

Relationship between the Bingham parameters and slump

Jon Elvar Wallevik *

The Norwegian University of Science and Technology, Department of Structural Engineering, Richard Birkelandsvei 1a, 7491 Trondheim, Norway

Received 19 May 2005; accepted 5 March 2006

Abstract

Viscometers in general have never been particularly popular at the jobsite. They are however well suited at the laboratory as they measure concrete consistency in terms of fundamental physical quantity, known as the yield stress and plastic viscosity. In contrast to viscometers, the slump cone is by far the most accepted tool for measuring consistency at the jobsite. This is due to its simplicity in handling. With the significance of both types of devices, it is clearly important to relate them to each other. The result of this study suggests a relationship between the yield stress and slump that depends on the concrete mixture proportions. More precisely, a particular trend line between the yield stress and slump seems to depend on volume fraction of matrix used in the concrete. The study shows a low correlation between the slump and plastic viscosity.

© 2006 Elsevier Ltd. All rights reserved.

Keywords: Fresh concrete; Rheology; Workability; Slump

1. Introduction

1.1. Terminology of the fresh concrete consistency

In practice, concrete that can be readily placed into formwork is referred to as workable. This is a rather loose description of the rheological properties of the fresh concrete and can also be very subjective, depending on the type of formwork, type of concrete and the means of compaction available at jobsite [1]. Terms like workability, consistency, flowability, mobility and pumpability have been used to describe the rheological behavior of fresh concrete. However, these terms more reflect personal viewpoints than scientific precision [1,2]. For example, an interesting discussion about the subjectiveness of the term *workability* is given in a textbook by Tattersall and Banfill [2]. Other similar terms are also discussed in the same literature. Many of these terms rely on visual observation and subjective assessment [2]. The primary problem is that there is no guarantee that these terms mean the same thing to different people. For example, there has been disagreement between

different workers about the exact meaning of the term *workability* [3].

1.2. The slump test

In the attempt to quantify the rheological behavior of fresh concrete, rheometers of different types and quality have been developed. One of the most famous and oldest tests is the slump test. Because of its simplicity, this method is used extensively in site work all over the world. The apparatus was developed in the USA around 1910 [4]. It is believed that it was first used by Chapman although in many countries the test apparatus is associated with Abrams [4].

The slump test gives only a single value, namely the slump value. Such single value workability test has been criticized on the basis that the same value may be produced by two concretes with quite different rheological characteristics [2,5,6]. It discusses the need for describing the rheological properties of fresh concrete in terms of fundamental physical quantities, not depending on the details of the apparatus with which they are measured. As a good first approximation, it is generally agreed that these fundamental physical quantities are the yield stress and plastic viscosity. More precisely, it is commonly agreed that the fresh concrete can with good accuracy be considered as

* Tel.: +47 73 59 47 00; fax: +47 73 59 47 01.

E-mail address: jon.wallevik@ntnu.no.

Bingham fluid [2,5–7]. The Bingham model can be represented with the following equation:

$$\tau = \mu \dot{\gamma} + \tau_0 \quad (1)$$

The term τ is the shear stress [Pa], τ_0 is the yield stress [Pa], μ is the plastic viscosity [Pa·s] and $\dot{\gamma}$ is the shear rate [s⁻¹].

1.3. The coaxial cylinders viscometer

To measure the Bingham parameters, namely the yield stress τ_0 and plastic viscosity μ , viscometers of different types and quality have been developed. One of such devices is the coaxial cylinders viscometer and it is such a device that is used in this work.

According to Tattersall and Banfill [2] a coaxial cylinders viscometer was not used for concrete until after about 1970. Motivated by the fact that such instrument had already been used for mortar and cement paste for quite a while before this, Tattersall made an attempt to apply coaxial cylinders geometry to measure the rheological properties of fresh concrete [2]. Unfortunately, he was not successful. After this in 1973, he introduced the use of a modified food mixer [2,5,8] to extract the Bingham parameters. This system is known as the Mk-I. A further development of the Mk-I resulted in the famous Mk-II and Mk-III devices. The Mk-systems are still being used and are continuously going through improvements as reported in different papers [6,7,9].

In the beginning of the 1970s, trials were made by others [10,11] using the coaxial cylinders system, which were reported to be somewhat more successful than the work done by Tattersall. To avoid slippage, the inner and outer cylinders consisted of protruding vanes. In the late 1980s, a further improvement of the coaxial cylinders system was made in Norway [12,13], which included that the bottom part of the inner cylinder did not measure torque. This was done to avoid the effect of shear stress generated from the bottom plate of the bucket that contains the fresh concrete. This approach is validated by numerical methods in Ref. [14]. Officially, this viscometer is named the ConTec BML Viscometer 3, but is commonly known as the BML viscometer.

1.4. Objectives

Viscometers in general (for example the BTRHEOM [15,16], Mk or the BML) have never been particularly popular outside the laboratory. This is due to their drawbacks, which are cost, immobility (weight and size) and difficulty in use. The slump cone is still by far the most accepted tool for measuring concrete consistency at building sites. Even for self-compacting concrete (SCC) this device is still being used; only the test procedure is changed. At least three different test procedures are used for SCC, named the slump flow test, the invert slump flow and the JRing test.

