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A concrete mix proportion design algorithm based on artificial neural networks

Tao Ji *, Tingwei Lin, Xujian Lin

College of Civil Engineering, Fuzhou University, Fuzhou, Fujian Province, 350002, People's Republic of China

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Abstract

The concepts of five parameters of nominal water–cement ratio, equivalent water–cement ratio, average paste thickness, fly ash–binder ratio, grain volume fraction of fine aggregates and Modified Tourfar's Model were introduced. It was verified that the five parameters and the mix proportion of concrete can be transformed each other when Modified Tourfar's Model is applied. The behaviors (strength, slump, et al.) of concrete primarily determined by the mix proportion of concrete now depend on the five parameters. The prediction models of strength and slump of concrete were built based on artificial neural networks (ANNs). The calculation models of average paste thickness and equivalent water–cement ratio can be obtained by the reversal deduction of the two prediction models, respectively. A concrete mix proportion design algorithm based on a way from aggregates to paste, a least paste content, Modified Tourfar's Model and ANNs was proposed. The proposed concrete mix proportion design algorithm is expected to reduce the number of trial and error, save cost, laborers and time. The concrete designed by the proposed algorithm is expected to have lower cement and water contents, higher durability, better economical and ecological effects.

Keywords: Concrete mix proportion design; Artificial neural network (ANN); Nominal water–cement ratio; Equivalent water–cement ratio; Average paste thickness (APT); Fly ash–binder ratio; Grain volume fraction of fine aggregates

1. Introduction

Traditional concrete mixture proportion algorithms [1–4] are from paste to aggregates. Namely, the contents of cement and water are determined firstly, then the content of fine and coarse aggregates. It results in more cement and water contents and less fine and coarse aggregate contents. Such gained concrete inclines to be neither economical nor durable.

In addition, the traditional concrete mixture proportion algorithms are based on a generalization of previous experience, available as tables or empirical formula. Due to the uncertain of concrete ingredients, such as fine and coarse aggregates, cement, chemical and mineral admixtures, the traditional concrete mixture proportion algorithms are a trial and error process, which results in the waste of cost, laborers and time.

Mix-design software programs were proposed to reduce trail batches [5–8]. Some products are essentially a programming of conventional mix-design methods like the ACI method or the French Dreux's method. Other programs are based on compressive packing model (CPM), a method from aggregates to paste. However, CPM applied parametric methods, not artificial neural networks (ANNs).

In recent years, ANNs have shown exceptional performance as regression tools, especially when used for pattern recognition and function estimation. They are highly nonlinear, and can capture complex interactions among input/output variables in a system without any prior knowledge about the nature of these interactions. The main advantage of ANNs is that one does not have to explicitly assume a model form, which is a prerequisite in the parametric approach. Indeed, in ANNs a relationship of possibly complicated shape between input and output variables is generated by the data points themselves. In comparison to parametric methods, ANNs tolerate relatively imprecise or incomplete data, approximate results, and are less vulnerable to outliers. They are highly parallel, that is, their numerous independent operations can be executed simultaneously [9]. Due to the complexity between concrete behaviors and concrete mix proportions, artificial neural network techniques have been adopted in concrete mix proportion design [10,11]. However,

* Corresponding author. Tel.: +86 591 87731361; fax: +86 591 83737442.
E-mail address: jt72@163.com (T. Ji).

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there were some shortcomings available: (1) the design process is from paste to aggregates, not from aggregates to paste; (2) the principle of a least paste content is not included; (3) the output neurons are the contents of concrete ingredients, which decreases the precision of concrete mix proportion design.

Based on the concepts of average paste thickness, nominal water–cement ratio, equivalent water–cement ratio, fly ash–binder ratio, grain volume fraction of fine aggregates and Modified Tourfar's Model, the relation between the five parameters and concrete mix proportion was studied in detail. By combining artificial neural networks, the principle of a least paste content and mathematical models, the output neuron of each artificial neural network is kept single. Then the design precision of concrete mix proportion is improved.

2. Basic concepts

2.1. Fly ash–binder ratio

The fly ash–binder ratio β_F is defined as

$$\beta_F = m_F/(m_C + m_F) \tag{1}$$

where m_C, m_F are the by weight contents of cement and fly ash in one cubic meter concrete, respectively, and m means by weight, not by volume.

2.2. Grain volume fraction of fine aggregates

The grain volume fraction of fine aggregates y_S is defined as

$$y_S = (m_S/\rho_S)/(m_S/\rho_S + m_G/\rho_G) \tag{2}$$

where m_S, m_G are the by weight contents of fine and coarse aggregates in one cubic meter concrete, respectively. ρ_S, ρ_G are the grain densities of fine and coarse aggregates, respectively.

2.3. Equivalent water–cement ratio

2.3.1. Equivalent water content $m_{W,E}$

The equivalent water content $m_{W,E}$ is defined as:

$$m_{W,E} = m_W + m_{W,R} + m_{W,A} - m_{W,S} \tag{3}$$

where

$$m_{W,R} = m_R(1 - C_R) \tag{4}$$

$$m_{W,A} = m_S C_S + m_G C_G \tag{5}$$

$$m_{W,S} = m_S C_{S0} + m_G C_{G0} \tag{6}$$

where m_W is the mixing water content in one cubic meter concrete; $m_{W,R}$ is the water content of superplasticizer (SP) in one cubic meter concrete; C_R is the solid content of SP; $m_{W,A}$ is the real water content of fine and coarse aggregates in one cubic meter concrete; C_S and C_G are the absorption coefficients of water for in situ fine and coarse aggregates, respectively; $m_{W,S}$ is the water content of saturated surface-dry (SSD) fine and coarse aggregates; C_{S0} and C_{G0} are the absorption coefficients of water for saturated surface-dry (SSD) fine and coarse aggregates, respectively; m_R is the content of SP in one cubic meter concrete. Herein, it is assumed that $m_{W,S}$ is independent of the slump and strength of concrete.

