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A new model for the estimation of compressive strength of Portland cement concrete

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Abstract

In this article, on the basis of the existing experimental data, an empirical equation for calculating the compressive strength of Portland cement concrete is developed. The determination of the compressive strengths by the equation described here relies on accurate determination of the water to cement ratio which gives maximum compressive strength and the analysis of its variation with the curing time. The results obtained for the plain (without admixture) and latex modified concretes at the age of 28 days show that this ratio ranges from 0.18 to 0.23. These values are reasonably close to the non-evaporable water content reported for the Portland cement. On the other hand, this range as determined by the above procedure limits the usefulness of the proposed equation for predicting the compressive strength of silica fume blended Portland cement concretes. However, a general method of solving problems of this type allows the determination of upper and lower bounds of this range. This method requires the measurement of at least two compressive strengths corresponding to two different water to cement ratios.

Keywords: Concrete; Estimation; Compressive strength; Non-evaporable water

1. Introduction

The various mathematical models have been developed to explain the dependence of mechanical properties of Portland cement concrete upon the water to cement ratio. It has been shown that within the range of applicability the Abrams equation [1] is a remarkably good representation of the experimental situation. This simple model which has only two experimental parameters demonstrates the great importance of water to cement ratio in determining the mechanical performance of Portland cement concrete. More complicated models with a large number of adjustable parameters have been used to account for the effects of other factors on the mechanical behaviour of Portland cement concrete with fairly satisfactory results. The augmented Abrams equations developed by Popovics [2] provide a specific information concerning this. However, in the proposed equations so far, the effect of non-evaporable water content has not been taken into account. Therefore, a new mathematical model is derived that contains not only the water to cement ratio and the quantities defining the compressive strength of the concrete but also the assumed values for the non-evaporable water content of Portland cement. The average values for these parameters are found by analyzing available experimental data [3–5] on the compressive strength of Portland cement concrete. Additionally, a brief discussion concerning the effects of formulation and environmental variables on the magnitude of these parameters is also given.

2. Derivation of the equation

The water to cement ratio dependence of compressive strength over small ranges of water to cement ratio can be represented by a hyperbolic-type relation

$$\sigma = \frac{\alpha \cdot \frac{w}{c}}{\left(\beta + \lambda \cdot \frac{w}{c}\right)^n} \tag{1}$$

where σ is the compressive strength of concrete (MPa); w/c is the water to cement ratio. This relation can be represented by a curve similar to that in Fig. 1. The data given on Fig. 1 were

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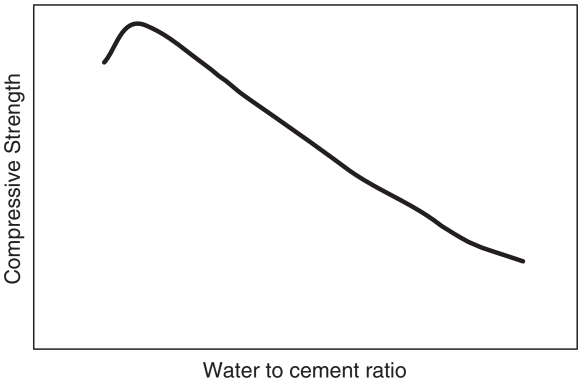

Fig. 1. Schematic representation of the compressive strength of Portland cement concrete as a function of water to cement ratio. The data were taken from Ref. [3].

reported by Neville [3]. The maximum point of this curve represents, for a given water to cement ratio, the maximum value of the compressive strength, σ_m. This means that

$$d\sigma/d(w/c)=0 \text{ and } \sigma=\sigma_m \qquad \text{at } w/c=(w/c)_m \tag{2}$$

The parameters α, β and λ in the general Eq. (1) are then obtained from the above conditions.