The large success of the slump cone is most likely due to its simplicity. Other empirical tests like the L-box or Orimet [4], which are a bit more complicated in operation, do not seem to be able to compete with the slump cone at the jobsite. Hence, it is likely that the slump device will dominate at the building site for many decades to come.

The BML viscometer is well suited for research. An example of such research is comparing different admixture types. Another example is to develop mixture proportions of SCC batch to be used for a very special application. It is clear to anyone that has used a viscometer for a while that it is irreplaceable in the laboratory, when the goal is to properly characterize the rheological behavior of the fresh concrete.

With the obvious benefits of the slump cone at the jobsite and the equally obvious benefits of the viscometer at the laboratory, it is clearly important to relate the two devices. Of course, since the slump cone gives only a single value, while the fresh concrete is a many parameter fluid, the same slump value S can in principle represent two different concrete consistencies. The objective of this article is to investigate the relationship between the slump S and the yield stress τ_0 . Also, the correlation between slump S and plastic viscosity μ will be addressed. Here, the yield stress and plastic viscosity are measured with the BML viscometer. As the outcome presented in this paper is based on limited number of results, a complete and universal relationship cannot be established and this is neither the intention. Rather, the aim is to give the reader an insight about what a particular slump value can mean, how to evaluate it from a scientific point of view and what are the limitations of this number. The purpose of this article is by no means to criticize the slump cone, which is both an elegant and simple device, but rather to understand the value it gives for the particular range of concrete mixes tested.

2. Experimental program

2.1. Fresh concrete as a suspension

The fresh concrete consist of particles with a broad range of mass, dimension, shape and surface texture. Hence, the distinction between matrix and suspended particles becomes a matter of choice, in contrast to the more traditional suspension of spheres submerged in a Newtonian liquid. Here, the filler modified cement paste (i.e. all particles with diameter below 0.125 mm) is defined to be the matrix. As such, aggregates above 0.125 mm are defined as the suspended particles. This definition is in accordance to the work done by Mørtzell et al. [17]. Here, the term V_m will designate the volume fraction of matrix in liters per cubic meter of concrete or l/m³. It should be noted that in Ref. [17] the entrained air is not included in the matrix phase. This approach is used in this work as well. As the air was measured about 2%, the total (or actual) volume fraction of matrix is $V_m + 20$ l/m³.

Three basic concrete mixture proportions are used. As shown in Table 1, they consist of different volume fraction of matrix V_m , different w/c -ratio and different amount of admixture. To generate concretes with large variety of rheological characteristics, up to eight different admixtures are used. These are plasticizers and superplasticizers of various types.

2.2. Materials

The eight admixtures include six types of lignosulfonates, one type of naphthalene based superplasticizer and one polycarboxylate based superplasticizer. The amount used is shown in Table 1

Table 1
Mixture proportions of the concrete batches

w/c	Matrix V_m (l/m ³)	Cement (kg)	% SP	Aggregate (kg)	Density (kg/m ³)
0.4	345	442	0.6	1762	2396
0.5	331	371	0.3	1802	2372
0.6	321	320	0.1	1829	2354

marked as % SP, meaning percentage of dry admixture relative to cement content (in weight). The admixture is premixed with water prior to water addition. The effect of each admixture is not the subject of this article and is therefore not discussed.

Norcem Standard Cement (CEM I 42.5R acc. to EN 197-1:2000) is used. It is Ordinary Portland cement with density and fineness (Blaine) of 3120 kg/m³ and 340 m²/kg, respectively. About 5% of the cement consists of gypsum and the loss of ignition is 2.4%. The clinker composition is nominally as follows: C₃S: 60%; C₂S: 15%; C₃A: 7.5%; C₃AF: 10%; Na₂O-eqv: 0.95%.

The aggregates are supplied by NorStone AS (Norway). It consists of gneiss/granite and its density is 2670 kg/m³. No less than 70% of the 0–8 mm fraction consist of uncrushed materials. Aggregates larger than 8 mm consist of about 30% of natural and about 70% of crushed materials. The grading curves are shown in Fig. 1. The moisture content of the 0–8 mm grading was kept at 3.5%, while the 8–16 mm grading was about 0%. The absorption of the 0–8 mm and 8–16 mm aggregates are 0.8% and 0.5%, respectively. The amount of aggregates shown in Table 1 is relative to (bone) dry condition.