2.3.2. Equivalent cement content $m_{C,E}$

Including the content and activity of fly ash, the equivalent cement content $m_{C,E}$ is defined as:

$$m_{C,E} = \frac{m_C}{1 - \alpha\beta_F} \tag{7}$$

where α is the activity ratio of fly ash to cement, and can be taken as 0.55 for class one fly ash (Chinese Standard) and grade 42.5 Portland cement based on a great deal of test data. α has relationship with the strength of concrete, but independent of the slump of concrete.

2.3.3. The equivalent water–cement ratio $(m_W/m_C)_E$ is defined as:

$$(m_W/m_C)_E = \frac{m_{W,E}}{m_{C,E}} = \left(\frac{m_{W,E}}{m_C}\right)(1 - \alpha\beta_F) \tag{8}$$

$(m_W/m_C)_E$ has relationship with the strength of concrete, but independent of the slump of concrete. If β_F is taken as zero, $(m_W/m_C)_E$ is transform into the normal water–cement ratio $(m_{W,E}/m_C)$. Eq. (8) can be transformed into

$$\left(\frac{m_{W,E}}{m_C}\right) = \frac{(m_W/m_C)_E}{(1 - \alpha\beta_F)} \tag{9}$$

2.4. Nominal water–cement ratio

2.4.1. Nominal water W_N

The nominal water W_N is defined as:

$$m_{W,N} = \frac{m_{W,E} - m_{W,R}}{1 - \mu} = \frac{m_W + m_{W,A} - m_{W,S}}{1 - \mu} \tag{10}$$

where μ is the reducing-water ratio of SP, namely the ratio of water reduced by superplasticizer to primary total water. $m_{W,R}$ is included in μ, so $m_{W,R}$ is subtracted from $m_{W,E}$. μ has relationship with the slump of concrete, but independent of the strength of concrete.

2.4.2. Needed water ratio λ_R

λ_R is the ratio of the needed by weight water of a paste composed by 300 g cement and 750 g standard sand to that of a paste composed by 300 g fly ash and 750 g standard sand when the slump-flows of both the two pastes reach 125–135 mm [12]. λ_R has relationship with the slump of concrete, but independent of the strength of concrete.

2.4.3. Nominal cement $m_{C,N}$

The nominal cement $m_{C,N}$ is defined as:

$$m_{C,N} = m_C + \lambda_R m_F \tag{11}$$

2.4.4. Nominal water–cement ratio $(m_W/m_C)_N$

The nominal water–cement ratio $(m_W/m_C)_N$ is defined as:

$$(m_W/m_C)_N = \frac{m_{W,N}}{m_{C,N}} = \frac{\frac{m_{W,E}-m_{W,R}}{1-\mu}}{m_C+\lambda_R m_F} = \frac{\frac{m_W+m_{W,A}-m_{W,S}}{1-\mu}}{m_C+\lambda_R m_F} \tag{12}$$

$(m_W/m_C)_N$ has relationship with the slump of concrete, but independent of the strength of concrete. If μ and m_F are taken as zero (namely SP and fly ash are not included in concrete), $(m_W/m_C)_N$ is transformed into the normal water–cement ratio $(m_{W,E}/m_C)$.

2.5. Average paste thickness

2.5.1. Basic assumptions

The theoretical models of aggregates are based on a number of assumptions

i) aggregates are perfect spheres
ii) aggregates are monosized
iii) fine and coarse aggregates are of a different size

The three assumptions conflict with the practical combination of realistic aggregates. The first two assumptions can be overcome by introducing characteristic diameters of fine and coarse aggregates d_{ST}, d_{GT} and by using the measured eigenpacking degrees of fine and coarse aggregates ϕ_{ST}, ϕ_{GT}. These two parameters compensate the deviations from the two assumptions.

The characteristic diameter d_{ST} (or d_{GT}) of fine aggregates (or coarse aggregates) can be chosen as the sieve size for which there is 36.8% residue [13].

The Eigenpacking degree ϕ_{ST} (or ϕ_{GT}) of fine aggregates (or coarse aggregates) is defined as the ratio of fine (or coarse) aggregate bulk density ρ_{ST} (or ρ_{GT}) to the fine (or coarse) aggregate grain density ρ_S (or ρ_G):

$$\phi_{ST} = \rho_{ST}/\rho_S \tag{13}$$

$$\phi_{GT} = \rho_{GT}/\rho_G \tag{14}$$

The fine (or coarse) aggregates are poured into a steel barrel. Then the steel barrel is vibrated for 30 s on the vibrating platform of a vibration device. The bulk density ρ_{ST} (or ρ_{GT}) can be obtained by dividing the by weight content of fine (or coarse) aggregates by the volume of the steel barrel occupied by the fine (or coarse) aggregates.

The third assumption can be overcome by dividing the aggregates into fine aggregates (namely, sands) and coarse aggregates (namely, natural or crushed gravels). In Chinese Standard, the maximum granular diameter of fine aggregates is less than 5 mm, while the minimum granular diameter of coarse aggregates is larger than 5 mm.

2.5.2. Modified Tourfar's Model

The packing model of aggregates proposed by Toufar et al. [14,15] was found to best fit for larger diameter ratios. However, later comparisons with test data show that the packing degree of a sample of particles predicted by this model does not increase when a small amount of fine particles is added to the coarse ones. The unrealistic behavior of the model was corrected by Goltermann et al. [13] as follows:

$$\phi_A = \frac{1}{\frac{y_s}{\phi_{ST}}+\frac{y_G}{\phi_{GT}}-y_G\left(\frac{1}{\phi_{GT}}-1\right)k_D k_s} \tag{15}$$

$$k_D = \frac{d_{GT}-d_{ST}}{d_{GT}+d_{ST}} \tag{16}$$

$$k_s = \begin{cases} (x/x_0)k_0 & x<x_0 \\ 1-(1+4x)/(1+x)^4 & x\geq x_0 \end{cases} \tag{17}$$

$$x = \frac{(y_S/y_G)(\phi_{GT}/\phi_{ST})}{1-\phi_{GT}} \tag{18}$$

$$y_s + y_G = 1 \tag{19}$$

where ϕ_A is the packing degree of aggregates. y_S and y_G are the grain volume fractions of the fine and coarse aggregates, respectively. k_D is a factor that determines the influence of a diameter ratio. k_s is a statistical factor. x is the ratio of the bulk volume of fine particles to the void volume of coarse aggregates. x_0=0.4753; k_0=0.3881. Concretes of less paste content and less shrinkage can be obtained by using Modified Tourfar's Model.

