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A novel indirect tensile test method to measure the biaxial tensile strength of concretes and other quasibrittle materials

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ABSTRACT

A novel indirect tensile test method, the biaxial flexure test (BFT) method, has been developed to measure the biaxial tensile strength of concretes. The classical modulus of rupture (MOR) test has been generalized to three dimensions. In this method, we use a circular plate as the new test specimen. This plate is supported by an annular ring. We apply an external load to this specimen through a circular edge. The centers of the specimen, the loading device and the support are identical. The biaxial tensile strength measured by this new method is about 19% greater than the uniaxial tensile strength obtained from the classical modulus of rupture test as reported by other researchers. However, at the same time, we also found that the stochastic deviation of the biaxial tensile strength is about 63% greater than the uniaxial strength.

1. Introduction

One of the most well-known mechanical properties of concrete is that the tensile strength is 8 to 10 times less than the compressive strength. For instance, the tensile strength of ordinary concrete is about 2.5 MPa; the compressive strength ranges from 20 to 27 MPa [1]. Because of such a low tensile strength, we commonly see many cracks on the surface of concrete structures. Generally, the development of cracks does not lead to collapse of reinforced concrete structures because of the reinforcement embedded in the concrete. In spite of that, cracking, or tensile failure, is still one of the most important issues because it influences the serviceability significantly. For some structures with no reinforcement such as concrete pavement, the tensile failure or crack development is directly related to the safety of the structures [2].

Tensile strength is one of the most important parameters used to evaluate tensile failure of a concrete together with the fracture energy [3,4]. The strength is not a constant parameter but rather depends on the stress state.[2] However, for practical reasons, the uniaxial strength is chosen as a reference value in many applications. The uniaxial tensile strength of a concrete can be measured by several methods.

The direct tension test shown in Fig. 1a is conceptually the simplest one. The two ends of the specimen are pulled until the concrete fails. Once the strain softening occurs at a material point, the strain gets localized to the point. This localization does not necessarily mean that the load is at its maximum value. The strain is never uniform through the width of the specimen after the localization. It is very difficult to control the position of crack initiation. Despite some success with the use of a lazy-tong grip, it is difficult to avoid secondary stresses such as those induced by grips or by embedded studs [8].

Therefore, in practice, it is preferred to use indirect methods for tensile strength such as the splitting test (Fig. 1b) and the modulus of rupture test (Fig. 1c). The strength determined in the splitting test is believed to be close to the direct tensile strength of concrete but being 5 to 12% higher. Many researchers suggest, however, that in the case of mortar and lightweight aggregate concrete, the splitting test yields too low a result [9]. With a normal aggregate, the presence of large particles near the surface where the load is applied may influence its behavior [10].

Unlike the direct tension method, the third method has a nonzero strain gradient from the bottom to the top surface. Because of the strain gradient, the concrete on the bottom surface always fails first. The region between the two loads where the applied stress is constant takes into account the stochastic nature of strength. Therefore, a crack can initiate at any point on the bottom surface between the two loads; the crack will initiate at the weakest point.

While various indirect methods are available for the uniaxial tensile strength, the biaxial tensile strength is still measured by the direct method [11]. To perform biaxial tensile test, generally four

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[2] In some of recently developed numerical methods, the loss of the hyperbolicity criterion is adapted to initiate cracks instead of the tensile strength [5–7]. The projection of the stress tensor to the crack normal at the crack initiation is considered as the strength.

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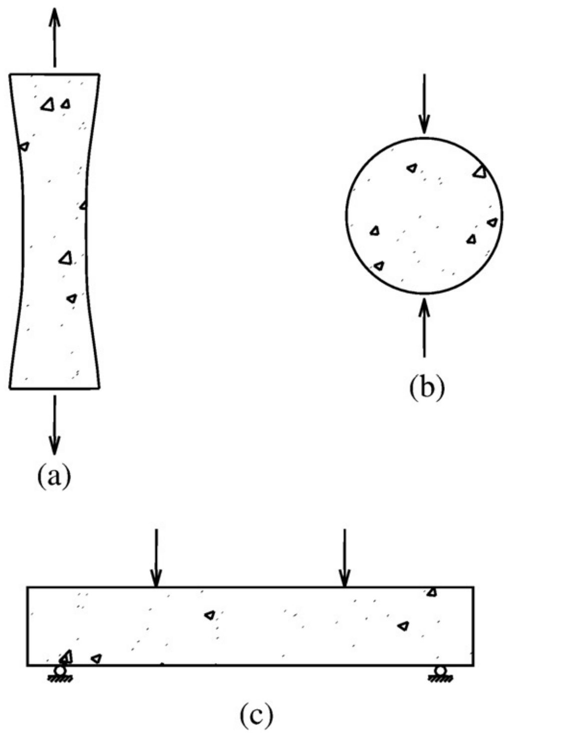

