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Short communication

Probabilistic evaluation of initiation time in RC bridge beams with load-induced cracks exposed to de-icing salts

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ABSTRACT

In this study, a reliability-based method for predicting the initiation time of reinforced concrete bridge beams with load-induced cracks exposed to de-icing salts is presented. A practical model for predicting the diffusion coefficient of chloride ingress into load-induced cracked concrete is proposed. Probabilistic information about uncertainties related to the surface chloride content and the threshold chloride concentration has been estimated from a wide review of previous experimental or statistical studies. Probabilistic analysis to estimate the time to corrosion initiation with/without considering the effect of the load-induced cracks on the chloride ingress into concrete has been carried out. Results of the analysis demonstrate the importance of considering the effect of the load-induced cracks for correct prediction of corrosion initiation in RC bridge beams exposed to chlorides.

1. Introduction

Chloride-induced corrosion of reinforcing steel in concrete is known to be one of the major causes of deterioration of reinforced concrete (RC) structures. Normally concrete protects embedded reinforcing steel against corrosion due to a thin iron oxide (Fe_2O_3) layer that forms on the steel surface and remains stable in the high alkaline environment provided by the concrete porous solution. However, if a RC structure is exposed to chlorides as exampled by highway bridges exposed to de-icing salts, chloride ions can penetrate the concrete cover and arrive at the surface of reinforcing bar. When the concentration of chloride ions in concrete at the surface of reinforcing bars reaches a critical level, known as threshold, the passive layer that protects the reinforcing bars from corrosion is broken down and hence the corrosion initiates. After initiation, corrosion may propagate relatively fast, which will result in the longitudinal cracking and delamination or spalling of the concrete cover and finally leading to structural failure. Usually, the time taken for the surface chloride to penetrate through the concrete cover and build up the threshold concentration at the reinforcing bars level to cause corrosion is termed the time to corrosion initiation (initiation time). Therefore, the initiation time represents an important parameter in controlling deterioration of RC structures and the estimation of this time period is important for the prediction of the service life of RC structures exposed to chlorides.

The initiation time depends on various factors including the surface chloride content, the rate of penetration of chloride ions in concrete, the depth of the concrete cover, as well as the threshold chloride concentration. Numerous models for ‘perfect’ and uncracked concrete developed from Fick's second law of diffusion [1] have been used to predict the initiation of chloride-induced reinforcement corrosion in concrete in the past decades [e.g., 2–10]. The influence of the rate of penetration of chloride ions in concrete due to the presence of cracks and its effect on the time to corrosion initiation have been also investigated in some recent researches [11–13]. However, all the previous models are developed within a deterministic framework. Because there is significant uncertainty associated with the factors related to the chloride ingress into concrete, it might be more appropriate to use a probabilistic approach to predict the initiation time.

In order to account for various sources of uncertainty and considering the problem of chloride ingress and corrosion initiation in RC structures in probabilistic terms, many probabilistic approaches and techniques are recently proposed for the evaluation of initiation time [14–19]. However, these applications using probabilistic approaches have been generally limited to the sound concrete without considering crack effect. Since the occurrence of visible macro-cracks is inevitable for RC structures under service loads, the influence of cracks on the time to corrosion initiation should be taken into account.

The main objective of this study is to present a reliability-based method for predicting the initiation time of RC bridge beams with load-induced cracks exposed to de-icing salts. A practical model for predicting the diffusion coefficient of chloride ingress into load-induced cracked concrete is proposed. Probabilistic information about

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uncertainties related to the surface chloride content and the threshold chloride concentration has been estimated from a wide review of previous experimental or statistical studies. A probabilistic analysis is then performed to estimate the probability of corrosion initiation in a highway bridge beam. Uncertainties in concrete strength, the depth of concrete cover, service loading, surface chloride content, and the threshold chloride concentration are taken into account. Results of the analysis demonstrate that the probability of corrosion initiation for the cases of considering the effect of cracks is higher than the corresponding cases of without considering the effect of cracks. The result clearly demonstrates that prediction of the time to corrosion initiation without considering the effect of load-induced cracks on the chloride ingress into concrete leads to the overestimation of the time to corrosion initiation and the underestimation of the probability of corrosion initiation.

2. Modelling chloride penetration

2.1. Modelling chloride penetration for uncracked concrete

Provided that concrete is in a saturated state, chloride ions enter the concrete by ionic diffusion due to the existing concentration gradient between the exposed surface and the pore solution of the cement matrix (diffusion driving force) [20]. This process is often described by Fick's first law of diffusion, according to which, the rate of transfer of chloride ions J_c through a plane perpendicular to the direction of diffusion is proportional to the concentration gradient $\partial C_f/\partial x$ [1], i.e.,

$$J_c = -D_e \frac{\partial C_f}{\partial x}, \tag{1}$$

where J_c is the flux of chloride ions due to diffusion (kg/m^{2}s); D_e is the effective diffusion coefficient (m^2/s); and C_f is the free chloride concentration (kg/m^3) at depth x (m). The negative sign in Eq. (1) indicates that diffusion occurs in the opposite direction to that of increasing chloride concentration.

It is assumed that the diffusion coefficient does not depend on the concentration. The mass balance leads to an expression for Fick's second law

$$\frac{\partial C_t}{\partial t} = -\frac{\partial J_c}{\partial x} = D_e \frac{\partial^2 C_f}{\partial x^2}, \tag{2}$$

in which C_t is the total chloride concentration (kg/m^3) in concrete at time t (s).