2.3. Apparatus, mixing procedure and measuring procedure

A 50 l mixer from Maschinenfabrik Gustav Eirich of type SKG1 is used in (re)mixing the concrete batches, before a rheological measurement. To measure the concrete consistency, the BML viscometer is used with the C-200 system. It is the outer cylinder (of radius $R_o=0.145$ m) that rotates at predetermined frequencies f_o [rps], while the inner cylinder (of radius $R_i=0.100$ m and height $h=0.199$ m) is stationary and measures torque T [Nm]. By measuring the applied torque T at different frequencies f_o , one can fit these values with a straight line. From the slope H and the point

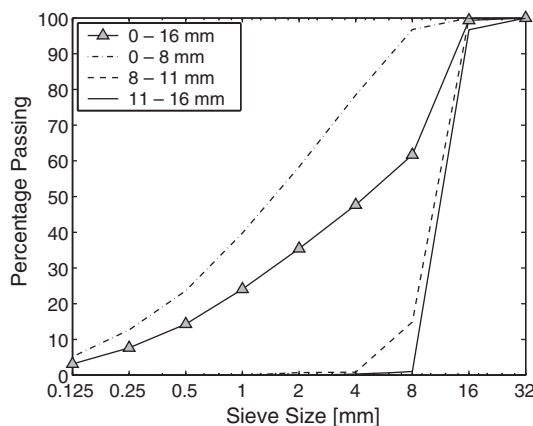


Fig. 1. The aggregate grading as percentage passing versus sieve size.

Table 2
Mixing and measuring procedure for the concretes

Time (min)	
–1	Dry mixing of aggregate and cement.
0	Addition of water (premixed with admixture).
1	Mixer stop.
3	Mixer start.
5	Mixer stop: mixing finished.
10	First measurement.
40	Second measurement.
70	Third measurement.
100	Fourth measurement.

of intersection with the ordinate G , the plastic viscosity μ and the yield stress τ_o can be calculated by the two following equations.

$$\mu = \frac{H(1/R_i^2 - 1/R_o^2)}{8\pi^2 h} \quad (2)$$

$$\tau_o = \frac{G(1/R_i^2 - 1/R_o^2)}{4\pi h \ln(R_o/R_i)} \quad (3)$$

The above equations are based on the *Reiner–Riwlin equation*. Further description about this equation can for example be found in Ref. [14]. It should be noted that G and H not only depend on the rheological properties, but also on the dimensions of the coaxial cylinders viscometer (i.e. on R_i , R_o and h). The purpose of Eqs. (2) and (3) is to filter out the dimensional effect from H and G so only the intrinsic flow properties τ_o and μ remain.

The slump apparatus consist of a mould in the shape of a truncated metal cone, open at both ends. The internal diameter of the slump cone is 200 mm at the base, 100 mm diameter at the top and has a height of 300 mm. Basically the test procedure consists of filling the metal cone in three layers with tamping. Thereafter, the metal cone is lifted, leaving the concrete sample behind which *slumps down* by the action of gravity. All slump values were recorded to the nearest 5 mm. A further description about the test can be found in ASTM C 143-90a and BS 1881:Part 102:1983.

The mixing and measuring procedure for each concrete batch is shown in Table 2. The yield stress τ_o , plastic viscosity μ and slump S were measured at 10, 40, 70 and 100 min after water addition. Air and density ρ were measured after the final rheological measurement at 100 min. Prior to each of the measurements at 40, 70 and 100 min, the concrete was remixed in the Gustav Eirich mixer for about 2 min. Some mixes were batched and measured more than once to test the reproducibility of the rheological results. Additional batches were mixed as well, to test and verify stability of air (about 2%) and density.

3. Results and discussion

The relationship between the slump S , yield stress τ_o and plastic viscosity μ for all the concrete batches are shown in Figs. 2, 3 and 4. The results are categorized relative to the volume fraction of matrix V_m (see also Table 1). In each figure, the main illustration shows the relationship between the slump S and the yield stress τ_o , while the

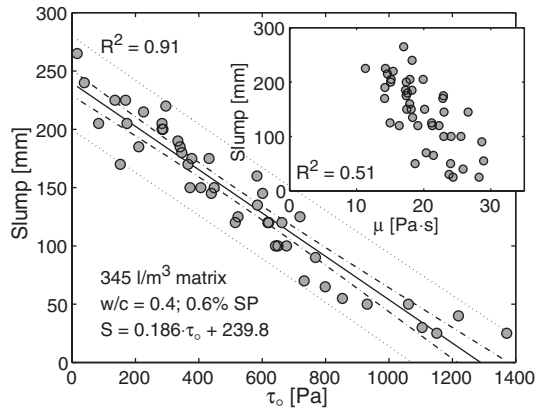


Fig. 2. Plot of slump as a function of yield stress τ_0 (main illustration) and plastic viscosity μ (small illustration) for the case of 345 liter matrix per cubic concrete.

small integrated illustration shows the relationship between the slump S and plastic viscosity μ . In each main illustration, the dashed dotted line represents the 95% confidence interval (based on t -distribution) for the regressed line $S = a \cdot \tau_0 + b$. That is, it is with 95% confidence that the true (and unknown) function $S = S(\tau_0)$ exists within the boundary of the two dashed dotted lines. The dotted lines represent the 95% prediction interval (also based on t -distribution); i.e. there is a 95% probability that the next measurement falls within the boundary of the two dotted lines of each illustration. For further readings about the statistical approach, see Walpole et al. [18].

