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Fatigue enhancement of concrete beam with ECC layer

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Abstract

The pseudo strain-hardening behavior of Engineered Cementitious Composites (ECC) is a desirable characteristic for it to replace concrete to suppress brittle failure. This widespread use of ECC in the industry is, however, limited by its high cost. To achieve higher performance/cost, ECC can be strategically applied in parts of a structure that is under relatively high stress and strain. In this paper, layered ECC-concrete beams subjected to static and fatigue flexural loads were investigated by experiments. The static test results showed that the application of a layer of ECC on the tensile side of a flexural beam increased its flexural strength and the degree of improvement increased with the thickness of ECC applied. In addition, the layer of ECC enhanced the ductility of the beam and the failure mode changed from brittle to ductile. Under four-point cyclic loading, the ECC layer significantly improved the fatigue life of the beam. Moreover, in comparison to plain concrete beams, layered ECC beams could sustain fatigue loading at a larger deflection without failure. The great improvement in fatigue performance was attributed to the effectiveness of ECC in controlling the growth of small cracks. The experimental findings reflect the feasibility of using ECC strategically in critical locations for the control of fatigue crack growth.

Keywords: Cementitious composites; Fiber reinforcement; Fatigue; Cracking

1. Introduction

Concrete, composing of cement, water, fine and coarse aggregates, is widely used in civil engineering construction. The brittle behavior of concrete, especially under tensile loading, is one of its major limitations. With the low toughness of concrete, cracks can propagate rapidly to result in failure at a low ultimate strain (around 0.01%) without warning. In order to improve the failure behavior, fiber reinforced concrete (FRC) is made by adding discrete short fibers into the concrete matrix. Fibers used currently include steel, glass, carbon and polymer fibers. These fibers act as bridges across the cracks to delay their propagation. Thus, a more ductile failure mode with a significant softening response could be attained.

Development of FRCs started in the 1970s. At that time, only glass fiber and steel fiber were investigated. During the past 10 years, with the development of micromechanical models, properties of fiber, matrix and fiber/matrix interface have been tailored to produce cementitious composites that exhibit a pseudo-strain hardening behavior similar to that of metals. These composites are called Engineered Cementitious Composites (ECC) [1]. With effective crack bridging provided by the fibers, failure of ECC occurs with the formation of multiple cracks that are very fine (the crack opening at ultimate strain is normally below 100 μm). The ultimate tensile strain can reach a value as high as 2–6%, which is 200 to 600 times greater than that of concrete.

A typical mix of PVA–ECC contains about 2% of PVA fibers. Also, the binder content is often above 1000 kg/m^3. As a result, the cost of PVA–ECC can be 4 to 5 times that of normal concrete [2]. In order to achieve better economy, ECC should be used selectively in parts of a structure where their advantages can be fully exploited.

The flexural fatigue performance of concrete is important for many applications such as highway and airfield pavements, bridge decks and pavement overlays. Consider a plain concrete beam subjected to cyclic flexural load. The first crack will form when the maximum principal tensile stress exceeds the cracking strength of the material. Crack propagation will initially occur at

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a slow rate, and increase rapidly with crack size. The number of cycles to failure is hence governed by the 'small crack' regime, when crack propagation is slow. Once the crack propagates beyond a certain size, it will grow very rapidly and failure occurs after a small number of additional cycles. Following this argument, mechanisms that can delay crack propagation at an early stage should be particularly effective in extending the fatigue life. The incorporation of an ECC layer (which possesses very high resistance to crack growth) on the tensile side of the concrete beam (where cracks will initiate) may hence be a cost-effective alternative to improve the fatigue performance of a concrete beam.

The fatigue performance of beams made entirely of ECC has been investigated in the past [3,4]. As expected, the ECC beams exhibited significantly higher fatigue life than plain concrete members. In the present investigation, layered ECC/concrete beams will be made with two different ECC layer thickness. Both monotonic and fatigue flexural tests will be performed. As control specimens, plain concrete members are also cast and tested. The test results, including the number of cycles to failure and the growth of deflection with cyclic loading, will be compared for various cases and discussed.

2. Experimental details

2.1. Specimens, materials and mix

Fatigue behavior of layered ECC–concrete beam was investigated through conducting four-point cyclic bending tests. In each batch, six specimens with dimensions 100×100×500 mm were cast. Among the specimens, three of them were tested under static flexural load; another three were tested under fatigue flexural load. The strength values obtained in the static tests were used to determine the range of fatigue loading when conducting the bending fatigue tests. The mix constituents and their mix proportions are shown in Table 1. As the sand used in our investigation was much coarser than that in other studies, no superplasticizer was required. In this study, specimens with two different ECC layer thickness (25 mm and 50 mm) were prepared. During the beam casting, the ECC layer was cast first at the bottom of the mould. One hour after casting the ECC layer, concrete was added. The reason for not casting concrete immediately after the ECC was to prevent the mixing of the two materials. Besides the layered beams, plain concrete beams were also cast as references. For both plain concrete and ECC, 100×200 mm cylinders for compressive strength test were also cast. During specimen preparation, all members were vibrated to ensure proper compaction. After casting, the layered specimens were kept wet in the mold for another 24 hours before they were demolded and transferred to a curing room at 20 °C and 95% relative humidity. The type and number of specimens cast and tested are summarized in Table 2.

