into the matrix.
- The elastic constants of nonporous matrix were calculated in Ref. [12], from cement paste samples with the same w/c ratio and cement type as used in this work. These constants were calculated using destructive and nondestructive measurements and applying a two-phase micromechanical model [5]. Table 4 shows these elastic constants.

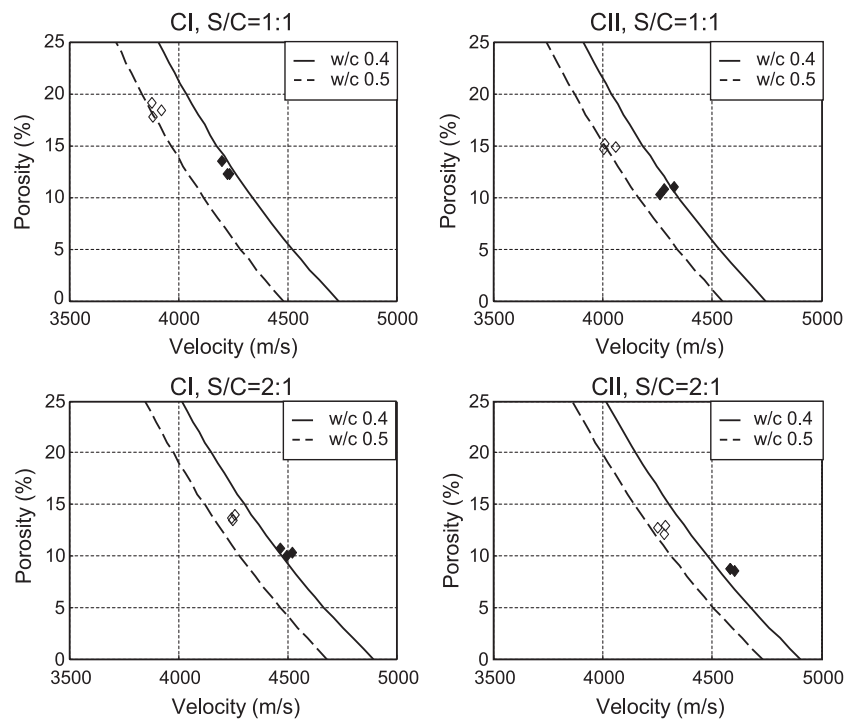


Fig. 9. Graphics of the measured porosity by destructive methods related to the ultrasonic velocity and theoretical curves from the micromechanical model (◆: w/c=0.4; ◇: w/c=0.5).

Table 5  
Mean error on the estimation of porosity of mortar samples

Groups of samples	CI-04 1/1	CI-05 1/1	CII-04 1/1	CII-05 1/1	CI-04 2/1	CI-05 2/1	CII-04 2/1	CII-05 2/1
Error (%)	7.7	4.5	9.7	2.3	8.8	20.7	18.1	11.9

From the assumptions above, the micromechanical model of three phases has been applied to mortar samples by coupling Eqs. (2) and (3) with Eqs. (7) and (8). The theoretical results are shown in Fig. 8. Eight curves were drawn representing the theoretical relation between porosity and longitudinal velocity, each one being a function of the volume fraction of sand, type of cement and water/cement ratio. The theoretical porosity obtained from the velocity measurements is also shown.

Fig. 8 shows that the curves are well separated by w/c and sand/cement ratios (s/c), while the separation by cement type is not clear. Furthermore, the samples with the lowest volume fraction of sand and the lowest water-to-cement ratio (CI-04 1/1 and CII-04 1/1) are similar to the samples with the highest volume fraction of sand and w/c ratio (CI-05 2/1 and CII-05 2/1). In the figure, it can also be observed that an increase in porosity causes a decrease in longitudinal velocity, while an increase in the volume fraction of sand and the compressive strength of cement induces the opposite effect.

Fig. 9 shows the porosity of the samples, measured by destructive testing, in relation to the ultrasonic velocity. The porosity can also be estimated using the micromechanical model of three phases. The porosity error is the vertical distance from the measured porosity to its theoretical curve. The mean error for every group of samples is shown in Table 5. From these results, it can be seen that the error in estimation of porosity for the groups of mortars with the lowest proportion of sand (1/1) are lower than 10%. The mean error in the estimation of porosity of mortar samples using the model of three phases is 10.56%. These estimations are good if we consider that in cementitious materials, the errors in destructive testing are often of the order of 10%. Moreover, these predictions were made based on an estimation of elastic constants of the nonporous matrix from the destructive testing, which has its own margin of error of 4%.

## 5. Conclusions

In this investigation, a new methodology for estimating the porosity in cement-based materials by means of nondestructive testing has been proposed. This methodology is based on coupling the micromechanical model of three phases with ultrasonic velocity measurements.

The influence of volume fraction and geometry of pores, and the geometry and elastic properties of sand, on the ultrasonic velocity was studied using the micromechanical model of three phases. This study suggests

that an increase in porosity causes a decrease in the ultrasonic velocity and that an increase in the volume fraction or elastic constant of sand results in the opposite effect.

To validate and contrast the predictions of the micromechanical model, destructive (porosity and density) and nondestructive tests (ultrasonic velocity measurements) were conducted on mortar samples with varying degrees of porosity, determined by the water/cement ratio of the sample, cement type and volume fraction of sand.

The application of the model of three phases to mortar samples suggests that an increase in water/cement ratio results in a reduction in the ultrasonic velocity, while an increase in the compressive strength of cement and the volume fraction of sand increases the ultrasonic velocity. The porosity (as determined using the nondestructive testing technique outlined in this paper) was estimated with a mean error of 10%.

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