3.1. Relationship between the slump and yield stress

As the slump cone is lifted, the concrete sample slumps down by the action of gravity. Being a yield stress fluid, the concrete stops flowing downward when the applied shear stress (by gravity) becomes less than the yield stress τ_0 . Hence, for traditional concrete, it is not unexpected that there is a strong relationship between the measured slump value S and the yield stress τ_0 . As shown in Figs. 2, 3 and 4, such relationship is found. For the slump S and plastic viscosity μ , there is no such good correlation.

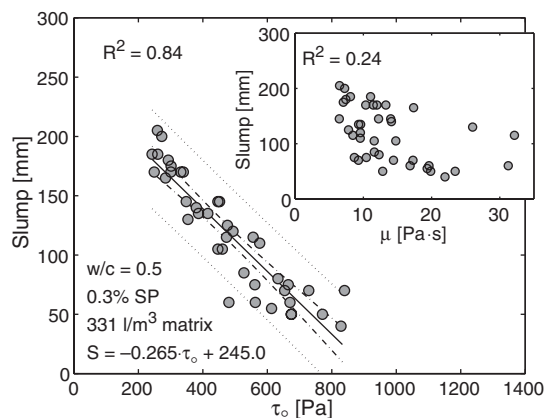


Fig. 3. Plot of slump as a function of yield stress τ_0 (main illustration) and plastic viscosity μ (small illustration) for the case of 331 l matrix per cubic concrete.

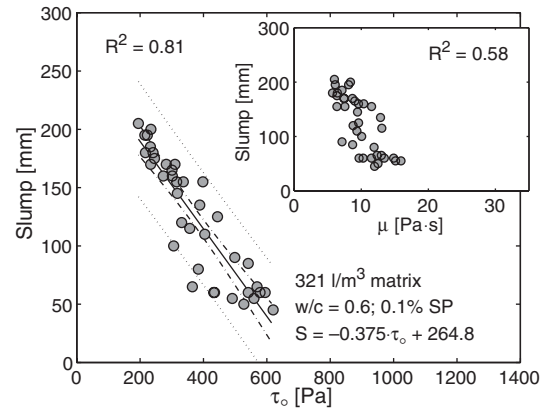


Fig. 4. Plot of slump as a function of yield stress τ_0 (main illustration) and plastic viscosity μ (small illustration) for the case of 321 liter matrix per cubic concrete.

There have been several attempts to find a relationship between the plastic viscosity μ , yield stress τ_0 and slump S . Tanigawa et al. [19,20] proposed a graph showing this relationship. Their study indicates that the slump value S is more sensitive to the yield stress τ_0 than to the plastic viscosity μ , which is in an agreement with the present result.

In this paper, an equation will be presented that directly relates slump S to the yield stress τ_0 . This equation is empirical and based on the experimental data shown here. The idea of creating such relationship is not new. For example Murata and Kukawa [21] have proposed such a relationship, shown with Eq. (4). This equation is based on numerous experiments, where the yield stress τ_0 is extracted from a coaxial cylinders viscometer.

$$\tau_0 = 714 - 473 \log(S/10) \quad (4)$$

The unit of the slump in the above equation is in mm. According to de Larrard [22], a slump equation (like shown above) should include the density ρ of the concrete. More precisely, he argues that a slump equation should be governed by the quantity $\tau_0/(g \cdot \rho_{sg})$, where g is gravity and $\rho_{sg} = \rho/\rho_{ref}$ is the specific gravity. The term ρ_{ref} is the density of a reference fluid, which is in this case the density of water at 4 °C, equal to 1000 kg/m³. In his doctoral thesis, Hu [15] proposed a relationship between slump S and yield stress τ_0 that includes density ρ , shown with Eq. (5). This equation is based on a number of numerical simulations.

$$S = 300 - 270 \frac{\tau_0}{\rho} = 300 - 0.27 \frac{\tau_0}{\rho_{sg}} \quad (5)$$

In the above equation, the gravity ($g = 9.81 \text{ m/s}^2$) is included in the constant used (i.e. $0.27 = 2.65/g$). The same consideration applies for all the following equations.

In a NIST report [23] Ferraris and de Larrard suggest a modification to Eq. (5) based on numerous experiments, using the BTRHEOM viscometer. This modification is given by Eq. (6).

$$S = 300 - 0.347 \frac{(\tau_0 - 212)}{\rho_{sg}} \quad (6)$$

In his textbook, de Larrard [22] adopts this equation for the range of slump above 100 mm. With a slight modification, the

above equation can be used for the data shown in Fig. 2. More specifically, the following relationship can be extracted from this illustration:

$$S = 300 - 0.416 \frac{(\tau_o + 394)}{\rho_{sg}} \quad (7)$$

The above equation is not identical to the equation shown in Fig. 2, but it falls within the boundary of the two dashed dotted lines. The reason for the use of “ $(\tau_o + 394)$ ” instead of “ $(\tau_o - 212)$ ” as in Eq. (6), is due to the difference in yield stress τ_o that the BTRHEOM viscometer and the BML viscometer gives. Comparison of these two viscometers, including the Mk-system, revealed that the BTRHEOM measures a higher yield stress than either the BML viscometer or the Mk-system [24]. A good agreement was shown to exist between the Mk-system and the BML viscometer [24].