Table 1
Mix proportion of concrete and ECC

Constituent composition (by weight)	Cement	Water	Sand	Stone	PVA (by volume)
Plain concrete	1	0.55	2	2	–
ECC	1	0.32	0.4	–	1.7%

Cement: ordinary Portland cement.
Sand: river sand with maximum size around 4 mm.
PVA: Polyvinyl Alcohol Fiber from Kurarey Corp. (diameter=0.04 mm, length=12 mm, Young's modulus=4 GPa, tensile strength=1.6 GPa).

Table 2
Types and number of specimens under investigation

Type of specimen	ECC thickness	Number of specimen cast	Number of specimen tested (static/fatigue)
Static concrete 1, (CON_1)	0 mm	6	3/3
Static concrete 2, (CON_2)	0 mm	6	3/3
Static concrete 3, (CON_3)	0 mm	6	3/3
Static layered 25 mm 1, (LAY_ECC_25_1)	25 mm	6	3/3
Static layered 25 mm 2, (LAY_ECC_25_2)	25 mm	6	3/3
Static layered 25 mm 3, (LAY_ECC_25_3)	25 mm	6	3/2
Static layered 50 mm 1, (LAY_ECC_50_1)	50 mm	6	3/2
Static layered 50 mm 2, (LAY_ECC_50_2)	50 mm	6	3/3
Static layered 50 mm 3, (LAY_ECC_50_3)	50 mm	6	3/2

2.2. Testing configuration

Standard four-point flexural test was conducted. The experimental setup was as shown in Fig. 1. The loading span was 100 mm and the support span was 300 mm. During the test, both loading and mid-point deflection were recorded. The mid-point deflection was recorded with two linear variable differential transducers (LVDT) and it represents the relative displacement between the top of the beam and the loading fixture. The employed displacement sensor is of the non-contact type and displacement measurement with this sensor is illustrated in Fig. 2. The analog data of the mid-point deflection from the displacement sensor was recorded and converted to digital data through an analog-to-digital (A-to-D) card. The converted digital signal was then transmitted to a computer for storage.

In the static test, deflection control was used; the loading rate was 0.15 mm/min. Fatigue testing, on the other hand, was conducted under load control. A typical load vs time curve for the fatigue test is shown in Fig. 3. The load was first ramped up to the maximum value $P_{\max}$. Cyclic loading with a sinusoidal waveform was then applied with a loading range of ($P_{\max}$–$P_{\min}$) and at a frequency of 3 Hz or 10 Hz. The types of specimen tested and the corresponding stress level and loading frequency are shown in Table 3. The stress level, SL, is defined by:

$$\mathrm{SL} = \frac{P_{\max}}{P_{\mathrm{ult}}} \tag{1}$$

where $P_{\max}$ is the maximum load during the loading cycle, and P_{ult} is the static ultimate load of the member.

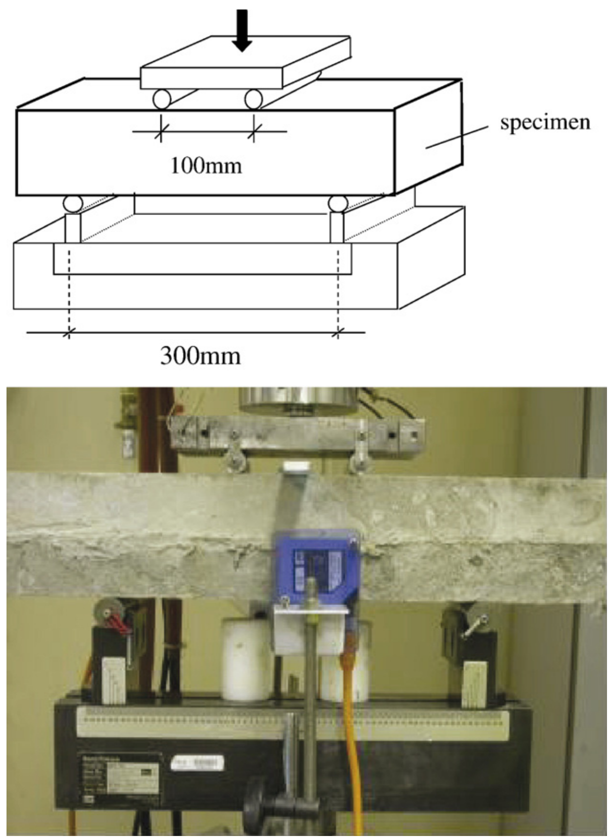

Fig. 1. Experimental set-up for four-point bending fatigue tests.

