



## Short communication

## Quasi-static compressive strength of cement-based materials

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## ABSTRACT

Wet cementitious materials show a noticeable dependence on the rate of quasi-static loading. While dry cementitious materials are almost independent of loading rate in the quasi-static region, the mechanical strength of wet materials increases with increasing rate of loading. Therefore, the Abrams' formula for the static mechanical strength cannot provide reliable values with wet materials at higher rates and should be corrected. Some possibilities for its improvement have been discussed.

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## 1. Introduction

It has long been recognized that the behavior of cement-based materials under dynamic loading differs from their behavior under static loading. Results of loading tests have confirmed an increase in compressive strength of concrete subjected to dynamic loading. This general result has been confirmed by many researchers in the course of many decades. For example, Abrams [1] even in 1917 reported that an increase in the rate of loading was accompanied by an increase in the compressive strength of concrete. The same finding was announced by Jones and Richard [2] in 1936 or Glanville [3] in 1938. Similar results were published by Watstein [4] in 1953 and Atchley and Furr [5] in 1967 or Hughes and Gregory [6] in 1972. A comprehensive overview of more recent works of this topic can be found in several surveys and books [7–10].

A further general conclusion which resulted from experiments concerned the different behavior of wet and dry materials. The wet in contrast to dry materials show a higher increase in compressive strength. This was confirmed even as early as in 1966 by Cowell [11] or by Kaplan [12] in 1980 who pointed out that the moisture content of concrete is one of the main factors influencing the relationship between strength and loading rate. More recently Rossi and Toutlemonde showed that it is the viscous effects of pore water that are responsible for the strength increase in the quasi-static region ( $10^{-4}\text{s}^{-1}$ ,  $10^0\text{s}^{-1}$ ). On the other hand, only weak or no relationship between the strength increase and the type of aggregate, water-to-cement ratio or age at testing has been found.

The purpose of this paper is to present some analytical consequences which follow from the results published about the quasi-static compressive strength of cement-based materials.

## 2. Factors influencing dynamic strength of cementitious materials

There is a wide scatter in the magnitudes of strength increase with increasing rate of loading. These discrepancies exist mainly because the behavior of concrete is dependent on many variables such as the static strength, curing conditions, size and shape of samples and type of loading. As soon as the rate of loading is further increased, other factors such as viscous effects of pore water, inertia forces, stress-wave propagation or plain stress effects become important.

Fig. 1 schematically shows some dynamic regions and their typical factors influencing the dynamic compressive strength of concrete. In the quasi-static region, which is of our primary concern, the porous water plays a decisive role as to the dynamic behavior of concrete. This fact was repeatedly demonstrated by Rossi, Toutlemonde and others [13–17] within the series of tensile experiments. Rossi and Toutlemonde expressed a hypothesis [16] concerning the mechanism of viscous resistance which pore water develops against the tensile disruption. The mechanism is based on the physical phenomenon called the Stéfán effect and may be described as a viscous adhesion of pore water to the pore walls. Similar although not identical viscous effects were observed by Scherer [18] in flexure experiments when studied the time-dependent relaxation process during three-point bending of cement-based samples. Scherer's viscous effects may be described as “hindered viscous flow” of pore water that tends to act against loading. The faster the loading, the larger hindering is established. During the static loading ( $\sim 10^{-5}\text{s}^{-1}$ ) the hindered

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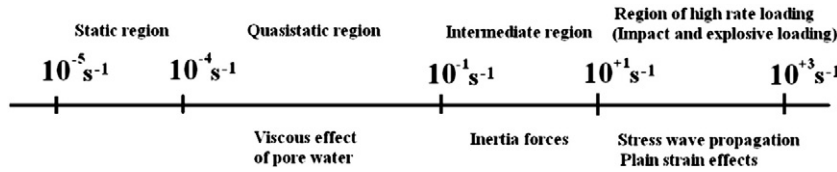


Fig. 1. Distribution of dynamic regions.

resistance is negligible small but at quasi-static loadings ( $10^{-4}\text{s}^{-1}$ ,  $10^0\text{s}^{-1}$ ) the viscous effects become preponderant [16]. It is very likely that the same mechanism of hindered viscous flow of pore water occurs within purely compressive tests in the quasi-static region. Nevertheless, at still higher dynamics ( $\sim 10\text{s}^{-1}$ ) other influences, e.g. inertial forces, become more important, as was mentioned by Rossi and Toutlemonde [16].