2.5.3. Average paste thickness APT

The average paste thickness wrapping fine and coarse aggregates can be calculated as follows:

$$\mathrm{APT} = \frac{V_P-V_{P0}}{\pi(n_S d_{ST}^2+n_G d_{GT}^2)} = \frac{V_P-V_{P0}}{6\left(\frac{m_S}{d_{ST}\rho_S}+\frac{m_G}{d_{GT}\rho_G}\right)} \tag{20}$$

$$V_P = \frac{m_C}{\rho_C}+\frac{m_F}{\rho_F}+\frac{m_{W,E}}{\rho_W}+\theta \tag{21}$$

$$V_{P0} = \left(\frac{m_S}{\rho_S}+\frac{m_G}{\rho_G}\right)\cdot\frac{1-\phi_A}{\phi_A} \tag{22}$$

$$n_S = \frac{m_S}{\frac{1}{6}\pi d_{ST}{}^3\cdot\rho_S} \tag{23}$$

$$n_G = \frac{m_G}{\frac{1}{6}\pi d_{GT}{}^3\cdot\rho_G} \tag{24}$$

where V_P is the total paste volume; V_{P0} is the paste volume filling the void of aggregates; n_S, n_G are theoretical numbers of fine and

Table 1
Material properties

ρ_C (kg/m^3)	ρ_F (kg/m^3)	ρ_W (kg/m^3)	ρ_S (kg/m^3)	ρ_G (kg/m^3)	ρ_{ST} (kg/m^3)	ρ_{GT} (kg/m^3)	θ
3008	2212	1000	2632	2630	1471	1494	2%

d_{ST} (mm)	d_{GT} (mm)	C_R	C_{SO}	C_{GO}	C_S	C_G	λ_R
0.562	12.43	0.4	0.8%	1.0%	0	0	0.95

coarse aggregates, respectively; ρ_C, ρ_W, ρ_F are the specific densities of cement, water, fly ash, respectively; θ is the air content.

3. Experimental program

3.1. Material properties

All materials used in the experiments are produced in China. The cement is a grade 42.5 Portland cement. A class one fly ash (Chinese Standard) is used. The superplasticizer TW-7 is naphthalene-type. The coarse aggregate used is continuous grading crushed gravels, with the maximum particle size of 16 mm. The fine aggregate is river sand, with a fineness modulus of 2.1. Material properties can be found in Table 1.

3.2. Test results

Eighteen-group concrete mix proportions were designed as listed in Table 2. Groups 1 through 9 are normal concrete. Groups 10 through 18 are fly ash concrete mixed with fly ash and SP. μ can be calculated according to the specification provided by SP companies. The 28d strength f_{cu} and slump sl of the eighteen-group concrete are listed in Table 3.

Table 2
Mix proportion of concrete

No.	m_C (kg/m^3)	m_W (kg/m^3)	m_F (kg/m^3)	m_S (kg/m^3)	m_G (kg/m^3)	m_R (kg/m^3)	μ (%)
1	537	214	0	495	1100	0	0
2	502	226	0	495	1100	0	0
3	472	236	0	495	1100	0	0
4	566	226	0	431	1108	0	0
5	530	238	0	431	1108	0	0
6	498	249	0	431	1108	0	0
7	592	237	0	388	1102	0	0
8	554	250	0	388	1102	0	0
9	521	261	0	388	1102	0	0
10	461	203	81	495	1101	2.01	5.8
11	461	203	81	495	1101	5.64	16.3
12	461	203	81	495	1101	9.08	24.6
13	486	214	86	431	1108	2.11	5.8
14	486	214	86	431	1108	5.94	16.3
15	486	214	86	431	1108	9.57	24.6
16	509	224	90	388	1102	2.21	5.8
17	509	224	90	388	1102	6.22	16.3
18	509	224	90	388	1102	10.02	24.6

Table 3
Calculated five parameters and test results

No.	$(m_W/m_C)_N$	$(m_W/m_C)_E$	APT (10^{-6} m)	y_S	β_F	sl (cm)	f_{cu} (MPa)
1	0.371	0.371	62	0.31	0	2.5	50.5
2	0.420	0.420	62	0.31	0	5.5	42.6
3	0.468	0.468	62	0.31	0	12.5	36.4
4	0.374	0.374	83	0.28	0	5.5	47.4
5	0.422	0.422	83	0.28	0	8.5	43.1
6	0.471	0.471	83	0.28	0	12	35
7	0.376	0.376	103	0.26	0	9.5	44.7
8	0.426	0.426	103	0.26	0	13	44.5
9	0.474	0.474	103	0.26	0	19	36.8
10	0.371	0.377	63	0.31	0.15	4.5	49.7
11	0.418	0.381	64	0.31	0.15	20.5	55.2
12	0.464	0.385	64	0.31	0.15	21	50.8
13	0.373	0.379	83	0.28	0.15	9.5	50.7
14	0.420	0.383	85	0.28	0.15	21	53.1
15	0.466	0.387	86	0.28	0.15	23	50.5
16	0.375	0.381	104	0.26	0.15	10	51.3
17	0.422	0.385	105	0.26	0.15	21.5	53
18	0.468	0.389	107	0.26	0.15	23.5	50.4

4. Equivalency between five parameters and mix proportion of concrete

4.1. From mix proportion of concrete to five parameters

According Eqs. (1), (2), (10), (12) and (20), the five parameters of the eighteen-group concrete can be calculated as listed in Table 3, and other parameters are listed in Table 4.