Among the six beams cast for each specimen type, three of them were tested under static four-point bending, and P_{ult} was obtained as the averaged flexural load capacity. The minimum load (P_{min}) applied in the fatigue test is given by:

$$P_{min} = 0.2 * P_{max} \quad (2)$$

Note that a non-zero minimum load, P_{min}, was applied to ensure that the beam would not be unloaded during the cyclic bending test. For plain concrete specimens, the 3 Hz-loading rate was high enough to finish the test within a reasonably short period of time. However, when layered ECC beams were under investigation, if the loading rate was kept at 3 Hz, it would take a long time to complete the test. Hence, a higher frequency (10 Hz) was employed for these specimens. In order to verify that the change of loading frequency would not affect the results significantly, a set of concrete specimens was also tested at 10 Hz and a stress level of 80%. The results were then compared to similar specimens tested at 3 Hz. The fatigue life of plain concrete beam in logarithmic scale was found to be between 3.6 to 4.9 at 3 Hz, and between 4.3 to 5.2 at 10 Hz. Since the two ranges are comparable, it is acceptable to replace the loading rate of 3 Hz by 10 Hz.

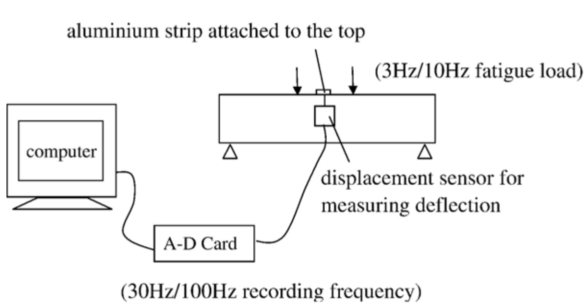

Fig. 2. Data recording and converting system used in bending test.

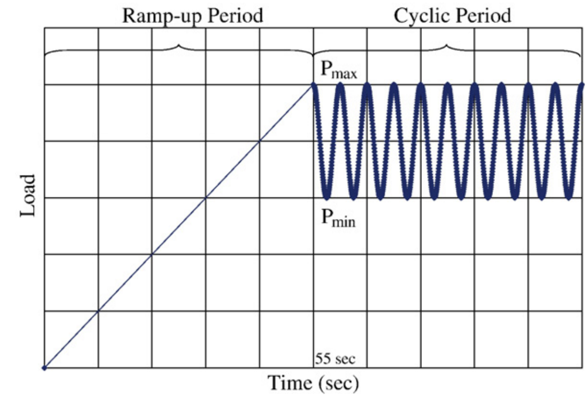

Fig. 3. Loading procedures used in fatigue test.

During the fatigue test, the data acquisition card collected the data (including both load and displacement) at frequencies of 30 Hz and 100 Hz for loading frequency of 3 Hz and 10 Hz respectively. There were hence 10 data points for each loading cycle. The test was terminated either at the time of failure of the specimen or when the number of applied loading cycles reached million.

3. Results and discussion

3.1. Layered ECC–concrete beam under static flexural loading

Static load-deflection curves for the different kinds of beams are displayed in Fig. 4. The values of compressive strength for the materials in each batch are given in Table 4. In Table 5, the static flexural strength (or modulus of rupture) of the beams is

Table 3
Type of specimens under fatigue tests and the frequencies applied

Type of specimen	ECC thickness	Stress level (P_{max}/P_{ult})/%	Frequency applied/Hz
Fatigue concrete 1, (CON_1)	0 mm	90	3
Fatigue concrete 2, (CON_2)	0 mm	80	3
Fatigue concrete 3, (CON_3)	0 mm	70	3
Fatigue layered 25 mm 1, (LAY_ECC_25_1)	25 mm	93	10
Fatigue layered 25 mm 2, (LAY_ECC_25_2)	25 mm	80	10
Fatigue layered 25 mm 3, (LAY_ECC_25_3)	25 mm	70	10
Fatigue layered 50 mm 1, (LAY_ECC_50_1)	50 mm	95	10
Fatigue layered 50 mm 2, (LAY_ECC_50_2)	50 mm	90	10
Fatigue layered 50 mm 3, (LAY_ECC_50_3)	50 mm	83	10