In each main illustration of Figs. 2, 3 and 4, there is a linear regression through the data, shown with a solid line (also shown, is the corresponding equation for each such line). These solid lines and the corresponding confidence intervals are joined into the main illustration of Fig. 5. In this illustration, one can clearly see how different volume fraction of matrix gives a different relationship between the slump S and yield stress τ_o . For example, the yield stress of $\tau_o = 600$ Pa corresponds (in the nearest 5 mm) to 130 mm in slump at $V_m = 345 \text{ l/m}^3$ ($w/c = 0.4$), to $S = 90$ mm at $V_m = 331 \text{ l/m}^3$ ($w/c = 0.5$) and to $S = 40$ mm at $V_m = 321 \text{ l/m}^3$ ($w/c = 0.6$). This is shown with arrows. The plastic viscosity μ cannot be used to explain this behavior, as it is generally decreasing from $\mu_{av} = 20 \text{ Pa s}$ at $V_m = 345 \text{ l/m}^3$, to $\mu_{av} = 14 \text{ Pa s}$ at $V_m = 331 \text{ l/m}^3$ and to $\mu_{av} = 10 \text{ Pa s}$ at $V_m = 321 \text{ l/m}^3$. The term μ_{av} is the average plastic viscosity, calculated from the data shown in each corresponding Figs. 2, 3 and 4 (see also Fig. 8 for the distribution of the plastic viscosity values).

The small integrated illustration in Fig. 5 shows a comparison between the measured slump and the calculated slump by Eq. (7). It is clear how the majority of the data points are below the one-to-one line (applies mostly for the $V_m = 321 \text{ l/m}^3$ and $V_m = 331 \text{ l/m}^3$ batches).

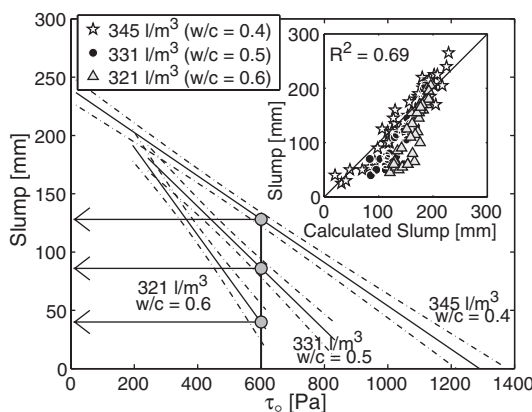


Fig. 5. The main illustration shows a plot of slump S as a function of yield stress τ_o for different mixture types (from Figs. 2–4). The small illustration shows a comparison between measured slump and calculated slump by Eq. (7).

3.2. Validity of the data

It should be clear that none of the batches in this work segregated as they were specifically designed to avoid this using continuous grading size (see Fig. 1) and sufficient fine particle contents. Hence, the slump values S can be considered to be reliable.

In a viscometer, there is always a possibility that some kind of experimental (and/or instrumental) error is present during a measurement. In particular a phenomenon called particle migration [14] can be present in viscometers of any type and brand. Basically, it is a shear rate induced segregation, and is responsible for moving particles away from the zone of highest shear rate [14]. In some cases, such phenomenon was observed during measurements with the BML viscometer. The concern was that this process could be responsible for the difference between the trend lines shown in Fig. 5. Therefore, to support (or validate) the generality of the result shown in this figure, mortars (with maximum aggregate size of $D_{max} = 2 \text{ mm}$) were designed from the concretes and batched separately. Mortars that are proportioned in this manner are very fat, stable and continuous, leaving little room for experimental error (cement contents are 770 kg/m^3 , 658 kg/m^3 and 574 kg/m^3 for $w/c = 0.4$, $w/c = 0.5$ and $w/c = 0.6$, respectively). The yield stress and plastic viscosity of these mortars were measured with a smaller version of the BML viscometer, called ConTec Viscometer 4 (same producer). As before, it is the outer cylinder (of radius $R_o = 0.101 \text{ m}$) that rotates at predetermined frequencies f_o [rps], while the inner cylinder (of radius $R_i = 0.085 \text{ m}$ and height $h = 0.116 \text{ m}$) is stationary and measures torque T [Nm] (for further description, see for example Ref. [14]). To avoid slippage, both the inner and outer cylinders are fitted with protruding vanes. Hence, the dimensions R_i and R_o are relative to the extremities of the vanes (this applies also for the BML viscometer).

The combination of large mortar stability, the use of protruding vanes and with relatively large gap width (i.e. $(R_o - R_i)/D_{max} = 8$), the presence of experimental error (if any) should be severely reduced for the ConTec Viscometer 4. Hence, knowing that the yield stresses of the mortars are reliable, one can plot these values against the slump from the equivalent concrete batches. This is done in the main illustration of Fig. 6. Although with a worse correlation, the same behavior is observed as in Fig. 5. The small integrated illustration shows a plot of the yield stress of the concretes (by the BML viscometer) versus the yield stress of the equivalent mortars (by the ConTec Viscometer 4). As demonstrated, there is a relatively good correlation between the two values. Thus, the BML results are considered to be reasonably valid.