Fig. 2 illustrates the dynamic behavior of concrete within compressive tests. As it is seen, the dynamic strength  $\sigma_d$  increases bi-linearly (two values of slopes  $\alpha$ ) in the log–log plot. This behavior may be described by the following function

$$\frac{\sigma_d}{\sigma_s} = \left( \frac{\dot{\sigma}_d}{\dot{\sigma}_s} \right)^\alpha \quad (1)$$

where  $\sigma_s$  is static strength,  $\dot{\sigma}_d$  and  $\dot{\sigma}_s$  are dynamic and static loading rates, respectively. Relation (9) was recommended by CEB [19]. The discontinuity appears at  $30\text{s}^{-1}$ . The point at  $10^{-5}\text{s}^{-1}$  represents a static reference. Recently Malvar and Ross published a curve for tensile strength possessing the discontinuity at  $1\text{s}^{-1}$  and the static reference at  $10^{-6}\text{s}^{-1}$ . The bi-logarithmic plots describing the dynamic behavior of compressive strength and tensile strength are not the only plots used in practice. The semi-logarithmic plots are used [17] as well especially for the expression of strength changes in the *quasi-static region*

$$\sigma_d = \sigma_s + \beta \cdot \log \left( \frac{\dot{\sigma}_d}{\dot{\sigma}_s} \right) = \sigma_s + \Delta\sigma. \quad (2)$$

### 3. Dynamic strength and water-to-cement ratio

Valuable results concerning the relationship between quasi-static tensile strength and the water-to-cement ratio  $r$  have been published by Rossi, Toutlemonde and others [13–17]. Fig. 3 shows some of their

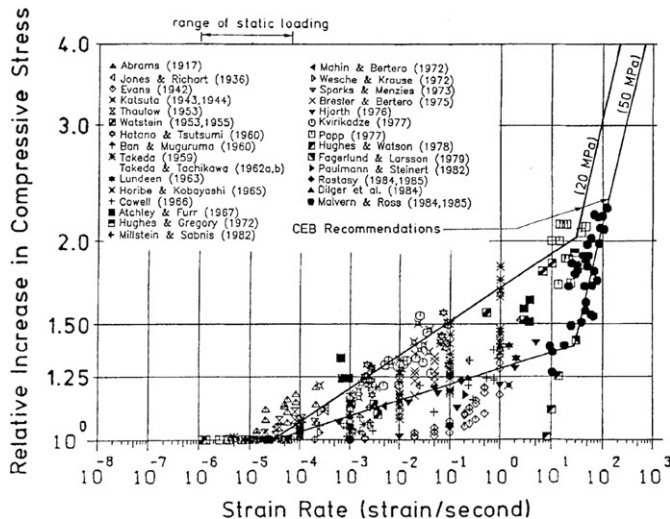


Fig. 2. Behavior of the dynamic compressive strength of concrete — reproduced from [8].

results. At first sight it is obvious that the tensile strength increment  $\Delta\sigma$  is not dependent on the water-to-cement ratio  $r$  since all the curves for various  $r$  are in parallel positions (they have the same slopes). However,  $\Delta\sigma$  does depend on loading rates  $\dot{\sigma}_d$  and  $\dot{\sigma}_s$ , i.e.  $\Delta\sigma(\dot{\sigma}_d, \dot{\sigma}_s)$ , since the slopes of the curves subsequently increase with increasing loading rates. On the other hand, the absolute positions of the curves are dependent on  $r$ , i.e.  $\sigma_d(r)$ ,  $\sigma_s(r)$ , so that the curves assume the following analytical form

$$\sigma_d(r, \dot{\sigma}_d, \dot{\sigma}_s) = \sigma_s(r) + \Delta\sigma(\dot{\sigma}_d, \dot{\sigma}_s). \quad (3)$$