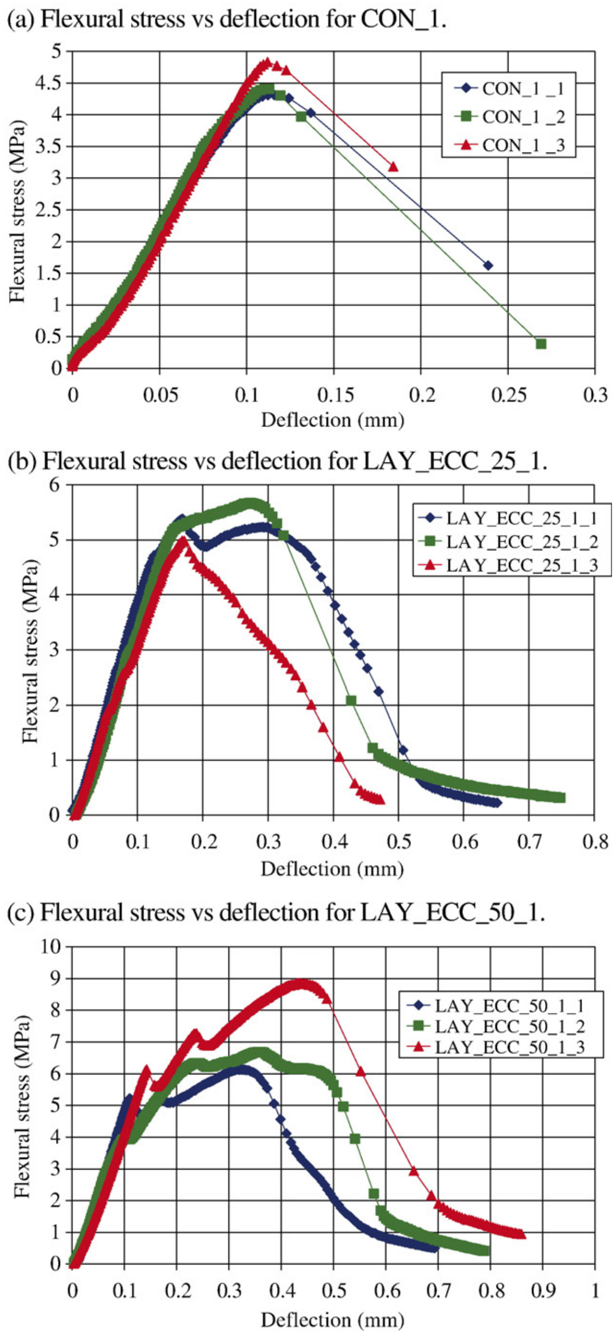

Fig. 4. Flexural stress versus deflection diagrams of layered ECC–concrete beams under four-point bending load, (a) plain concrete beams, (b) 25 mm ECC layered beams, (c) 50 mm ECC layered beams.

compared. The average flexural strength of concrete beam was 4.4 MPa. With the addition of ECC, the average flexural strength increased to 5.2 MPa for the 25 mm ECC layer and 6.3 MPa for the 50 mm layer. The percentage of strength improvement relative to the plain concrete member is 18.2% for the 25 mm layer and 43.2% for the 50 mm layer. As expected, the addition of ECC layer could improve the flexural strength. In addition, the percentage of improvement increased with the thickness of ECC layer applied.

In addition to the flexural strength improvement, there is also significant ductility improvement for the beams. A comparison on the deflection reached at ultimate load by different types of beam specimens is shown in Table 6. As indicated in Table 6, for the plain concrete beam, the average deflection at ultimate load was 0.11 mm. With the 25 mm ECC layer, this value increased to 0.18 mm, which is a 64% increase. Furthermore, the addition of a 50 mm ECC layer brought this value up to about 0.33 mm, representing a 200% increase over the plain concrete member. The increase in ductility is associated with a change in the failure mode. For plain concrete members, there is only one major crack propagating from the bottom of the beam. For the layered beams, multiple fine cracks are formed in the ECC layer first. Eventually, one of these cracks will open more widely than the others and propagate into the concrete layer. (Note: these observations are also found in the fatigue tests.) Multiple cracking enables large straining in the ECC layer before its failure, therefore leading to a higher ultimate deflection value and a ductile failure mode.

Table 4
Compressive strengths of plain concrete and ECC used in layered beams

Type of specimen	Material	Curing days	Average compressive strength (MPa)
Static concrete 1, (CON_1)	Concrete	56	54.8
Static concrete 2, (CON_2)	Concrete	56	54.7
Static concrete 3, (CON_3)	Concrete	56	59.6
Static layered 25 mm 1,	ECC	114	66.2
(LAY_ECC_25_1)	Concrete	114	52.8
Static layered 25 mm 2,	ECC	112	71.9
(LAY_ECC_25_2)	Concrete	112	59.2
Static layered 25 mm 3,	ECC	56	62.4
(LAY_ECC_25_3)	Concrete	56	44.2
Static layered 50 mm 1,	ECC	112	67.3
(LAY_ECC_50_1)	Concrete	112	55.5
Static layered 50 mm 2,	ECC	89	77
(LAY_ECC_50_2)	Concrete	89	57.3
Static layered 50 mm 3,	ECC	66	74.6
(LAY_ECC_50_3)	Concrete	66	47.4

3.2. Layered ECC concrete beam under constant amplitude fatigue loading

The results of the fatigue tests under varying maximum flexural stress level (SL) are summarized in Fig. 5 where

Table 5
Flexural strength comparison of different types of beam

Type of specimen	Flexural strength (MPa)			Average flexural strength for each batch (MPa)	Average value for each type of specimen (MPa)	Percentage increase compared to concrete (%)
	(1)	(2)	(3)			
CON_1	4.4	4.4	4.8	4.5	4.4	0
CON_2	4.7	4.0	3.7	4.1		
CON_3	4.5	5.0	4.5	4.7		
LAY_ECC_25_1	5.5	5.6	5	5.4	5.2	18.2
LAY_ECC_25_2	5.3	5.5	6	5.6		
LAY_ECC_25_3	5	4.5	4.7	4.7		
LAY_ECC_50_1	6	6.6	8.9	7.1	6.3	43.2
LAY_ECC_50_2	5.9	7.4	6.4	6.6		
LAY_ECC_50_3	5.2	6.5	4.5	5.4		