From the first of these conditions it follows that

$$\lambda=\frac{\beta}{(n-1)\cdot\left(\frac{w}{c}\right)_m} \tag{3}$$

and from the second it obtains

$$\alpha=\frac{\sigma_m\cdot\beta^n\cdot\left(1+\frac{1}{n-1}\right)^n}{\left(\frac{w}{c}\right)_m} \tag{4}$$

Substituting expressions (3) and (4) into Eq. (1) gives the following equation for calculating the compressive strength of concrete.

$$\sigma=\sigma_m\frac{\frac{w}{c}}{\left(\frac{w}{c}\right)_m}\left[\frac{1+\frac{1}{n-1}}{1+\frac{\frac{w}{c}}{(n-1)\cdot\left(\frac{w}{c}\right)_m}}\right]^n \tag{5}$$

As the age of concrete increases, the ratio $(w/c)_m$ approaches a limiting value 0.23 which is very close to the non-evaporable water content of Portland cement. If the magnitude of σ_m for this value of the ratio $(w/c)_m$ is known, the compressive strength of concrete for any value of the water to cement ratio can be calculated easily from Eq. (5). Attempts can be made to experimentally determine the magnitude of σ_m. However, an important conclusion regarding the magnitude of σ_m can be obtained from Eq. (5). By the substitution of σ_a for σ and $(w/c)_a$ for w/c in this equation the following expression for σ_m is found.

$$\sigma_m=\sigma_a\frac{\left(\frac{w}{c}\right)_m}{\left(\frac{w}{c}\right)_a}\left[\frac{1+\frac{\left(\frac{w}{c}\right)_a}{(n-1)\cdot\left(\frac{w}{c}\right)_m}}{1+\frac{1}{n-1}}\right]^n \tag{6}$$

in which $(w/c)_a$ represents any value of the water to cement ratio; σ_a is the compressive strength which corresponds to the ratio $(w/c)_a$. The value of σ_a must be obtained experimentally by performing tests on standard samples. Then, with the use of Eq. (6), the value of σ_m can be calculated for various values of σ_a and $(w/c)_a$. Such calculations show that in general the experimental value of σ_a results in a reasonable value for σ_m and should be taken as a basis for the design of the compressive strength—water to cement ratio diagram of the concrete. The final form of Eq. (1) is found by substituting the relation for σ_m from Eq. (6) into Eq. (5), yielding

$$\sigma=\sigma_a\cdot\frac{\frac{w}{c}}{\left(\frac{w}{c}\right)_a}\cdot\left[\frac{\left(\frac{w}{c}\right)_m(n-1)+\left(\frac{w}{c}\right)_a}{\left(\frac{w}{c}\right)_m(n-1)+\frac{w}{c}}\right]^n \tag{7}$$

This hyperbolic function offers numerous advantages from an analytical mechanics point of view. In this relation, the parameter n is a presumably characteristic of the binder and it was found to be 12 for the Portland cement. This value was obtained from the simple curve-fitting procedure. The data reported by Popovics [2] and Neville [3] for Portland cement concretes were used for these purposes. The value of the empirical characterization parameter, $(w/c)_m$, could be determined by choosing two σ–w/c points on a strength curve, writing Eq. (7) using these values and then solving the resulting equation. It should be noted that one of the σ–w/c points represents σ_a–$(w/c)_a$. This method of procedure eliminates the necessity of solving two equations. However, the value of $(w/c)_m$ can be obtained to a reasonable degree of accuracy from an analytic expression. In this case, the solution to this problem is a relatively simple one, since it can be reduced to only one unknown. One of the goals in this design is to develop a reliable model for characterizing $(w/c)_m$ in terms of an appropriate parameter. This approach is based on a combination of mathematical and physical arguments with an eye toward the interpretation of such a relation for use in the analysis.

3. Results and discussion

Eq. (7) is a mathematical formulation designed to approximately describe the compressive strength of the Portland cement concrete over a certain restricted range of the variables involved. This equation provides a clear indication of the range of compressive strengths obtainable by simply varying the