As has been mentioned in Section 2, the strength increment  $\Delta\sigma$  in the quasi-static region is caused by the viscous effects of pore water and similar effects may be expected for compressive tests in the same region. It means that a relation analogous to Eq. (3) may be assumed for the dynamic compressive strength as well. In such a case the static compressive strength may be represented by Abrams' relation [20]

$$\sigma_s(r) = \sigma_o \exp \left( -\frac{r}{r_o} \right) \quad (4)$$

where  $\sigma_o$ ,  $r_o$  are material constants. The analytical form of  $\Delta\sigma(\dot{\sigma}_d, \dot{\sigma}_s)$  in the quasi-static region can be inferred from Fig. 4 presenting experimental results of dynamic compressive tests of microconcrete and miniconcrete. From this figure it is clear that  $\Delta\sigma(\dot{\sigma}_d, \dot{\sigma}_s)$  may be well approximated by the logarithmic function as in the case of tensile strength increment (see Eq. (2))

$$\Delta\sigma \approx \beta \cdot \log \left( \frac{\dot{\sigma}_d}{\dot{\sigma}_s} \right). \quad (5)$$

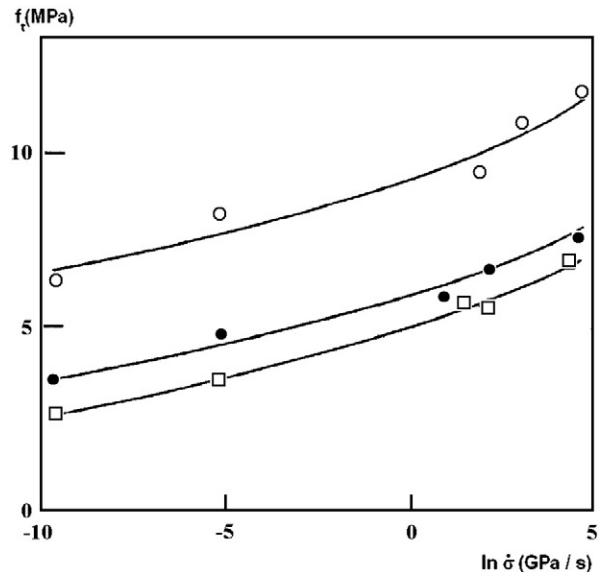


Fig. 3. Dependence of tensile strength on the water-to-cement ratio — reproduced from [15].

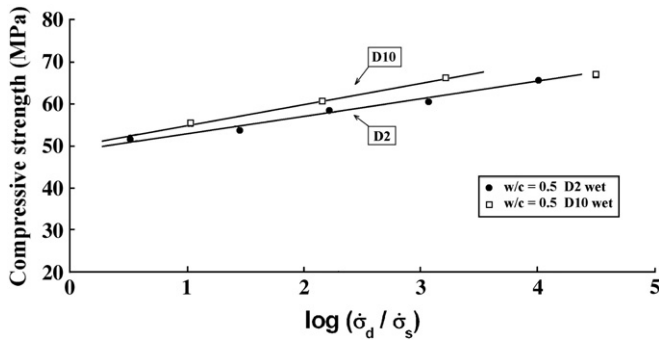


Fig. 4. The compressive strength of concrete in the quasi-static region – data taken from [17].