Table 6
A comparison on the deflection at peak load for different types of beam

Type of specimen	Deflection at ultimate load (mm)			Average value of the deflection (mm)	Average valuefor each type of specimen (mm)	Percentage increase compared to concrete (%)
	(1)	(2)	(3)			
CON_1	0.11	0.12	0.12	0.12	0.11	0
CON_2	0.08	0.09	0.13	0.10		
CON_3	0.11	0.13	0.1	0.11		
LAY_ECC_25_1	0.17	0.17	0.26	0.20	0.18	64
LAY_ECC_25_2	0.12	0.11	0.15	0.13		
LAY_ECC_25_3	0.12	0.30	–	0.21		
LAY_ECC_50_1	0.31	0.36	0.44	0.37	0.33	200
LAY_ECC_50_2	0.28	0.40	0.29	0.32		
LAY_ECC_50_3	0.41	0.30	0.22	0.31		

maximum flexural stress in the fatigue experiment is plotted against the logarithm of number of cycles to failure. Along with the data points, linear least square fits are shown for each beam type. In performing the regression, the lifetime for any specimen that does not fail after 2 million cycles is taken to be 2 million cycles. As mentioned above, $P_{\min}$ is set at 0.2 $P_{\max}$ for all the tests. In Fig. 6, the results for each beam type are shown in the normalized form, by dividing the value in Fig. 5 with the corresponding average static flexural strength for the same batch of specimens. For example, if the specimens are taken from the batch CON_1 (see Table 5), the fatigue load is divided by 4.5 MPa. However, if the specimen is from batch CON_2, the fatigue load is divided by 4.1 MPa instead. Fig. 7 shows typical variation of mid-point deflection with time during the test. At a particular time, the mean value of mid-span deflection is calculated as the average over the time period ($t-0.5$ s to $t+0.5$ s). For specimens with different ECC layer thickness, the variations of this mean mid-point deflection with the number of cycles (in logarithmic scale) are shown in Figs. 8–10.

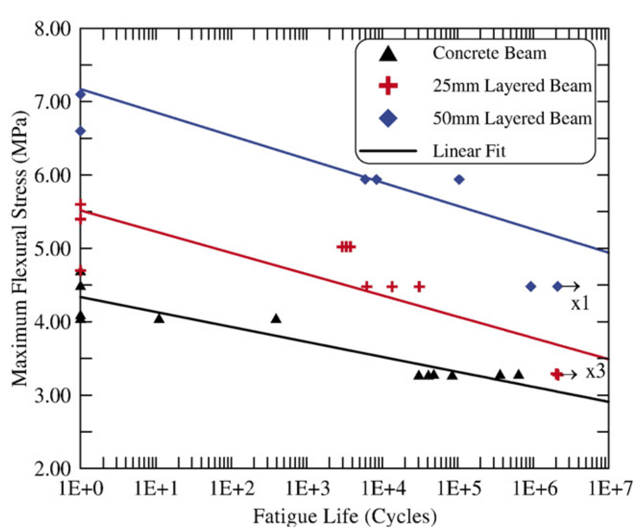

Fig. 5. Maximum stress versus fatigue life diagrams for different beam types, linear fits showing the trends of fatigue sustainability of the beams.

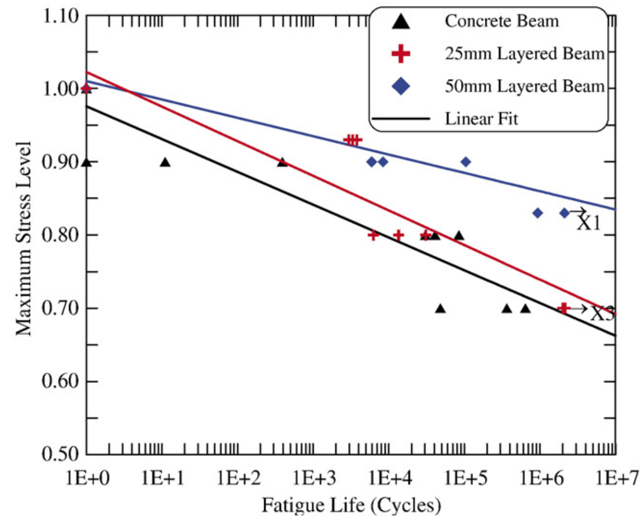

Fig. 6. Maximum fatigue stress level versus fatigue life curves of different beam types, linear fits showing the trends of fatigue sustainability of the beams.

From Figs. 5 and 6 the effects of ECC layer on the fatigue life of concrete beam can be evaluated in two different ways, depending on whether the improvement in static performance is taken into account or not. If the change in static performance is not considered (Fig. 5), i.e. comparing beams under the same magnitude of maximum flexural stress, it can be concluded that the ECC layer significantly improves the fatigue life and that the fatigue life is improving with increasing ECC layer thickness. When taking the static strength improvement into account (by normalizing the results as in Fig. 6), that is, comparing results for specimens under similar relative stress levels (e.g. 90%, 93% of the static strength), an increase in the fatigue life was also observed in specimens with a layer of ECC applied. In addition, with a thicker layer of ECC applied, the fatigue life at the same stress level was higher. Specifically, one can look at the results in Fig. 6. The regression lines indicate that the rate of strength deterioration with number of cycles decreases when the ECC layer thickness is increased. As an example, in comparison to the plain concrete beam under 90% stress level, the application of a 25 mm ECC layer increased the fatigue lifetime by almost 2