Fig. 1. Various test methods for the unidirectional tensile strength of concretes; (a) the direct tension test, (b) the splitting (or Brazilian) test and (c) the modulus of rupture test.

actuators are needed and also a big frame. In a biaxial fatigue test, the oil pressure of the actuators should be changed according to a prescribed period to keep the variation of individual loads proportional to each other. It is very difficult to control the oil pressure of all actuators at the same time. Because of such a problem, Muzyka [12] proposed a special device which controls the stresses in both loading directions identical. However it is still an expensive test.

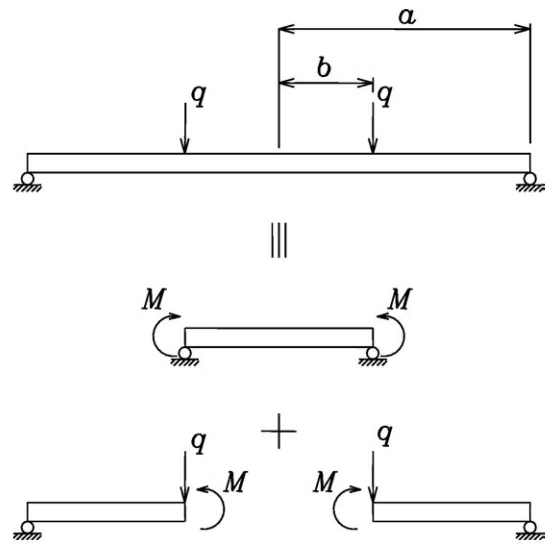

Fig. 3. The decomposition of an axisymmetrically loaded circular plate [13].

In our research described in this paper, we have proposed a new indirect method to measure the biaxial tensile strength of concrete and other quasibrittle materials. In Section 2, we describe the idea behind the new method. We show that our experimental data of the biaxial tensile strength obtained from the new test method is followed in Section 3. A possible application of this method to the biaxial fatigue is briefly mentioned in Section 4. After that, we draw conclusion in Section 5.

2. The biaxial flexure test (BFT) method

In the classical modulus of rupture method (Fig. 1c), the nominal stress caused by the load is uniform on the bottom surface between the two loads, which is important because of the stochastic nature of the strength. The concrete must fail at the weakest point between the