3.3. A suggested hypothesis

As mentioned in Section 2.1, the filler modified cement paste is defined to be the matrix and aggregates above 0.125 mm are defined as the suspended particles. This definition and terminology will be used in the text below, when discussing the possible reason for the result shown in Fig. 5.

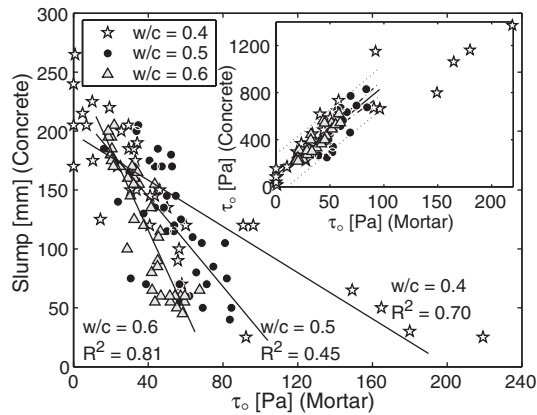


Fig. 6. The main illustration shows a plot of slump S (of concrete) as a function of yield stress τ_0 (of equivalent mortar). The small illustration shows the yield stress of concrete plotted as a function of the yield stress of equivalent mortar.

During the deformation of slump, the suspended particles must bypass one another. The stop-flow condition of the slump (which gives the final slump value S) is attained by the situation that the suspended particles can no longer bypass one another. As shown in Fig. 5 (with arrows), then for a given yield stress τ_0 , this stop condition occurs sooner if the volume fraction of matrix V_m is low. As a suggestion, this early stop-flow condition is attributed to the granular properties of the concrete: With reduced volume fraction of matrix V_m , the suspended particles must further deviate from a straight path when they are bypassing one another. That is, with smaller V_m , the distance between these particles is reduced, forcing them to take a more curvature path alongside each others surfaces. Hence, during the deformation of slump, the suspended particles are forced to go into almost a stair like trajectory path, where each “step” represents an obstacle against deformation, which gravity must overcome. More to the point, with low V_m , the difference between the slump and a viscometer is that the gravity cannot always overcome this type of obstacle (due to insufficient weight in further pushing the concrete cake down), while the engine of a viscometer always can. This results in a larger sensitivity of the slump value S to the volume fraction of matrix V_m , which results in an earlier stop-flow condition in slump than anticipated for a given yield stress (c.f. Fig. 5). Specifically, the granular properties of the test sample will affect the slump value S more relative to the yield stress τ_0 , due to the characteristic difference in measuring method used in retrieving those two values.

Of course, different volume fraction of matrix V_m will influence the measured yield stress τ_0 and plastic viscosity μ as it will affect the degree of particle–particle interactions inside the viscometer. But the idea presented here is that such relationship between the concrete consistency and matrix content will not be influenced in the same manner as in the slump cone. This is because of the more controlled shear rate condition that is present inside the viscometer relative to the slump cone. With such controlled condition, the engine of the viscometer will always over win the above-mentioned obstacle (and hence force the suspended particles to bypass one another), when gravity cannot in slump.

In Fig. 5, one can see how each trend line starts to overlap each other between and around 100 to 200 Pa in yield stress. For example, at the yield stress of 200 Pa, each corresponding trend line gives the following slump value (to the nearest 5 mm): 205 ($V_m = 345 \text{ l/m}^3$), 190 ($V_m = 331 \text{ l/m}^3$) and 190 mm ($V_m = 321 \text{ l/m}^3$). Hence, as the yield stress becomes lower (as a result of better matrix lubrication), the three different trend lines seems to converge towards one single trend line. This could indicate that with reduced apparent viscosity η of the matrix (i.e. better matrix lubrication), the suspended particles will slide past one another with the same or similar ease, regardless of the amount of matrix used, reducing the granular properties of the fresh concrete. More precisely, the trend line between the slump and yield stress becomes less dependent on the matrix volume fraction V_m as the concrete becomes more workable.

3.4. Relevance of viscometers

Just as for slump, the gravity plays an important role when casting traditional concrete into formwork. Hence, when considering the characteristic difference in measuring method used in retrieving the slump value S and the yield stress τ_0 , one cannot help wondering if the slump device is better suited to measure the concrete consistency. Although this is a fair question, one should be careful in making such claim. This is because of the different types of compaction devices available and used at the jobsite (like the vibrator). In similar manner as the engine of a viscometer, these different compaction devices will introduce mechanical energy into the concrete, which reduces the effect of its pre-mentioned granular properties. Furthermore, as mentioned in the previous paragraph, for very workable concretes where little or no mechanical compaction is needed, the relationship between slump and yield stress becomes stronger (i.e. more independent of matrix volume fraction V_m). This means that the granular properties of the fresh concrete will affect the yield stress τ_0 and the slump value S to the same or similar degree for such cases. Hence, one can conclude that viscometers are at least as relevant measuring tool as the slump device (especially when bearing in mind that the viscometers can give two flow parameters, while the slump only one).