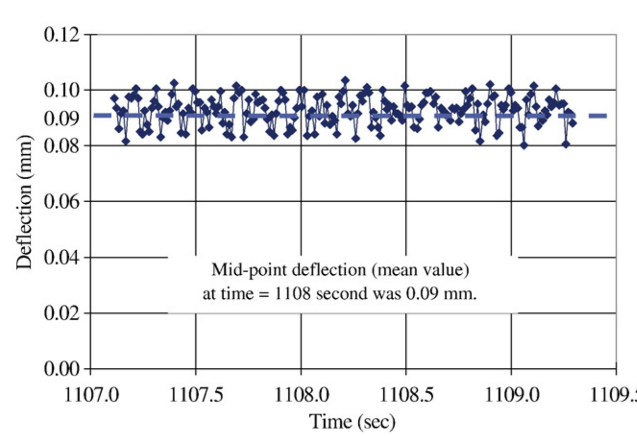

Fig. 7. Mid-point deflection (mean value) and the corresponding time record in second.

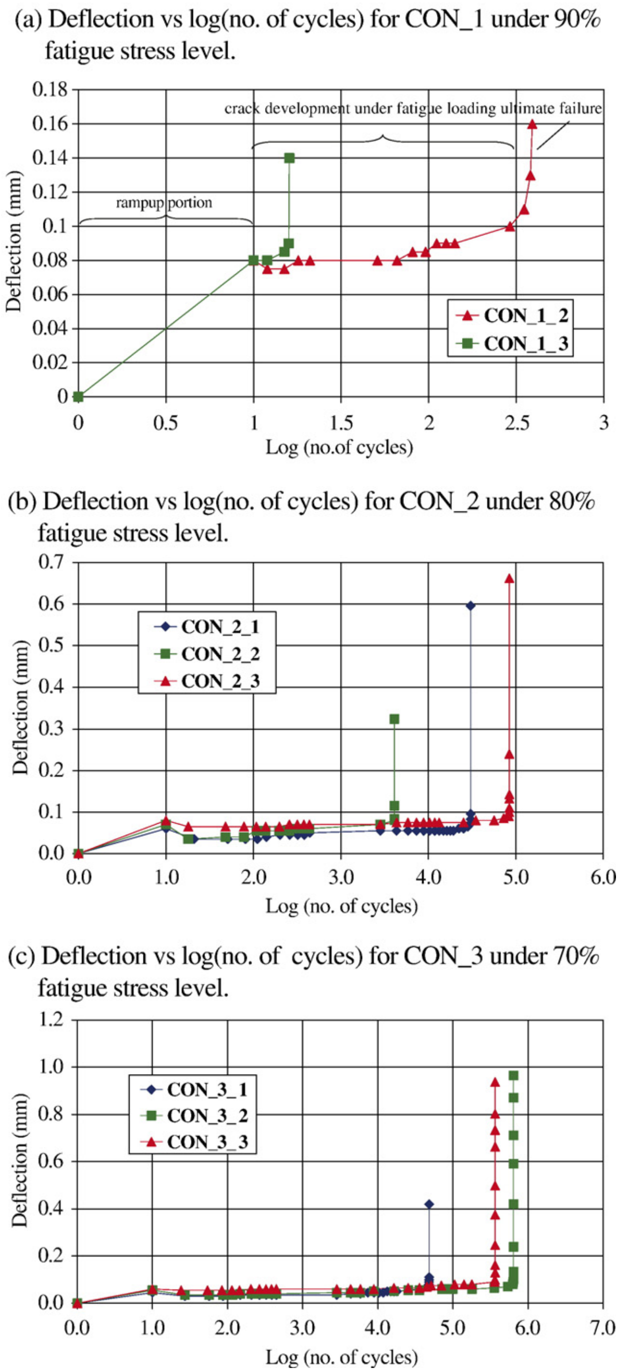

Fig. 8. Mid-point deflection versus fatigue cycles (in log) diagram for concrete beam under different fatigue stress level, (a) SL = 0.90 (b) SL = 0.80 (c) SL = 0.70.

orders of magnitude and a 50 mm ECC layer increased the fatigue lifetime by about 3 orders of magnitude. In addition, according to Figs. 5 and 6, three specimens with 25 mm ECC layer subjected to 70% stress level and one specimen with 50 mm ECC layer subjected to 83% stress level reached 2 million fatigue cycles without failure. On the other hand, none of the concrete specimens, even for the ones subjected to 70% stress level, could reach 2 million cycles without failure. Hence, the addition of an ECC layer on the lower portion of the beam is shown to be effective in decreasing the rate of crack propagation in the member and thus improved the overall fatigue performance of the beam.