4.2. From five parameters to mix proportion of concrete

For a mix proportion of concrete, the grade of cement and the nature of aggregates et al. are constant. If the five parameters y_S,

Table 4
Other parameters

No.	$m_{W,S}$ (kg/m^3)	$m_{W,R}$ (kg/m^3)	$m_{W,E}$ (kg/m^3)	ϕ_A	V_P (m^3/m^3)	V_{P0} (m^3/m^3)
1	15.0	0	199.0	0.699	0.413	0.261
2	15.0	0	211.0	0.699	0.413	0.261
3	15.0	0	221.0	0.699	0.413	0.261
4	14.5	0	211.5	0.694	0.434	0.258
5	14.5	0	223.5	0.694	0.434	0.258
6	14.5	0	234.5	0.694	0.435	0.258
7	14.1	0	222.9	0.688	0.454	0.256
8	14.1	0	235.9	0.688	0.454	0.256
9	14.1	0	246.9	0.688	0.454	0.256
10	15.0	1.2	189.2	0.699	0.413	0.261
11	15.0	3.4	191.4	0.699	0.413	0.261
12	15.0	5.4	193.5	0.699	0.413	0.261
13	14.5	1.3	200.7	0.694	0.434	0.258
14	14.5	3.6	203.0	0.694	0.434	0.258
15	14.5	5.7	205.2	0.694	0.434	0.258
16	14.1	1.3	211.2	0.688	0.454	0.256
17	14.1	3.7	213.6	0.688	0.454	0.256
18	14.1	6.0	215.9	0.688	0.454	0.256

Table 5
Mix proportion of concrete calculated from five parameters

No.	m_C (kg/m^3)	m_W (kg/m^3)	m_F (kg/m^3)	m_S (kg/m^3)	m_G (kg/m^3)	m_R (kg/m^3)
1	534.81	213.13	0	493.08	1095.7	0
2	499.76	225	0	492.9	1095.3	0
3	469.88	234.95	0	492.89	1095.3	0
4	563.23	224.9	0	428.99	1102.8	0
5	527.39	236.83	0	428.98	1102.8	0
6	495.36	247.68	0	428.82	1102.4	0
7	588.23	235.5	0	385.65	1095.3	0
8	550.27	248.32	0	385.5	1094.9	0
9	517.47	259.24	0	385.5	1094.9	0
10	458.19	201.76	80.509	492.14	1094.6	2.01
11	457.15	201.28	80.326	491.08	1092.3	5.64
12	456.17	200.83	80.153	490.08	1090	9.08
13	482.84	212.61	85.441	428.33	1101.1	2.11
14	481.69	212.07	85.237	427.36	1098.6	5.94
15	480.6	211.56	85.045	426.43	1096.3	9.57
16	505.01	222.24	89.295	385.1	1093.8	2.21
17	503.75	221.66	89.072	384.19	1091.2	6.22
18	502.57	221.1	88.863	383.32	1088.7	10.02

β_F, $(m_W/m_C)_E$, $(m_W/m_C)_N$, APT are given, the mix proportion of concrete can be calculated. The algorithm is provided as follows:

1. From Eqs. (13)–(14), ϕ_{ST}, ϕ_{GT} can be obtained.
2. From Eq. (15), ϕ_A can be obtained.
3. The total paste volume of concrete mixtures corresponding to a unit volume of aggregates V'_P can be obtained according to Eq. (25).

$$V'_P = V'_{P0} + S' \cdot \mathrm{APT} = \frac{(1-\phi_A)}{\phi_A} + 6\left(\frac{y_S}{d_{ST}} + \frac{(1-y_S)}{d_{GT}}\right) \cdot \mathrm{APT} \tag{25}$$

$$V'_{P0} = \frac{(1-\phi_A)}{\phi_A} \tag{26}$$

$$S' = y_S \frac{\pi d_{ST}^2}{\pi d_{ST}^3/6} + (1-y_S)\frac{\pi d_{GT}^2}{\pi d_{GT}^3/6} = 6\left(\frac{y_S}{d_{ST}} + \frac{(1-y_S)}{d_{GT}}\right) \tag{27}$$

where V'_{P0} and S' are the void volume and the total surface area of a unit volume of aggregates, respectively.

4. The total paste volume corresponding to a unit volume of concrete mixture can be obtained according to Eq. (28).

$$V_P = V'_P/(1 + V'_P) \tag{28}$$

5. The by weight contents of fine and coarse aggregates corresponding to a unit volume of concrete mixture are shown in Eqs. (29) and (30), respectively.

$$m_S = \rho_S y_S/(1 + V'_P) \tag{29}$$

$$m_G = \rho_G(1-y_S)/(1 + V'_P) \tag{30}$$

6. From Eq. (9), $(m_{W,E}/m_C)$ can be obtained.
7. From Eqs. (1) and (21), m_C can be obtained.

$$m_C = (V_P - \theta)\Big/\left(\frac{1}{\rho_C} + \frac{\beta_F}{\rho_F(1-\beta_F)} + \left(\frac{m_{W,E}}{m_C}\right)\cdot\frac{1}{\rho_W}\right) \tag{31}$$