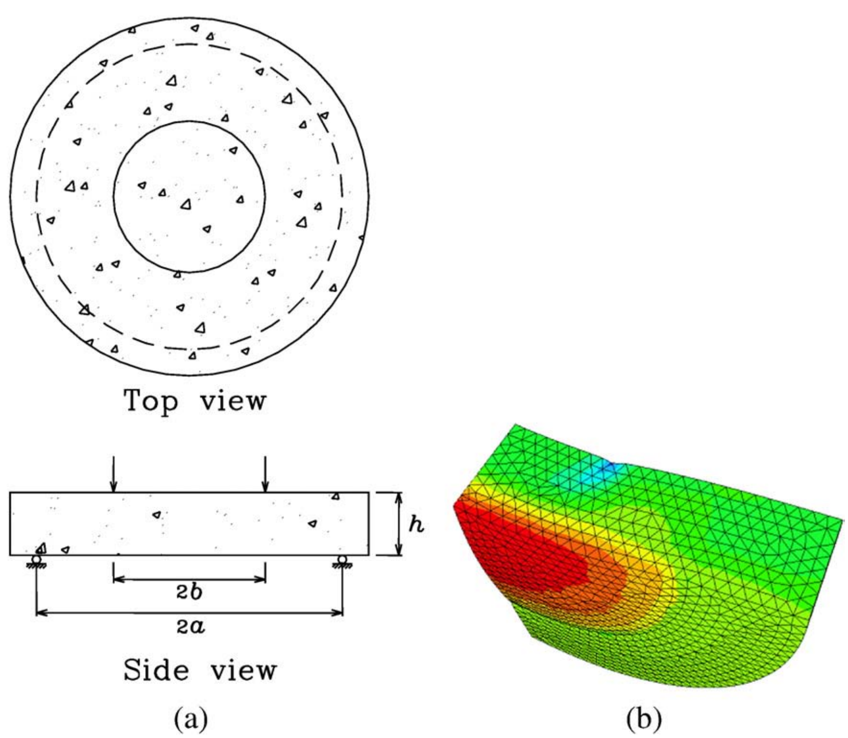

Fig. 2. (a) The specimen for the biaxial flexure (BFT) method and (b) the uniform principal stress on the bottom surface.

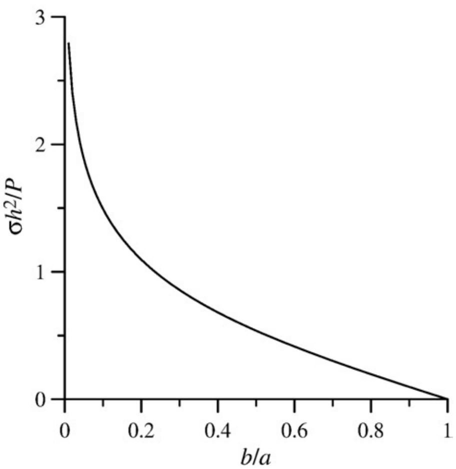

Fig. 4. The relation between the stress and the applied load for various plates.

two loads. This test method can be generalized to three dimensions for the new test method. Instead of a prismatic specimen, we use a circular plate. The plate is supported on the top of an annular support. The external loading is applied to the specimen through a circular edge. This new type of specimen is shown in Fig. 2a. Because of the axisymmetry of the specimen and the theory of elasticity, it is obvious that on the bottom surface of the concrete plate within the circle on which the load is applied, the stress is constant in any direction in the region. The uniform distribution of the principal stress calculated by a commercial finite element program is shown in Fig. 2b.

It is useful to have an analytical expression of the stress on the bottom surface caused by the applied load. If the specimen is simplified to a circular plate, the solution can easily be obtained from the governing differential equation of an axisymmetrically loaded circular plate. An axisymmetrically loaded circular plate can be decomposed to two systems as shown in Fig. 3 [13]. The first system is simply a supported circular plate which is loaded by a uniformly distributed moment M along its edge. The second system is simply a supported annular plate. The inner edge is loaded by the same moment M and uniformly distributed vertical load q. The integration of q is equal to the applied load.

The analytical solutions of the two systems can be found in the literature [13]. Using the two solutions and the displacement consistency condition that the vertical displacement w and the slope $\partial w/\partial r$ must be the same at the connection of the two plates, we can obtain the relation between the applied force P and the moment M.

Table 1
The dimensions of the specimens

Specimen type		Diameter or length [mm]	2a [mm]	2b [mm]	h [mm]	Thickness [mm]	Maximum aggregate size [mm]	Number of specimens
Biaxial test	B3	187.5	125	65	30	–	5	28
	B6	375.0	250	130	60	–	10	29
	B12	750.0	500	260	120	–	20	32
Uniaxial test	U3	187.5	125	65	30	60	5	28
	U6	375.0	250	130	60	60	10	28
	U12	750.0	500	260	120	60	20	27

$$M = \frac{1}{8\pi}\left\{(1-\nu)\left[1-(b/a)^2\right] - 2(1+\nu)\log(b/a)\right\}P \quad (1)$$

in which ν = Poisson's ratio, a, b = radii to the support and the load and P=2πbq = applied load, respectively. Since the moment is constant in any direction, the stress is constant, too. From the plate elasticity theory, the stress on the bottom surface is given by

$$\sigma = \frac{6M}{h^2} \quad (2)$$

where h = the thickness of the plate. Therefore the maximum stress caused by the applied load P is

$$\sigma = \frac{3}{4\pi h^2}\left\{(1-\nu)\left[1-(b/a)^2\right] - 2(1+\nu)\log(b/a)\right\}P. \quad (3)$$