The foregoing considerations enable to rewrite Eq. (3) into the following form

$$\sigma_d(r, \dot{\sigma}_d, \dot{\sigma}_s) = \sigma_o \exp\left(-\frac{r}{r_o}\right) + \beta \cdot \log\left(\frac{\dot{\sigma}_d}{\dot{\sigma}_s}\right). \quad (6)$$

The estimation of  $\beta$  from Fig. 4 provides  $\beta_{D2} \approx 4.4$  MPa for microconcrete and  $\beta_{D10} \approx 5.8$  MPa for miniconcrete. The value of  $\beta$  seems to decrease with decreasing size of aggregates and thus  $\beta_o$  for hydrated cement paste will be likely still smaller than that for microconcrete  $\beta_{D2} > \beta_o \approx 4$  MPa.

#### 4. Experimental verification

In the preceding section the dynamic compressive strength of cement-based materials has been derived in the form which is convenient as a fitting function

$$\sigma_d = \sigma_o \exp\left(-\frac{r}{r_o}\right) + \Delta\sigma \quad (7)$$

where  $\sigma_o$ ,  $r_o$  and  $\Delta\sigma$  are fitting parameters. The first term on the right-hand side of expression (7) actually is the original Abrams' formula (4) that is valid for static loading and, therefore, the parameters  $\sigma_o$  and  $r_o$  should be fitted to the data measured under static conditions ( $\sim 10^{-5} \text{ s}^{-1}$ ). As soon as these static values of  $\sigma_o$  and  $r_o$  are known, they can be inserted in expression (7) as fixed parameters and then this expression with the only free parameter  $\Delta\sigma$  can be fitted to the dynamic data in order to obtain a shift  $\Delta\sigma$  between the static  $\sigma_s(r)$  and dynamic  $\sigma_d(r)$  curves. For this reason two series of experiments were performed. In the first series the wet samples of hydrated Portland cement pastes were loaded statically ( $\dot{\sigma}_s = 5 \cdot 10^{-5} \text{ s}^{-1}$ ) and then in the second series other samples of the same kind were loaded quasi-dynamically ( $\dot{\sigma}_d = 2.5 \cdot 10^{-4} \text{ s}^{-1}$ ). Each of the seven values of the water-to-cement ratios was represented by 9 samples, i.e. 63 samples underwent static tests and the same number quasi-dynamical tests, which means that 126 samples were the subject of the present study.

Fig. 5A shows the results of the static tests that enabled determining the static parameters  $\sigma_o \approx 314$  MPa and  $r_o \approx 0.25$ . As a fitting procedure the Levenberg–Marquardt procedure was used. The values of the static parameters  $\sigma_o$  and  $r_o$  were employed as fixed parameters within the fitting procedure imposed on the quasi-dynamic data (Fig. 5B) in order to determine the shift  $\Delta\sigma \approx 2.74$  MPa. This value is in a good agreement with the shift 2.46 MPa determined as an average difference between the static and quasi-dynamic experimental data. In addition, the shift can be calculated directly from Eq. (5) using the input values  $\beta_o \approx 4$  MPa,  $\dot{\sigma}_s = 5 \cdot 10^{-5} \text{ s}^{-1}$  and  $\dot{\sigma}_d = 2.5 \cdot 10^{-4} \text{ s}^{-1}$  which gives the value  $\Delta\sigma \approx 2.8$  MPa again in a satisfactory agreement with the fitted shift 2.75 MPa.