Table 1
Comparison of the experimental and the theoretical values of compressive strength of ordinary Portland cement concrete at different cure times

w/c	Experimental values (MPa)				Theoretical values (MPa)			
	1 day	7 days	28 days	1 year	1 day	7 days	28 days	1 year
0.4	22	43	53	65	22	43	53	65
0.5	13	30	42	55	13.66	31.80	41.55	53.40
0.6	8	22	32	45	8.46	23.08	31.83	42.71
0.7	5	16	26	37	5.27	16.62	24.09	33.66
0.8	3	10	18	28	3.32	11.95	18.13	26.31
0.9	2	8	14	23	2.12	8.60	13.62	20.48
For the ordinary Portland cement concrete, the value $n=12$ can be taken with good accuracy.					$\sigma_m=60.18$ MPa $(w/c)_a=0.4$ $\sigma_a=22$ MPa	$\sigma_m=67.20$ MPa $(w/c)_a=0.4$ $\sigma_a=43$ MPa	$\sigma_m=71.44$ MPa $(w/c)_a=0.4$ $\sigma_a=53$ MPa	$\sigma_m=79.14$ MPa $(w/c)_a=0.4$ $\sigma_a=65$ MPa

The data were taken from Ref. [3].

water to cement ratio. However, the formulation variables such as the Portland cement and admixture types employed and the environmental variables such as temperature and relative humidity as well as cure conditions will generally result in a change in σ, the magnitude of the change being largely dependent upon the values of σ_a and $(w/c)_m$. For example, the data in Table 1 show that for an ordinary Portland cement concrete with a water to cement ratio of 0.4, σ_a changes between 22 and 65 MPa are possible over cure times ranging from 1 day to 1 year. Perhaps of greater importance is the pronounced influence of these variables on $(w/c)_m$. From the theoretical point of view, $(w/c)_m$ approaches a constant value, with an increase in the cure time. Fig. 2 shows the variation of $(w/c)_m$ as a function of the square root of the cure time. This curve can be represented with sufficient accuracy by the following equation.

$$\left(\frac{w}{c}\right)_m = 0.23\frac{\sqrt{t}}{(1+\sqrt{t})} \tag{8}$$

where 0.23 is a constant for an ordinary Portland cement concrete; t is the cure time (days) and $(w/c)_m$ is the water to cement ratio which corresponds to σ_m at any instant of cure. This equation is valuable in determining how the magnitude of $(w/c)_m$ changes with a change in the cure time. By calculating $(w/c)_m$ from Eq. (8) and by using the experimental values of σ_a corresponding to the water to cement ratio of 0.4, the compressive strengths of concrete at any instant of cure are calculated for each value of the water to cement ratio from Eq. (7). The results of such calculations are given in Table 1. It can be seen that the experimental results are in good agreement with the results obtained theoretically by using Eq. (7). In the same manner, the values of σ can be found for any other assumed value of the ratio $(w/c)_a$ and the corresponding values of σ_a. This theory is also applied to account for the changes in the magnitude of σ_m during cure. In this analysis, the values of σ_m are found by taking $(w/c)_a=0.4$ and by determining the corresponding values of σ_a from Table 1 and then by calculating the values of $(w/c)_m$ from Eq. (8) and finally by substituting these selected and calculated values into Eq. (6). The values of σ_m calculated in this way vary between 60.18 and 79.14 MPa over cure times ranging from 1 day to 1 year. It is interesting to note that the value of σ_m for the concrete at the age of 1 year is only 32% higher than the value obtained for the concrete at the age of 1 day. From this comparison, it can be seen how important is the use of $(w/c)_m$ and σ_a in determining the magnitude of σ_m. However, it may be difficult to determine the values of $(w/c)_m$ for the rapid hardening Portland cement concrete without carrying out an extensive series of tests on standard samples. The procedure of calculation in this particular case will be as follows: the value of $(w/c)_m$ for the rapid hardening Portland cement concretes is determined from Eq. (7) by taking two values for the water to cement ratio and by using the corresponding values of the compressive strength. In this analysis, one of the water to cement ratios is denoted by $(w/c)_a$ and the corresponding value of the compressive strength by σ_a.

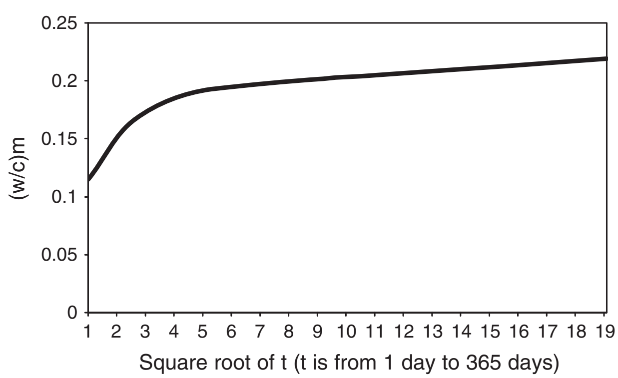

Fig. 2. Variation of $(w/c)_m$ as a function of the square root of the cure time [Eq. (8)] for ordinary Portland cement concrete.