The stress σ is a nonlinear function of the aspect ratio of b/a. The change of the dimensionless stress $\sigma h^2/P$ with respect to the dimensionless size of the specimen is shown in Fig. 4 where Poisson's ratio is assumed to be 0.18. The dimensionless stress changes rapidly if the inner radius b is less than 20% of the outer radius a. Note that in the classical modulus of rupture test, the relation is linear. To minimize the sensitivity of the test result to the aspect ratio, it is better to have the inner radius on which the load is applied greater than 30% of the outer radius on which the specimen is supported. Of course, the expression of Eq. (3) gets invalidated for too small b/a, because of the three-dimensional effect, where the solution based on the plate bending theory cannot be used.

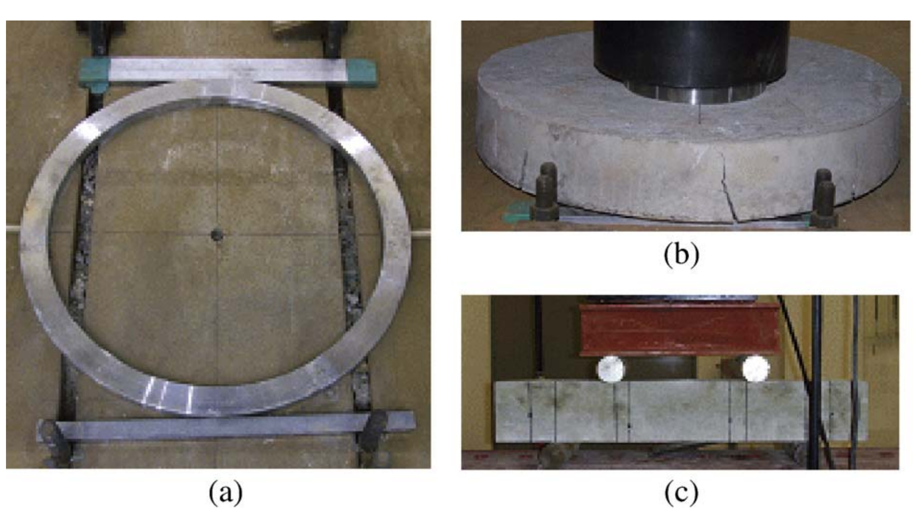

Fig. 5. (a) The annular steel support, (b) the biaxial flexure test and the initial crack development, and (c) the uniaxial modulus of rupture test for the reference test result.

3. Experiment

3.1. The materials and the experimental method

The specimens were prepared in two series, B and U, for the biaxial strength and the uniaxial strength. The circular specimen mentioned above was used for series B. Three different sizes of specimens were used. The diameters were 18.75, 37.5, and 75 cm, and the heights were 3, 6, and 12 cm, respectively. To investigate the difference of the biaxial strength to the uniaxial strength, one-dimensional specimens were prepared to be the same sizes as the circular specimens (Fig. 5c). The thickness of the classical MOR test specimens was fixed as 6 cm. The gage length $2a$ was 67% of the diameter or the length of the specimens. The distance b to the load was equal to $0.52a$. The detailed dimensions of the specimens are given in Table 1. Because of the stochastic nature of the strength, about 30 specimens were prepared for each type.

To minimize the effect of anisotropy caused by too big aggregates, the size of the maximum aggregates was limited to a sixth of the height of the specimens. The sizes were 5, 10 and 20 mm, respectively, which meant that three different concretes were used for the experiment. The mix proportion of the concretes is given in Table 2. Because of such a large number of specimens, the B and U series were cast from two different concrete batches for each size. Although the mix proportion was the same, the compressive strength could be different. Therefore, three standard cylindrical specimens were prepared for each type to measure the compressive strength.

The biaxial flexure test was performed on the circular specimens supported by an annular steel ring as shown in Fig. 5a and loaded at the center through a circular loading device placed on the top of the specimen as shown in Fig. 5b. Both the support and the loading device were carefully lathed to have the given geometry by skilled professionals. Although only the maximum load was of interest, the increment of the displacement was controlled instead of the load. The velocity of the loading plate was 1 mm/min. The increase of the load and the development of cracks were carefully monitored.