Actually, while the ECC layer can improve the static strength (as shown in the previous section), the enhancement of fatigue performance is even more impressive. This finding is in support of our argument that the delay of crack propagation at its earlier stage (by the ECC layer at the lower part of the beam) is particularly helpful in extending the fatigue life. Figs. 8–10 show the mid-point deflection (mean value) against fatigue

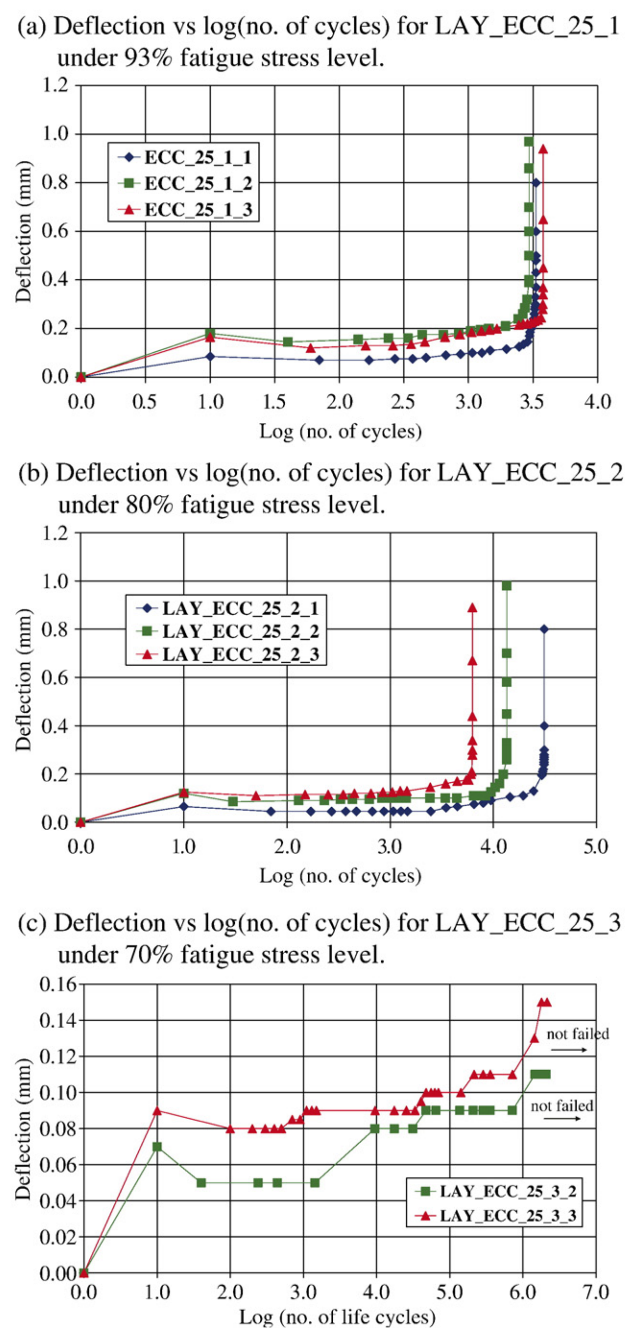

Fig. 9. Mid-point deflection versus number of cycles (in log) diagram for 25 mm ECC layered beams under different fatigue stress level, (a) SL = 0.90 (b) SL = 0.80 (c) SL = 0.70.

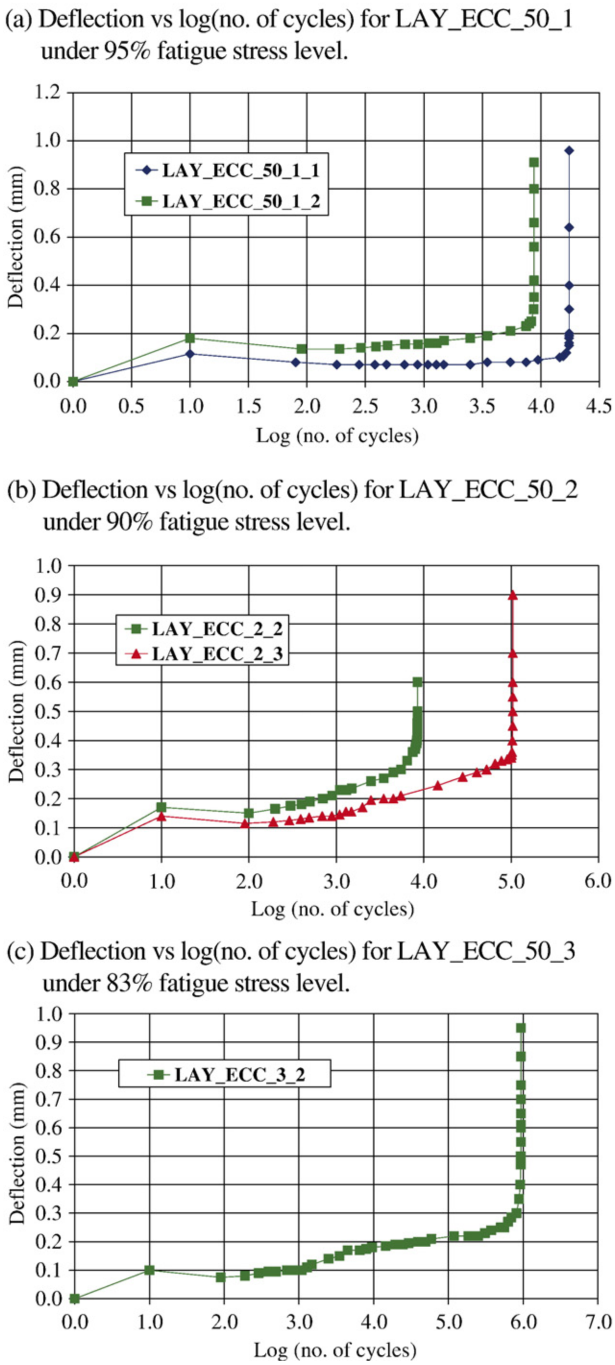

Fig. 10. Mid-point deflection versus number of cycles (in log) diagram for 50 mm ECC layered beams under different fatigue stress level, (a) SL = 0.95 (b) SL = 0.90 (c) SL = 0.83.