8. From Eq. (1), m_F can be obtained.

$$m_F = \frac{\beta_F m_C}{1-\beta_F} \tag{32}$$

9. According to Eq. (33), $m_{W,E}$ can be obtained.

$$m_{W,E} = m_C \cdot \left(\frac{m_{W,E}}{m_C}\right) \tag{33}$$

10. According to Eqs. (5) and (6), $m_{W,A}$ and $m_{W,S}$ can be obtained.

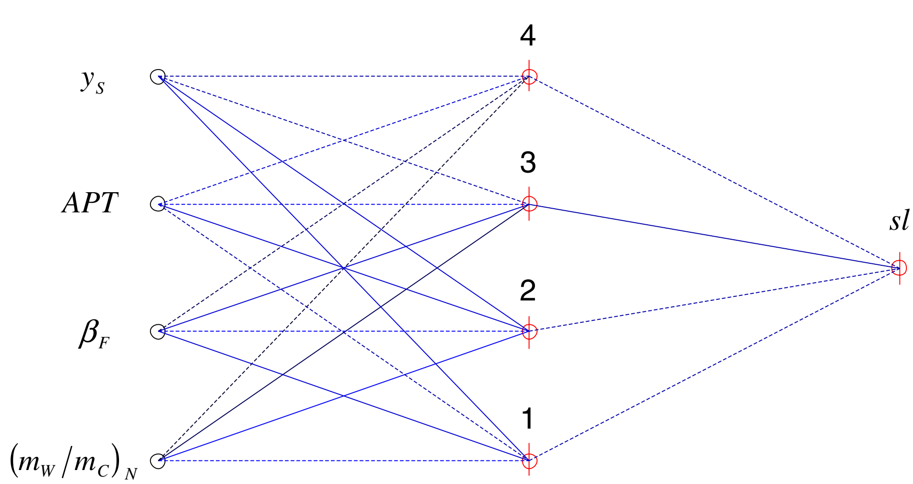

Fig. 1. Architecture of ANN used to predict the slump of concrete.

Table 6
Connection weights and biases used to predict the slump of concrete

Neuron	Connection weights				Biases
	$(m_W/m_C)_N$	β_F	APT	y_S	
1	−0.2316	0.7073	−1.1085	0.9906	3.2272
2	0.1225	−0.0736	0.6811	0.7762	0.0083
3	2.4880	0.7266	−0.4756	−1.0891	2.5211
4	−2.7809	−2.2917	−0.9557	−1.2410	1.4302

Neuron	1	2	3	4	Biases
sl	−0.8751	−1.2372	0.3517	−1.0945	0.8321

11. According to Eqs. (4) and (12) and the test data provided by SP companies, μ and m_R can be obtained.
12. According to Eq. (3), m_W can be obtained.

The calculation results of the eighteen-group concrete based on the five parameters listed in Table 3 are shown in Table 5. By comparing the values of Table 2 with that of Table 5, the correctness of the proposed algorithm above is verified, and the equivalency between the five parameters and the mix proportion of concrete is confirmed.

5. Application of ANNs

5.1. Neural networks methodology

A neural network is an information processing system whose architecture essentially mimics the biological system of the brain. The neural network technique is a relatively new computational tool that is particularly useful for evaluating systems with a multitude of nonlinear variables. A neural network consists of a number of interconnected processing units. These units are commonly referred to as neurons. Each neuron receives an input signal from neurons to which it is connected. Each of these connections has numerical weights associated with them. These weights determine the nature and strength of the influence between the interconnected neurons. The signals from each input are then processed through a weighted sum *on* the inputs. The processed output signal is then transmitted to another neuron via a transfer function. The transfer function adopted in this study is $f(x)=1-2/(1+e^{2x})$. The transfer function modulates the weighted sum of the inputs so that the output approaches unity when the input gets larger and approaches zero when the input gets smaller. The architecture of a typical neural network consists three layers of interconnected neurons. Each neuron is connected to the neurons in the next layer. There is an input layer where data is presented to the neural network, and an output layer that holds the response of the network to the input. It is the intermediate layers, also known as hidden layers, which enable these networks to represent and compute complicated associations between patterns. Currently, there is no rule for determining the optimal number of neurons in the hidden layer or the number of hidden layers, except through experimentation. A single hidden layer has been found to be satisfactory for many problems.

Training of the neural network is essentially carried out through the presentation of a series of example patterns of associated input and observed output values. The neural network "learns" what it is to compute through the modification of the weights of the interconnected neurons. The most commonly used learning system is the back-propagation model. To simplify the learning process of the back-propagation neural network and to reduce the time required for training, the learning algorithm adopted to train the network model in this study is the Levenberg–Marquardt algorithm. The learning algorithm processes the patterns in two stages. In the first stage, the input pattern generates a forward flow of signals from the input layer to the output layer. The error of each output neuron is then determined from the difference between the computed values and the observed (experimental) values. The second stage involves the readjustment of the weights and biases in the hidden and output layers to reduce the difference between the computed

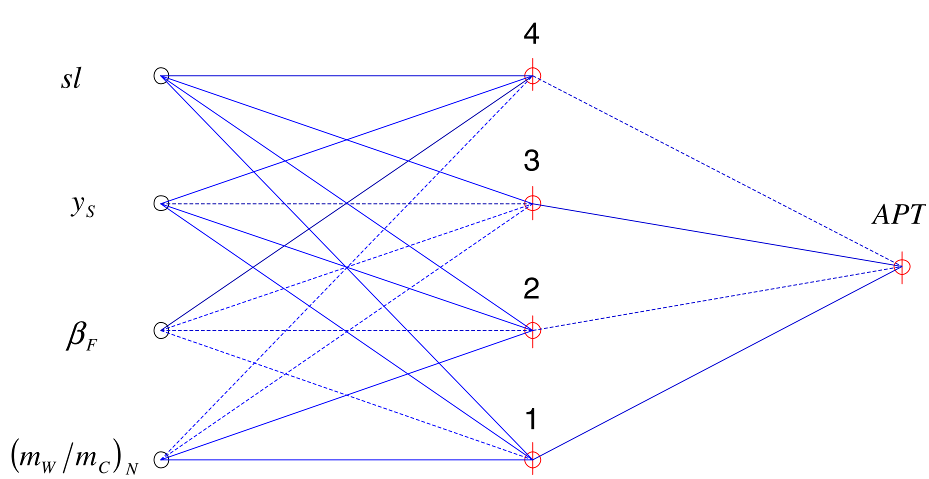

Fig. 2. Architecture of ANN used to calculate APT.