3.2. The results and discussion

Some of the load–displacement relations are shown in Fig. 6 for different sizes. In the case of the MOR test, the U series, the load increased monotonically with the increase of the displacement. As the load reached the maximum value, the specimen failed by a crack cutting the specimen completely (Fig. 6a). The initial behavior of the B series was similar to that of the U series. As the first crack formed, the load dropped suddenly as shown in Fig. 6b. The points where the first crack formed are marked by circles in the figure. The direction of the first crack was completely arbitrary. The first crack might form in the direction where the strength was a minimum. By a further increase of the displacement, the load increased again because the plate could still resist the load until the specimen became completely fractured. The final failure pattern of the B series specimens is illustrated in Fig. 7b.

The uniaxial and biaxial tensile strengths obtained from this experiment are given in Table 3 in ascending order. The histogram of the strength and the number of occurrence is given in Fig. 8. Thanks to the simplicity of the new experimental method, about 30 biaxial tests per each type could easily be performed.

Table 2
The mix proportion of the concretes

w/c	Slump	Air	S/a	Weight [kgf]				SP	AE
[%]	[mm]	[%]	[%]	w	c	S	G		
35	120	5	39.5	193.9	553.9	642.4	1036.3	0	0

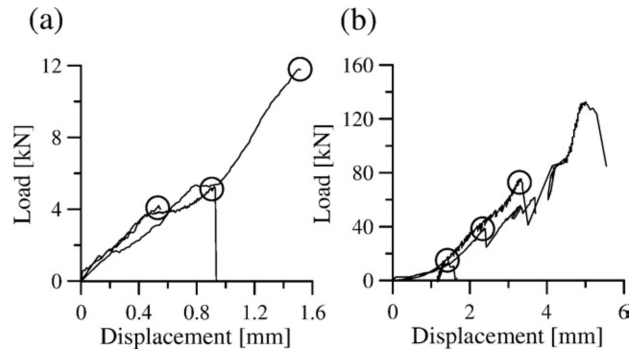

Fig. 6. The typical load–displacement relations of (a) the uniaxial modulus of rupture test and (b) the BFT test.

The tensile strength was calculated from the maximum load, using Eq. (3). It was found from the data in Table 3 that the average biaxial tensile strength was about 19% greater than the average uniaxial strength. A similar trend was also reported by others [14]. An interesting point was found: The deviation of the biaxial tensile strength was significantly higher than the uniaxial one for all cases. The standard deviation of the biaxial strength measured by this method was about 63% greater than the uniaxial strength. If the strengths were normalized by the square roots of the individual average compressive strengths, the trend was still same. It might be considered that the probability of a crack could be higher for a certain critical stress value

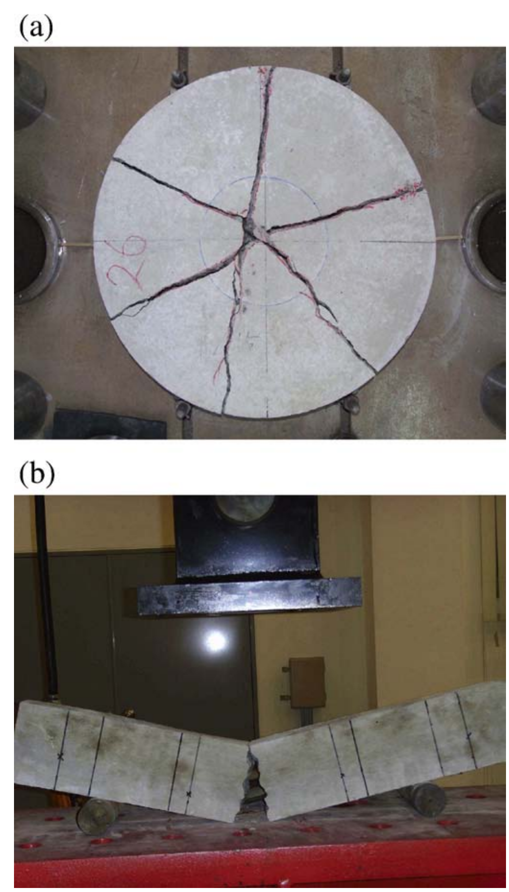

Fig. 7. The failure patterns of (a) the BFT specimen and (b) the classical modulus of rupture test specimen.