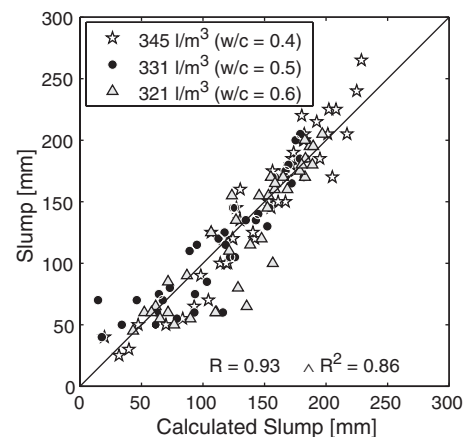


Fig. 7. Comparison between measured slump and calculated slump by Eq. (8).

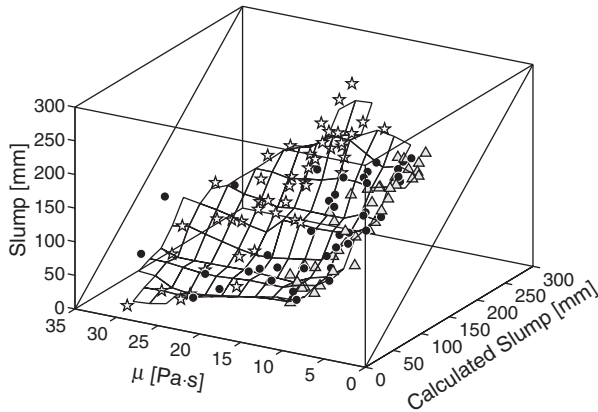


Fig. 8. Three dimensional presentation of Fig. 7, including the plastic viscosity μ .

3.5. Lubrication and magnitude of matrix

It is possible to normalize Eq. (7) with respect to the volume fraction of matrix and the degree of matrix lubrication. This is done here empirically by applying an addition as shown with Eq. (8).

$$S = 300 - 0.416 \frac{(\tau_o + 394)}{\rho_{sg}} + \alpha(\tau_o - \tau_o^{\text{ref}})(V_m - V_m^{\text{ref}}) \quad (8)$$

The term $(\tau_o - \tau_o^{\text{ref}})$ regulates to the above-mentioned lubrication effect, while the term $(V_m - V_m^{\text{ref}})$ treats the effect of the distance between the suspended particles (i.e. the effect of matrix volume fraction). Of course, as the addition term $\alpha(\tau_o - \tau_o^{\text{ref}})(V_m - V_m^{\text{ref}})$ is only based on empirical consideration from the data of this work, it is unlikely that it is generally valid. Other research programs might reveal another type of addition term, for example because of different angularity, density and grading of the aggregates used.

Plotting the measured slump against calculated slump by Eq. (8), gives the result shown in Fig. 7. In this figure, the following values are used: $\alpha = 7.7 \cdot 10^{-3}$ mm/(Pa l), $V_m^{\text{ref}} = 345$ l/m³, $\tau_o^{\text{ref}} = 200$ Pa. This figure shows a relatively good one-to-one correspondence between measured and calculated slump. A three dimensional version of Fig. 7 is shown in Fig. 8, which includes the corresponding plastic viscosity values. The mesh curve is a linear interpolation between the data points. In this figure, it is evident that the slump value is more or less independent of plastic viscosity μ . However, it should be noted that this result is relative to traditional concrete where the yield stress is usually much higher than the plastic viscosity.

4. Conclusions

The result of this study suggest a relationship between the slump S and yield stress τ_o that depends on the volume fraction of matrix V_m . As a proposal, this dependency is attributed to a certain granular-type obstacle against flow that increases with reduced V_m . Due to the characteristic difference in measuring method used in retrieving the slump S and yield stress τ_o , this type of obstacle will affect the former variable more relative to the latter. The result of this study also suggest that the trend line between the slump S and

yield stress τ_o becomes less dependent on the matrix volume fraction V_m as the concrete becomes more workable. The suggested reason for this is that with better matrix lubrication between the aggregates, the granular-type obstacle against flow becomes less evident.

A slump equation is presented in this work that takes into account for both the matrix lubrication effect and the granular properties and is given by Eq. (8). This equation also contains the quantity $\tau_o/(g \cdot \rho_{sg})$, as proposed by de Larrard (in this equation, the gravity $g = 9.81$ m/s² can be considered to be included in the constant used, i.e. $0.416 = 4.08/g$). As shown in Figs. 7 and 8, Eq. (8) works well in predicting the slump from a given yield stress. Since a low correlation is present between the slump S and plastic viscosity μ , no attempt was made to establish an equation between those two variables.

Acknowledgment

The work was carried out at the Norwegian University of Science and Technology, NTNU. This work was made possible by the financial support from Borregaard LignoTech, Norway and the Research Council of Norway.