Table 2
Comparison of the experimental and the theoretical values of the 7 day compressive strength of concrete made with a rapid hardening Portland cement

w/c	Experimental values (MPa)	Theoretical values (MPa)
0.4	42	42
0.5	34	34.56
0.7	21	21.85
0.9	13	13.33
1.1	8	8.07
For the rapid hardening Portland cement concrete, the value $n=12$ can be taken with good accuracy.		$\sigma_m=50.97$ MPa $(w/c)_m=0.22$ Eq. (7) $(w/c)_m=0.167$ Eq. (8) $(w/c)_a=0.4$ $\sigma_a=42$ MPa

The data were taken from Ref. [3].

Table 3
Comparison of the experimental and the theoretical values of the 28 day compressive strength of the silica fume blended Portland cement concrete

w/c	Silica fume replacement level (%)			
	8	16	8	16
	Experimental values (MPa)		Theoretical values (MPa)	
0.45	70	90	70	90
0.55	60	–	59.58	–
0.58	–	75	–	74.24
0.75	43	55	40.64	54.86
0.8	40	48	36.68	49.88
0.85	30	–	33.05	–
1	22	–	24.03	–
1.1	–	28	–	27.43
For the silica fume blended Portland cement concrete, the value $n=12$ can be taken with good accuracy.			$\sigma_m=83.04$ MPa $(w/c)_m=0.256$ Eq. (7) $(w/c)_m=0.193$ Eq. (8) $(w/c)_a=0.45$ $\sigma_a=70$ MPa	$\sigma_m=103.09$ MPa $(w/c)_m=0.271$ Eq. (7) $(w/c)_m=0.193$ Eq. (8) $(w/c)_a=0.45$ $\sigma_a=90$ MPa

The data were taken from Ref. [4].

Thus, the compressive strengths can be calculated easily from Eq. (7) for various values of the water to cement ratio. The results obtained for $(w/c)_m=0.22$ are given in Table 2. It is seen that for the rapid hardening Portland cement concrete cured for 7 days, this equation gives results which are in satisfactory agreement with the experiments. However, the value of $(w/c)_m$ calculated by this method is very different from that calculated by Eq. (8). This means that formulation variables can have as significant an effect on the magnitude of $(w/c)_m$ as the cure conditions highlighted above. This effect can be particularly severe with mineral admixtures which by necessity need incorporation in large quantities e.g. silica fume and fly ash. In Table 3, is given a comparison of the 28 day compressive strengths of silica fume blended Portland cement concretes as calculated by using Eq. (7) and as obtained by direct tests of standard samples. It is seen that a better agreement between the theory and the experiments is obtained when $0.256 \le (w/c)_m \le 0.271$. These values of $(w/c)_m$ calculated by Eq. (7) are generally higher than that found from Eq. (8) and therefore Eq. (7) can be recommended for silica fume blended Portland cement concretes. There are striking differences in the effects on $(w/c)_m$ and σ_a observed with silica fume replacement of Portland cement, both as to the effect of progressive modification and the direction of the change. One of the remarkable features of the changes in $(w/c)_m$ is that when the quantity of silica fume in the Portland cement concrete increases, the magnitude of $(w/c)_m$ begins to increase. This result should be expected since the non-evaporable water contains nearly all chemically combined water and also some water not held by chemical bonds [3]. In addition to silica fume, other admixture types are available, each capable of exerting a substantial influence on $(w/c)_m$. Although the effects of mineral admixtures on $(w/c)_m$ are well recognised, the