Table 7
Connection weights and biases used to obtain APT

Neuron	Connection weights				Biases
	$(m_W/m_C)_N$	β_F	y_S	sl	
1	0.3355	−0.1715	0.1243	0.1187	0.6521
2	0.3785	−0.9375	0.0907	0.0209	−0.2016
3	−0.0051	−0.0089	−1.3627	0.0115	−0.2440
4	−0.4928	1.1050	0.3907	0.8619	−0.6941

Neuron	1	2	3	4	Biases
APT	0.6169	−0.5876	1.2932	−0.3578	−0.3813

Table 8
Connection weights and biases used to predict the strength of concrete

Neuron	Connection weights				Biases
	$(m_W/m_C)_E$	β_F	APT	y_S	
1	−4.5782	−2.6180	2.9929	−1.6614	0.3565
2	6.8325	3.4674	−1.0428	−0.1714	−1.0693
3	−6.7848	1.4290	−4.1855	−2.9634	−0.9992
4	1.4802	0.3597	0.7409	1.2836	−0.2810

Neuron	1	2	3	4	Biases
f_{cu}	2.4719	−3.5213	2.5945	7.8429	−0.9255

and desired outputs. The modification of the weights is carried out using a "generalized delta rule" through the gradient descent on the error. Training is carried out iteratively until the average sum-squared errors over all training patterns are minimized.

On the satisfactory completion of the training phase, verification of the performance of the neural network is then carried out using patterns that were not included in the training set. This determines whether the neural network can generalize correct responses for patterns that only broadly resemble the data in the training set. This is often called the testing phase. Since no additional learning or connection weight changes occur during this phase, the run time is almost instantaneous [9].

5.2. Prediction model for the slump of concrete

The architecture of prediction model for the slump of concrete consists three layers as shown in Fig. 1. The input layer includes four neurons $(m_W/m_C)_N$, β_F, APT, y_S. The output layer includes one neurons sl. The hidden layer consists of four neurons. It has been verified that the five parameters y_S, β_F, $(m_W/m_C)_E$, $(m_W/m_C)_N$, APT and the mix proportion of concrete m_C, m_F, m_W, m_S, m_G, m_R can be transformed each other. The behaviors (strength, slump, et al.) of concrete primarily determined by the mix proportion of concrete now depend on the five parameters. It is also explained above that the slump of concrete has no relationship with $(m_W/m_C)_E$. So the slump of concrete is now determined only by four parameters, as Fig. 1 shows. Training of the neural network is carried out through the example patterns in Table 3 [16]. The average sum-squared error after 22 cycles is 4.6×10^{-14}. The connection weights and biases of neurons after training are shown in Table 6 and Fig. 1. The solid lines in Fig. 1 represent the positive values of connection weights in Table 6, while the dashed lines in Fig. 1 represent the negative values of connection weights. The verification phase was omitted in this study due to the less example patterns. However, the correctness of the neural work has been verified by theoretical analysis and calculation.

Other parameters also affect the slump of concrete, such as the grade of cement and the nature of aggregates. However, for a ready-mix concrete company, these parameters are often kept constant. They have been considered in the connection weights and biases of the prediction model for the slump of concrete. If these parameters change, another prediction model based on ANNs should be built.

5.3. Calculation model for APT

From the prediction model for the slump of concrete, the calculation model for APT can be obtained, as Fig. 2 shows. Training of the neural network is carried out through the

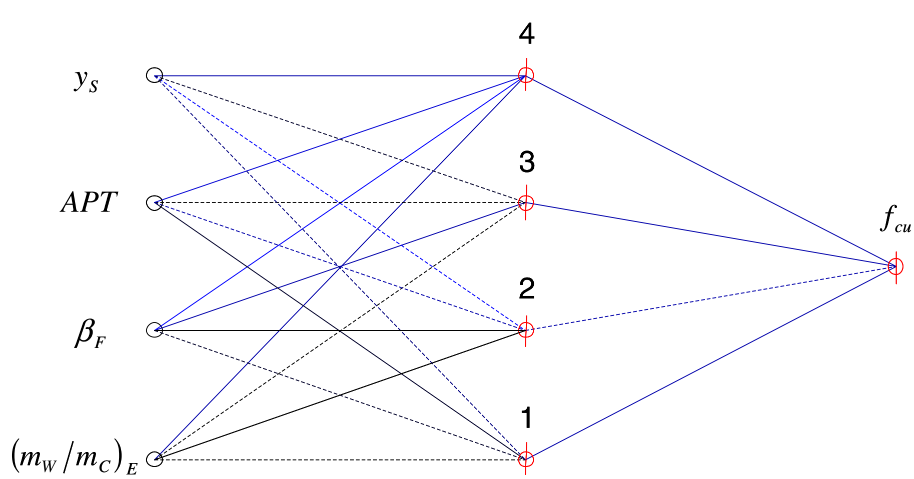

Fig. 3. Architecture of ANN used to predict the strength of concrete.

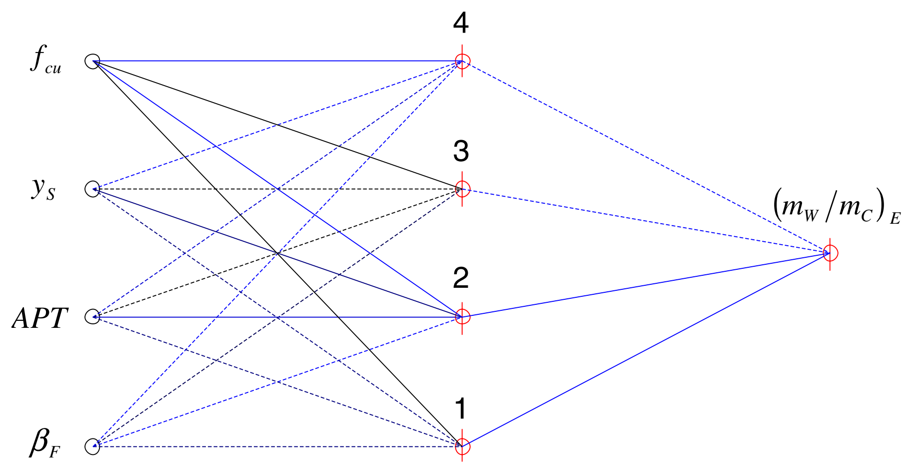

Fig. 4. Architecture of ANN used to calculate $(m_W/m_C)_E$.

example patterns in Table 3. The average sum-squared error after 35 cycles is 1.4×10^{-14}. The connection weights and biases of neurons after training are shown in Table 7 and Fig. 2.