References

- [1] A.M. Neville, Properties of Concrete, Addison Wesley Longman Limited, Great Britain, 1995.
- [2] G.H. Tattersall, P.F.G. Banfill, The Rheology of Fresh Concrete, Pitman Books Limited, Great Britain, 1983.
- [3] A.M. Neville, Chairman's summary, fresh concrete: important properties and their measurement, Proceedings of a RILEM Seminar Held 22–24 March 1973, University of Leeds, Great Britain, 1973.
- [4] P.J.M. Bartos, M. Sonebi, A.K. Tamimi (Eds.), Workability and Rheology of Fresh Concrete: Compendium of Tests; Report of RILEM Technical Committee TC 145-WSM, Workability of Special Concrete Mixes, RILEM Publications S.A.R.L., Cachan Cedex, France, 2002.
- [5] G.H. Tattersall, Workability and Quality Control of Concrete, E & FN Spon, London, Great Britain, 1991.
- [6] G.H. Tattersall, S.J. Bloomer, Further development of the two-point test for workability and extension of its range, Mag. Concr. Res. 31 (109) (1979) 202–210.
- [7] P.L.J. Domone, X. Yongmo, P.F.G. Banfill, Developments of the two-point workability test for high-performance concrete, Mag. Concr. Res. 51 (3) (1999) 171–179.
- [8] G.H. Tattersall, Relationships between the British standard test for workability and two-point test, Mag. Concr. Res. 25 (84) (1973) 169–174.
- [9] G.H. Tattersall, Progress in measurement of workability by the two-point test, Properties of Fresh Concrete, Proceedings of the RILEM Colloquium, University Press, Cambridge, Great Britain, 1990, pp. 203–212.
- [10] J. Murata, H. Kikukawa, Studies on rheological analysis of fresh concrete, Fresh Concrete: Important Properties and their Measurement, Proceedings of a RILEM Seminar held 22–24 March 1973, University of Leeds, Great Britain, 1973, pp. 1.2-1–1.2-33.
- [11] O.J. Uzomaka, A concrete rheometer and its application to a rheological study of concrete mixes, Rheol. Acta 13 (1974) 12–21.
- [12] O.H. Wallevik, The Rheology of Fresh Concrete and its Application on Concrete with and without Silica Fume, The Norwegian Institute of Technology, Dr.ing. thesis no. 1990:45, Trondheim, Norway, 1990.
- [13] O.H. Wallevik, O.E. Gjrv, Development of a coaxial cylinder viscometer for fresh concrete, Properties of Fresh Concrete, Proceedings of the RILEM Colloquium, University Press, Cambridge, Great Britain, 1990, pp. 213–224.

- [14] J.E. Wallevik, Rheology of Particle Suspensions — Fresh Concrete, Mortar and Cement Paste with Various Types of Lignosulfonates (Ph.D.-thesis), Department of Structural Engineering, The Norwegian University of Science and Technology, 2003.
- [15] C. Hu, Rhéologie des Bétons Fluides, PhD thesis of ENPC. Études et Recherches des LPC. Série ouvrages d'art no. 16, La France, 1995.
- [16] C. Hu, F. de Larrard, T. Sedran, C. Boulay, F. Bosc, F. Deflorenne, Validation of BTRHEOM, the new rheometer for soft-to-fluid concrete, *Mater. Struct.* 29 (194) (1996) 620–631.
- [17] E. Mørtzell, S. Smeplass, T.A. Hammer, M. Maage, Flowcyl — how to determine the flow properties of the matrix phase of high performance concrete, Fourth International Symposium on the Utilization of High Strength/High Performance Concrete, Presses Ponts et Chaussées, Paris, France, 1996, pp. 261–268.
- [18] R.E. Walpole, R.H. Myers, S.L. Myers, Probability and Statistics for Engineers and Scientists, 6th edition, Prentice-Hall Inc., New Jersey, 1998.
- [19] Y. Tanigawa, H. Mori, Analytical study on deformation of fresh concrete, *J. Eng. Mech.* 115 (3) (1989) 493–508.
- [20] Y. Tanigawa, H. Mori, K. Watanabe, Computer simulation of consistency and rheology tests of fresh concrete by viscoplastic finite element method, Properties of Fresh Concrete, Proceedings of the RILEM Colloquium, University Press, Cambridge, Great Britain, 1990, pp. 301–308.
- [21] J. Murata, H. Kukawa, Viscosity equation for fresh concrete, *ACI Mater. J.* 89 (3) (1992) 230–237.
- [22] F. de Larrard, Concrete Mixture Proportioning, A Scientific Approach, F & FN Spon, New York, 1999.
- [23] C.F. Ferraris, F. de Larrard, Testing and Modelling of Fresh Concrete Rheology (NISTIR 6094), National Institute of Standard and Technology (NIST), Gaithersburg, USA, 1998.
- [24] P.F.G. Banfill, D. Beaupre, F. Chapdelaine, F. de Larrard, P. Domone, L. Nachbaur, T. Sedran, O.H. Wallevik, J.E. Wallevik, in: F. Ferraris, L.E. Brower (Eds.), Comparison of Concrete Rheometers: International Tests at LCPC (Nantes, France) in October, 2000 (NISTIR 6819), National Institute of Standard and Technology (NIST), Gaithersburg, USA, 2001.