Table 4
Comparison of the experimental and the theoretical values of the 28 day compressive strength of the latex modified Portland cement concrete

w/c	Latex content (%)					
	2	2.3	2.6	2	2.3	2.6
	Experimental values (MPa)			Theoretical values (MPa)		
0.38	–	–	61.9	–	–	61.9
0.41	–	–	57.3	–	–	57.51
0.42	–	55.6	–	–	55.6	–
0.44	–	–	53.3	–	–	53.25
0.46	–	51.4	–	–	51.49	–
0.47	–	–	48.9	–	–	49.16
0.49	51.4	–	–	51.4	–	–
0.50	–	47.1	–	–	47.43	–
0.53	47	44.5	–	46.57	44.47	–
0.57	43.2	–	–	42.05	–	–
0.61	37.2	–	–	37.89	–	–
For the latex modified Portland cement concrete, the value $n=12$ can be taken with good accuracy.				$\sigma_m=82.57$ MPa $(w/c)_m=0.1998$ Eq. (7) $(w/c)_m=0.193$ Eq. (8) $(w/c)_a=0.49$ $\sigma_a=51.4$ MPa	$\sigma_m=69.96$ MPa $(w/c)_m=0.22$ Eq. (7) $(w/c)_m=0.193$ Eq. (8) $(w/c)_a=0.42$ $\sigma_a=55.6$ MPa	$\sigma_m=83.58$ MPa $(w/c)_m=0.183$ Eq. (7) $(w/c)_m=0.193$ Eq. (8) $(w/c)_a=0.38$ $\sigma_a=61.9$ MPa

The data were taken from Ref. [5].

influence of organic admixtures such as latex is somewhat less predictable due to its ability to influence several factors. However, it is found that $(w/c)_m$ is virtually independent of the latex concentration from approximately 2% to 2.6%. i.e. the common range of concentration traditionally employed in practise. Moreover, the values of $(w/c)_m$ obtained from Eq. (7) are usually similar in magnitude to those found for plain (without admixture) Portland cement concrete from Eq. (8). In Table 4 is given a comparison of the values of $(w/c)_m$ as calculated by using these equations. For example, $(w/c)_m$ of the latex modified Portland cement concrete cured for 28 days is found to be 0.1998 for the latex concentration of 2%, 0.22 for the latex concentration of 2.3% and 0.183 for the latex concentration of 2.6%. These values of $(w/c)_m$ correspond usually to the non-evaporable water contents of fully or well-hydrated Portland cements. Proceeding as in the previous cases and using the above-calculated values of $(w/c)_m$, the theoretical values of the compressive strength can be calculated with sufficient accuracy from Eq. (7). The results of calculations made for $(w/c)_m = 0.1998$ and $(w/c)_m = 0.22$ and $(w/c)_m = 0.183$ are given in Table 4. It is seen that the values obtained in this manner are usually in a very satisfactory agreement with the experimental values.

4. Conclusions

Eq. (7) can be used for calculating the compressive strength of Portland cement concrete as long as the ratio $(w/c)_m$ does not exceed the limiting value 0.23. In this case, the value of the ratio $(w/c)_m$ at any instant of cure is obtained from Eq. (8). This method used for the analysis of plain Portland cement concrete can also be applied to the analysis of the latex modified Portland cement concrete. The value of the ratio $(w/c)_m$ for the 28 days cured concretes containing latex between 2% and 2.6% can vary from about 0.18 up to 0.23, these again being the typical values for the non-evaporable water content of Portland cement. However, the selection of a proper value for the ratio $(w/c)_m$ presents considerable difficulty in the analysis of the silica fume blended Portland cement concretes. The simplest method, based on measuring the compressive strengths corresponding to two different water to cement ratios, could be used to calculate the values of the ratio $(w/c)_m$ from Eq. (7). This method gives $(w/c)_m$ values which are accurate enough for the silica fume blended Portland cement concretes.

Nomenclature

w/c	Water to cement ratio
σ	Compressive strength of Portland cement concrete
σ_m	Maximum compressive strength of Portland cement concrete
$(w/c)_m$	Water to cement ratio corresponding to the maximum compressive strength of Portland cement concrete, σ_m.
$(w/c)_a$	Any value of the water to cement ratio.
σ_a	Compressive strength of Portland cement concrete which corresponds to the ratio $(w/c)_a$.
t	Cure time.
n	A parameter that is presumably characteristic of the binder.
α, β, λ	Parameters in Eqs.(1), (3) and (4)

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