5.4. Prediction model for the strength of concrete

The architecture of prediction model for the strength of concrete consists three layers as shown in Fig. 3. The input layer includes four neurons $(m_W/m_C)_E$, β_F, APT, y_S. The output layer includes one neurons f_{cu}. The hidden layer consists of four neurons. It has been explained above that the strength of concrete has no relationship with $(m_W/m_C)_N$. So the architecture of prediction model for the strength of concrete does not include $(m_W/m_C)_N$ Training of the neural network is carried out through the example patterns in Table 3. The average sum-squared error after 50 cycles is 3.5×10^{-14}. The connection weights and biases of neurons after training are shown in Table 8 and Fig. 3.

5.5. Calculation model for $(m_W/m_C)_E$

From the prediction model for the strength of concrete, the calculation model for $(m_W/m_C)_E$ can be obtained, as Fig. 4 shows. Training of the neural network is carried out through the example patterns in Table 3. The average sum-squared error after 56 cycles is 1.1×10^{-14}. The connection weights and biases of neurons after training are shown in Table 9 and Fig. 4.

Table 9
Connection weights and biases used to obtain $(m_W/m_C)_E$

Neuron	Connection weights				Biases
	β_F	APT	y_S	f_{cu}	
1	−1.6634	−1.8491	−1.7076	5.7111	0.9425
2	−0.3270	0.7373	1.6555	0.1617	1.5846
3	−2.0299	−4.7128	−3.5931	11.5674	4.0508
4	−0.1892	−0.6869	−0.4764	0.1727	0.6795

Neuron	1	2	3	4	Biases
$(m_W/m_C)_E$	5.7245	1.5142	−6.8989	−2.3798	0.7655

6. Concrete mix proportion design algorithm

The concrete mix proportion design process is shown in Fig. 5. The design algorithm of concrete mix proportion based ANNs is provided in detailed as follows:

(1) Given f_{cu}, sl, β_F, μ, θ.
(2) Given C_{S0}, C_{G0}, C_S, C_G, ρ_C, ρ_F, ρ_W, ρ_S, ρ_G, ρ_{ST}, ρ_{GT}, d_{ST}, d_{GT}, λ_R, C_R, α.
(3) Assume $(m_W/m_C)_N$.
(4) Assume y_S.
(5) Solve APT through the calculation model of APT as shown in Fig. 2 and Table 7.
(6) From Eqs. (15) and (25), the grain volume fraction of fine aggregates corresponding to the least total paste volume of concrete mixtures for a unit volume of aggregates can be obtained. However, the fraction of fine aggregates corresponding to the least paste content is not necessarily a best target for mix designers. An increase of 5% in the grain volume fraction of fine aggregates is reasonable in the aggregate combination, because the reduction of packing degree is very limited (as shown in Fig. 6, where $e_S=1-\phi_A$) and the consistency of concrete mixtures can be improved. Then the modified grain volume fraction of fine aggregates y_S and the corresponding total paste volume of concrete mixtures for a unit volume of aggregates V'_P can be calculated.
(7) Check if the calculated y_S in the step (6) agrees with the assumed y_S. If not, assume the new y_S as the mean of the calculated y_S and the assumed y_S, and return to step (5).
(8) Solve $(m_W/m_C)_E$ through the calculation model of $(m_W/m_C)_E$ as shown in Fig. 4 and Table 9.
(9) From Eq. (9), $(m_{W,E}/m_C)$ can be obtained.
(10) The total paste volume corresponding to a unit volume of concrete mixture can be obtained according to Eq. (28).
(11) The by weight contents of fine and coarse aggregates corresponding to a unit volume of concrete mixture m_S, m_G can be obtained according to Eqs. (29) and (30), respectively.

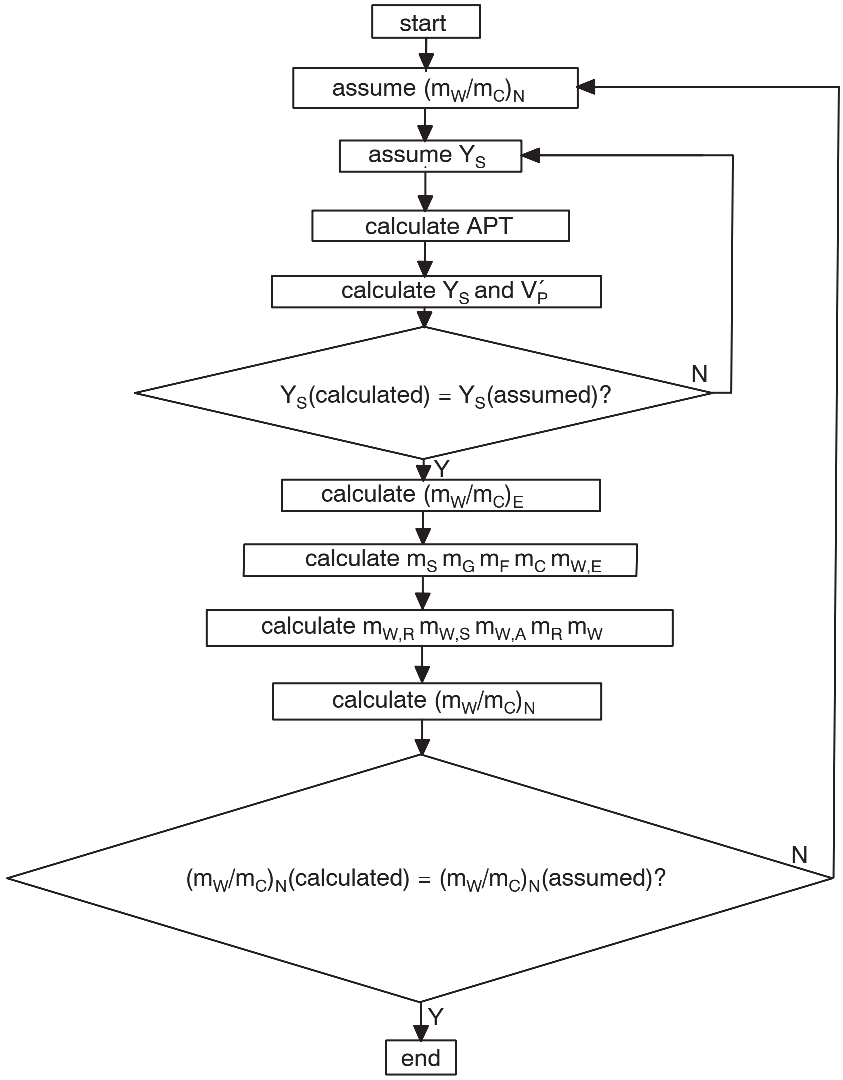

Fig. 5. Flow-chart of mix proportion design algorithm.