Table 3
The biaxial and uniaxial tensile strengths obtained from the BFT test and the MOR test

Number	Measured strength [MPa]						Strength normalized by $\sqrt{\bar{f}_{ck}}$ [%]					
	Biaxial test			Uniaxial test			Biaxial test			Uniaxial test		
	B3	B6	B12	U3	U6	U12	B3	B6	B12	U3	U6	U12
1	2.73	4.32	2.66	5.06	4.00	2.71	44.95	74.48	47.22	87.49	67.52	49.61
2	5.00	4.47	3.05	5.06	4.08	2.79	82.41	77.17	54.14	87.49	68.92	51.15
3	5.45	4.94	3.09	5.06	4.17	3.17	89.89	85.26	54.78	87.49	70.33	58.01
4	5.62	5.10	3.27	5.42	4.25	3.21	92.71	87.95	57.92	93.73	71.74	58.77
5	5.79	5.10	3.94	5.42	4.42	3.33	95.51	87.95	69.89	93.73	74.55	61.06
6	5.91	5.21	4.08	5.78	4.50	3.46	97.39	89.92	72.40	99.98	75.96	63.35
7	6.02	5.28	4.08	5.96	4.50	3.67	99.25	91.14	72.40	103.09	75.96	67.17
8	6.36	5.33	4.30	6.14	4.67	3.79	104.87	91.88	76.18	106.22	78.77	69.46
9	6.42	5.52	4.37	6.32	4.75	3.79	105.81	95.30	77.44	109.34	80.18	69.46
10	7.21	5.88	4.37	6.32	4.83	3.79	118.92	101.42	77.44	109.34	81.58	69.46
11	7.44	5.94	4.44	6.32	5.08	3.88	122.68	102.41	78.70	109.34	85.80	70.98
12	7.95	6.02	4.45	6.32	5.08	3.92	131.10	103.87	78.82	109.34	85.80	71.75
13	8.01	6.28	4.51	6.68	5.08	3.96	132.04	108.29	79.96	115.60	85.80	72.50
14	8.01	6.36	4.65	6.68	5.08	3.96	132.04	109.75	82.48	115.60	85.80	72.50
15	8.18	6.43	4.69	6.86	5.17	4.00	134.84	110.98	83.12	118.72	87.21	73.27
16	8.24	6.62	4.90	6.86	5.17	4.08	135.78	114.17	86.89	118.72	87.21	74.79
17	8.29	6.73	4.94	7.04	5.42	4.21	136.72	116.14	87.51	121.85	91.43	77.08
18	8.52	6.73	5.01	7.04	5.67	4.46	140.46	116.14	88.84	121.85	95.65	81.66
19	8.69	6.87	5.04	7.22	5.67	4.54	143.27	118.59	89.41	124.96	95.65	83.20
20	8.75	7.24	5.13	7.40	5.67	4.67	144.21	124.95	90.92	128.10	95.65	85.49
21	8.86	7.60	5.18	7.58	5.67	4.67	146.09	131.08	91.93	131.21	95.65	85.49
22	8.98	7.61	5.22	7.58	5.67	4.96	147.95	131.32	92.55	131.21	95.65	90.82
23	9.03	7.84	5.30	7.76	5.75	4.96	148.89	135.24	93.93	134.34	97.05	90.82
24	9.60	7.87	5.30	7.94	5.92	5.00	158.25	135.74	94.06	137.46	99.87	91.59
25	10.28	7.94	5.40	7.94	6.00	5.04	169.49	136.96	95.71	137.46	101.27	92.36
26	10.57	8.29	5.53	7.94	6.00	5.21	174.18	143.09	98.03	137.46	101.27	95.40
27	10.62	8.61	5.57	8.31	6.58	5.42	175.11	148.47	98.84	143.72	111.11	99.23
28	11.30	8.99	5.67	10.11	7.25	–	186.34	155.09	100.55	174.95	122.37	–
29	–	9.73	6.09	–	–	–	–	167.82	107.91	–	–	–
30	–	–	6.25	–	–	–	–	–	110.81	–	–	–
31	–	–	6.43	–	–	–	–	–	113.95	–	–	–
32	–	–	7.56	–	–	–	–	–	134.10	–	–	–
$\bar{f}_r$	7.78	6.58	4.83	6.79	5.22	4.10	1.28	1.14	0.86	1.17	0.88	0.75
STD	1.96	1.40	1.03	1.16	0.78	0.73	0.32	0.24	0.18	0.20	0.13	0.13
COV	0.25	0.21	0.21	0.17	0.15	0.18	0.25	0.21	0.21	0.17	0.15	0.18
$\bar{f}_{ck}$	36.80	33.60	31.80	33.40	35.10	29.80	–	–	–	–	–	–