cycles. All the figures could be divided into three stages, the ramp-up stage, the crack development stage and the final failure stage. In the ramp-up stage, the specimen was loaded up to the maximum applied load, P_{max}. Since ultimate failure under static loading occurs after the crack has propagated, loading up to a certain percentage of the static strength can result in nonlinear damage of the member. The deterioration state of the loaded specimen was indicated by the specimen deflection at the end of the ramp-up stage. If the specimen attained a larger deflection than other specimens of the same type, more initial deterioration had occurred. The beam was then subjected to fatigue loading as shown in the second portion of the graph up to the final failure as shown in the last portion. All beams exhibited a similar trend of development, in which the second portion of fatigue development was stable with little increase in deflection. The final failures of all the beams were indicated by a rapid increase in the ultimate deflection. In addition, it is shown in the figures that layered ECC beams under the same relative stress level (SL) reached a greater deflection at the final fatigue stage as compared to plain concrete beams. In other words, layered ECC beams could sustain fatigue loading at a greater deflection without failure. This indicates that the fatigue ductility is also improved by applying the ECC layer.

A general concept for concrete under cyclic loading is that of an envelope curve which provides a bound for compressive stress and strain values that can be attained under more general loading conditions [5–8]. Most authors agree that this envelope either coincides with the monotonic loading curve, or is at least very close to this curve for plain concrete. This concept may be applicable for fatigue in compression, but is inapplicable for fatigue in flexure or tension [9]. Fig. 11 displayed the relationship between deflection at the end of the second stage of fatigue failure (see Figs. 8–10), which is the maximum deflection that can be sustained before rapid crack growth, and the stress level. In the figure, the deflection value at peak load in the static test is represented by the point with SL = 1. From the figure, one can observe that with the decrease of fatigue stress level, the deflection decreases slightly. Also, the deflection is closer to the value at peak load in the static test than the value on the descending branch of the static load-deflection curve that corresponds to the maximum load in the fatigue test (see Fig. 4). The above findings agree well with the theoretical predictions based on crack bridging degradation mechanism under fatigue load [9]. Under fatigue loading, crack-bridging stress degrades in the fracture zone [10,11]. This implies that the toughness of beam is gradually reduced with the number of cycles. Thus, it may be concluded that the deformation capacity, such as deflection, at fatigue failure will be less than the values

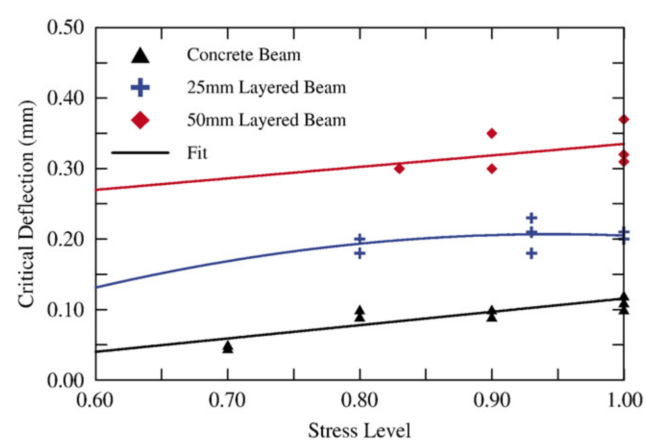

Fig. 11. Critical deflections at different stress level, showing test results of different kind of beam.

that can be attained under monotonic case. Again, from the figure we can conclude that the application of ECC layer can greatly enhance the ductility at fatigue failure. The degree of improvement is proportional to the thickness of the layer of ECC applied.

4. Conclusion

A thorough experimental investigation on the static and fatigue behaviors of ECC layered composite beam was performed through the conduction of four-point bending tests. From the static tests, it was found that the application of a layer of ECC on the tensile side of a flexural concrete beam increased its flexural strength and the degree of improvement increased with the thickness of ECC applied. From fatigue tests, it was found that the application of a layer of ECC was very effective in increasing the fatigue lifetime (e.g. at a stress level of 90%, the increase is around 2 orders of magnitude for a 100 mm beam with a 25 mm ECC layer and 3 orders of magnitude for a member with 50 mm ECC layer). The high ductility and multiple cracking capability of ECC are responsible for such an improvement. In addition, layered ECC beams could sustain fatigue loading at a larger deflection without failure. The ductility under cyclic loading is hence also improved. Based on our results, while the use of ECC layer can increase the static strength, its improvement on fatigue performance is even more impressive. The present investigation reveals the potential of the layered ECC/concrete beam as a cost-effective component for practical applications.

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