(12) From Eq. (31), m_C can be obtained.
(13) From Eq. (32), m_F can be obtained.
(14) According to Eq. (33), $m_{W,E}$ can be obtained.

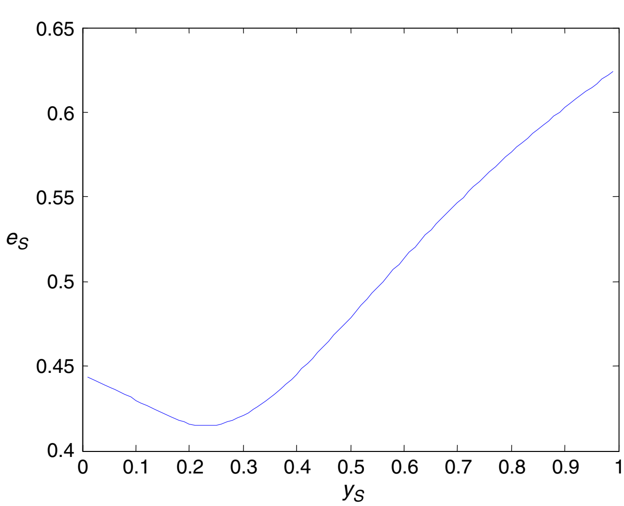

Fig. 6. Relation between y_S and e_S.

(15) According to Eqs. (5) and (6), $m_{W,A}$ and $m_{W,S}$ can be obtained.
(16) According to m_C, μ, and the test data provided by SP companies, m_R can be obtained.
(17) According to Eq. (4), $m_{W,R}$ can be obtained.
(18) According to Eq. (3), m_W can be obtained.
(19) According to Eq. (12), $(m_W/m_C)_N$ can be obtained.
(20) Check if the calculated $(m_W/m_C)_N$ in the step (19) agrees with the assumed $(m_W/m_C)_N$. If not, assume the new $(m_W/m_C)_N$ as the mean of the calculated $(m_W/m_C)_N$ and the assumed $(m_W/m_C)_N$, and return to step (4).

Table 10
Design parameters and results

β_F	μ	$(m_W/m_C)_N$	y_S	$(m_W/m_C)_E$	APT (10^{-6} m)
0.15	16.3%	0.4595	0.28	0.4203	82.86

m_C (kg/m^3)	m_W (kg/m^3)	m_F (kg/m^3)	m_S (kg/m^3)	m_G (kg/m^3)	m_R (kg/m^3)
457	220	81	429	1102	3.56

7. Example

A mix proportion of concrete with a 28d compressive strength f_{cu} of 53.1 MPa and a slump of 15 cm is designed. The selected μ and β_F are listed in Table 10. The other parameter values are the same as that of Table 1. According to the results of the previous tests on several mix-design concretes, two artificial neural networks (ANNs) are built as shown in Figs. 2 and 4 before being able to obtain the intended concrete in terms of slump and strength. The parameter values of $(m_W/m_C)_N$, y_S, $(m_W/m_C)_E$, APT can be obtained according to the designed process provided in Chapter 6, and are also listed in Table 10. The mix proportion of concrete m_C, m_W, m_F, m_G, m_S, m_R obtained by the proposed concrete mix proportion design logarithm is shown in Table 10.

8. Conclusions

1. A concrete mix proportion design algorithm based on a way from aggregates to paste, a least paste content, Modified Tourfar's Model and artificial neural networks (ANNs) was proposed. The proposed concrete mix proportion design algorithm is expected to reduce the number of trial and error, save cost, laborers and time. The concrete designed by the proposed algorithm is expected to have lower cement and water contents, higher durability, better economical and ecological effects.
2. The five parameters of nominal water–cement ratio, equivalent water–cement ratio, average paste thickness, fly ash–binder ratio, grain volume fraction of fine aggregates and the mix proportion of concrete can be transformed each other when Modified Tourfar's Model is applied. The behaviors (strength, slump, et al.) of concrete primarily determined by the mix proportion of concrete now depend on the five parameters when other parameters, such as the grade of cement and the nature of aggregates et al., are kept constant.
3. The prediction models of strength and slump of concrete were built based on artificial neural networks (ANNs). The calculation models of average paste thickness and equivalent water–cement ratio can be obtained by the reversal deduction of the two prediction models, respectively. And other parameters, such as the grade of cement and the nature of aggregates et al., are considered in the connection weights and biases of these ANN models as long as these parameters are kept constant. If these parameters change, another prediction model based on ANNs should be built.
4. For the grain volume fraction of fine aggregates corresponding to the least total paste volume of concrete mixtures, an increase of 5% in the grain volume fraction of fine aggregates is reasonable in the aggregate combination, because the reduction of packing degree is very limited and the consistency of concrete mixtures can be improved.
5. Artificial neural networks of slump, strength, equivalent water–cement ratio and average paste thickness should be built in each ready-mixed concrete company of different areas. Test data (namely, example patterns of ANNs) should be accumulated in order to update the connection weights and biases of ANNs. Then the prediction precision of the artificial neural networks can be improved.
6. To complement this type of approach, more research is needed to predict the adiabatic temperature rise, creep and shrinkage, durability of concrete, and to design the mix proportion of high performance concrete (HPC), whose strength, workability and durability satisfy specific requirements.

Acknowledgments

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