for the biaxial loading condition due to such a high stochastic deviation in spite of the higher strength in the biaxial condition. However, it is not clear at this moment if the scatter was due to the biaxial loading condition or there were other sources influencing the experimental data obtained from the BFT method.

4. A comment on a biaxial fatigue test using this method

If a concrete is subjected to a continuously changing load, the fatigue damage accumulates gradually in the concrete. Although the maximum magnitude of the load is less than the critical load, the concrete fails eventually by the accumulated fatigue damage after a certain number of loading cycles. It is still a difficult problem of completely understanding such a fatigue behavior. Because of its complicated nature, experimental approaches are often taken to study the fatigue response of a concrete in practice.

Regarding a biaxial fatigue test, it is very important to keep the same amount of the stress variation in the both directions. Otherwise it would not the biaxial loading condition. This new test method is especially useful for the biaxial fatigue test. Because only one actuator is used for this method, the control of the stress variation cannot be a problem. If multiple actuators have to be used, it is a very difficult task to control the variation of the oil pressure in the individual actuators.

One may ask how much the biaxial fatigue life is different from the uniaxial one. It must be one of the interesting future research topics. At this point, however, we can obtain qualitative information from the failure probability of a material containing a penny-shaped crack with arbitrary orientations. Depending on the loading conditions, the failure probability is given by [15]

$$P_f = \begin{cases} 1 - \sqrt{\sigma_{cr}/\sigma} \text{ for uniaxial loading} \\ \sqrt{1 - \sigma_{cr}/\sigma} \text{ for biaxial loading} \end{cases} \quad (4)$$

in which P_f = failure probability, σ_{cr} = critical stress at which the crack grows and σ = magnitude of the applied stress. The failure probability is obviously influenced by the loading condition. As can be seen in Eq. (4), the failure probability (or the damage[3]) increases significantly faster in the biaxial loading condition than the uniaxial one. When σ_{cr}/σ=0.9, the biaxial failure probability is 6 times greater than the uniaxial one. This means that the biaxial fatigue life would be much less than the uniaxial fatigue life.

5. Conclusions

(1) We have developed a new test method called the biaxial flexure test (BFT). This new method is a three-dimensional generalization of the classical modulus of rupture method. Using this method, we determined the tensile strength of a concrete subjected to isotropic biaxial loading condition.

[3] In some damage models for rocks, the damage is directly expressed by the failure probability of a material point having many penny-shaped cracks with different sizes and different orientations; see [16].

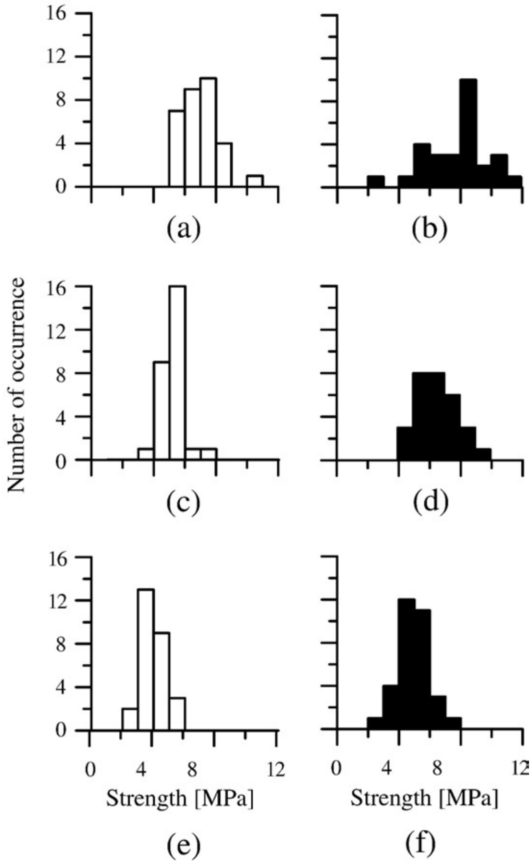

Fig. 8. The histogram of the uniaxial and biaxial tensile strengths for the experiment (a, b) U3, B3, (c,d) U6, B6 and (e,f) U12, B12 in which the white is for the uniaxial tension and the black the biaxial tension.