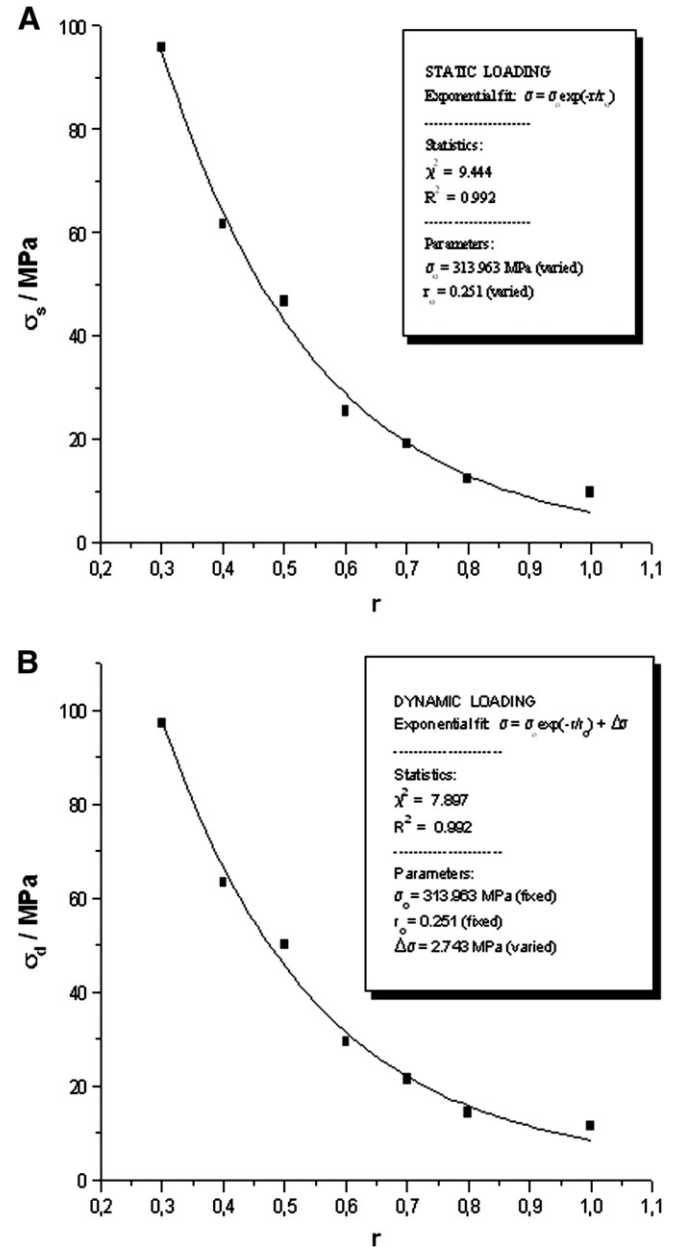


Fig. 5. The compressive strength of hydrated Portland cement paste with different water-to-cement ratios loaded statically (A) and quasi-statically (B).

Visual inspection of the curves in Fig. 5A and B could evoke impressions that those curves are identical. This is because their shapes are very similar and the shift  $\Delta\sigma$  is quite small in comparison with the range used on the vertical axis. Although the mentioned shift can be hardly recognizable by naked eyes, the existing differences can be safely revealed numerically.

A question may arise how the numerical uncertainty of the fitting procedure influences the accuracy of the parameter  $\Delta\sigma$ . If the fitting procedure with the fixed  $\sigma_o$  and  $r_o$  and free  $\Delta\sigma$  is imposed on the static data, the value  $\Delta\sigma \approx 0.29$  MPa is determined, which represents only about 10% of the found strength increase 2.74 MPa. Analogous fitting procedure with quasi-dynamic data provides a similar value 0.30 MPa, which also illustrates that the numerical uncertainty of the used fitting procedure does not play a crucial role.

There is another question, namely, whether a fit of Eq. (7) with all free parameters to the quasi-dynamic data would be capable to reproduce a realistic value of  $\Delta\sigma$ . Performing this procedure to our quasi-dynamic data, the following values were found:  $\sigma_o \approx 321$  MPa,

$r_0 \approx 0.24$  and  $\Delta\sigma = 4.8$  MPa. Such a value of  $\Delta\sigma$  is rather overestimated but, however, the essential part (more than 50%) of this shift is represented by the contribution associated with the dynamic loading.

## 5. Conclusion

The foregoing discussion and the performed experiments have showed that the performance of Abrams' formula (4) can be improved in the quasi-static region by including an additive term  $\Delta\sigma$  which is only dependent on the used loading rate but independent of the water-to-cement ratio  $r$ . In the static region the term  $\Delta\sigma$  is close to zero but with increasing loading rate its value becomes more relevant and represents an important non-negligible correction to original Abrams' formula (4).

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