(2) A circular plate specimen is used in the BFT method. The load is applied to the specimen through a circular edge, and the specimen is supported on an annular support. The stress on the bottom surface of the specimen in the region enclosed by the loading edge is constant in any direction.

(3) The relation between the applied load and the maximum stress can be obtained in an explicit way using the plate bending theory.

(4) According to the experimental result shown in this paper, the biaxial tensile strength of concrete was higher than the uniaxial one as reported by others. The stochastic deviation of the biaxial strength obtained from the BFT method was higher than the uniaxial strength from the MOR method.

References

[1] J.M. Raphael, Tensile strength, Concrete International 81 (2) (1984) 158–165.
[2] J. Sim, H. Oh, J.-M. Yu, J.-W. Shim, Theoretical assessment of the limit strengthening criterion of strengthened bridge decks based on failure characteristics, Cement and Concrete Research 35 (5) (2005) 999–1007.
[3] G. Zi, Z.P. Bažant, Eigenvalue method for computing size effect of cohesive cracks with residual stress, with application to kink bands in composites, International Journal of Engineering Science 41 (13–14) (2003) 1519–1534.
[4] Z.P. Bažant, Q. Yu, G. Zi, Choice of standard fracture test for concrete and its statistical evaluation, International Journal of Fracture 118 (4) (2002) 303–337.
[5] T. Belytschko, H. Chen, J. Xu, G. Zi, Dynamic crack propagation based on loss of hyperbolicity with a new discontinuous enrichment, International Journal for Numerical Methods in Engineering 58 (12) (2003) 1873–1905.
[6] G. Zi, T. Rabczuk, W. Wall, Extended meshfree methods without the branch enrichment for cohesive cracks, Computational Mechanics 40 (2) (2007) 367–382.
[7] T. Rabczuk, S. Bordas, G. Zi, A three-dimensional meshfree method for continuous multiple-crack initiation, propagation and junction in statics and dynamics, Computational Mechanics 40 (3) (2007), doi:10.1007/s00466-006-0122-1 in online.
[8] D.P. OĹaery, J.E. Byrne, Testing concrete and mortar in tension, Engineering (1960) 384–385.
[9] A.M. Neville, Properties of Concrete, Longman, England, 1995.
[10] D.J. Hannant, K.J. Buckley, J. Croft, The effect of aggregate size on the use of the cylinder splitting test as a measure of tensile strength, Materials and structures 6 (31) (1973) 487–494.
[11] S.-K. Lee, Y.-C. Song, S.-H. Han, Biaxial behavior of plain concrete of nuclear containment building, Nuclear Engineering and Design 227 (2004) 143–153.
[12] N.R. Muzyka, Equipment for testing sheet structural materials under biaxial loading. Part 2. Testing by biaxial loading in the plane of the sheet, Strength of Materials 34 (2) (2002) 206–212.
[13] S.P. Timoshenko, S. Woinowsky-Krieger, Theory of plates and shells, Engineering Mechanics Series, 2nd Edition, McGraw-Hill Book Company, Tokyo, 1989.
[14] P.K. Mehta, P.J.M. Monteiro, Concrete: Microstructure, Properties and Materials, 3rd EditionMcGraw Hill, 2006.
[15] S.B. Batdorf, J.G. Crose, A statistical theory for the fracture of brittle structures subjected to nonuniform polyaxial stresses, Journal of Applied Mechanics, ASME 41 (2) (1974) 459–464.
[16] L. Liu, P.D. Katsabanis, Development of a continuum damage model for blasting analysis, International Journal of Rock Mechanics and Mining Sciences 34 (2) (1997) 217–231.