The total concentration of chloride ions is given by Eq. (3).

$$C_t = C_f + C_b. \tag{3}$$

where C_b is the concentration of bound chlorides (kg/m^3).

The derivative of Eq. (3) with respect to t is denoted as

$$\frac{\partial C_t}{\partial t} = \frac{\partial C_b}{\partial t} + \frac{\partial C_f}{\partial t} = \left(1 + \frac{\partial C_b}{\partial C_f}\right)\frac{\partial C_f}{\partial t}, \tag{4}$$

where $\partial C_b/\partial C_f$ is the “binding capacity” of the concrete binder.

Based on the results of several long-term tests, Mohammed and Hamada [21] proposed linear relationships between free and total chloride concentrations for various types of cements. Under this circumstance, the “binding capacity” of the concrete binder is a constant and here is denoting as

$$\frac{\partial C_b}{\partial C_f} = k. \tag{5}$$

According to Eqs. (2), (4), and (5), one obtains

$$\frac{\partial C_f}{\partial t} = \frac{D_e}{1 + k}\frac{\partial^2 C_f}{\partial x^2} = D_a^f \frac{\partial^2 C_f}{\partial x^2}, \tag{6}$$

where D^f_a is the apparent diffusion coefficient (m^2/s) for free chloride.

Mangat and Molloy [22] pointed out that the apparent diffusion coefficient of free (water-soluble) chloride concentration D^f_a in the pore fluid shows a strong relationship of equality with the diffusion coefficient of total (acid-soluble) chloride concentration D^t_a (m^2/s) in concrete, i.e., $D^f_a = D^t_a$. Hereafter, the apparent diffusion coefficient is denoted as D_a (m^2/s). Then, Eq. (6) can be rewritten as

$$\frac{\partial C_f}{\partial t} = D_a \frac{\partial^2 C_f}{\partial x^2}. \tag{7}$$

The apparent diffusion coefficient D_a is considered depending on a large number of factors. In general, these factors may be classified into two major categories: internal factors (e.g., cement composition, supplementary cementing materials (SCM), water-to-binder ratio (w/b), the degree of hydration and aggregate size and grading) and external factors (e.g., relative humidity, temperature and stress states). Several models considering the influence of these factors have been developed for predicting the apparent diffusion coefficient, as shown in Table 1. In general, these existing empirical models can be divided into two categories according to whether the external factors are incorporated in their expressions or not, i.e., both the internal and external factors are incorporated in the first category; while in the second category, only the internal factors are considered. Although it is more reasonable to describe the chloride diffusion coefficient using models of the first category, there are few data (both laboratory and field) available at the referred conditions. Therefore, the chloride diffusion coefficients of the different concretes were estimated based on their water-to-binder ratios and the types of SCM selected for this study. The empirical expressions for chloride diffusion coefficient of ordinary Portland cement with/or without SCM developed by Maeda et al. [27], which were obtained from regression analysis of 415 specimens and 139 specimens, respectively, are used here:

$$D_a = 10^{-6.77(w/b)^2 + 10.10(w/b) - 14.64} \quad (\mathrm{m}^2/\mathrm{s}) \tag{8a}$$

(for concrete with no SCM),

$$D_a = 10^{-0.79(w/b)^2 + 3.40(w/b) - 13.10} \quad (\mathrm{m}^2/\mathrm{s}) \tag{8b}$$

(for concrete containing SCM).

The water-to-binder ratio is estimated from Bolomey's formula; namely,

$$w/b = \frac{27}{f_{cyl,150} + 13.5}, \tag{9}$$

where $f_{cyl,150}$ is the concrete compressive strength of a standard test cylinder in MPa.

For the initial condition $C_f(x>0, t=0) = 0$ and boundary condition $C_f\ (x=0,\ t>0) = \text{constant}$, a standard solution for Eq. (7) can be obtained as:

$$C_f(x, t) = C_s^f\left[1 - erf\left(\frac{x}{2\sqrt{D_a \cdot t}}\right)\right], \tag{10}$$

where $C_f(x,t)$ is the free chloride concentration (kg/m^3) at depth x at time t; C_s^f is the surface content (kg/m^3) of free chloride; and erf is the Gaussian error function.

Table 1
Some available analytical models for predicting the apparent chloride diffusion coefficient of concrete.

	Researchers	Equations
First category (both the internal and external factors are incorporated)	Saetta et al. [23]	$D_a = D_{a,ref} \cdot f_1(t) \cdot f_2(T) \cdot f_3(h)$ (m^2/s) in which $D_{a,ref}$ is evaluated at referred conditions of cement hydration degree, temperature, and relative humidity; $f_1(t)$ expresses the decrease of chloride diffusivity with increasing degree of hydration; $f_2(T)$ considers the dependence of D_a on temperature; $f_3(h)$ relates the diffusion coefficient D_a to the concrete pore relative humidity h.
	Tang and Nilsson [24]	$D_a = D_{a,ref} \cdot f_1(t) \cdot f_2(T) \cdot f_3(x)$ (m^2/s) in which $f_3(x)$ denotes the dependence of D_a on the concrete depth x.
	Thomas and Bentz [8]	$D_a = D_{a,ref} \cdot f_1(t) \cdot f_2(T)$ (m^2/s)
Second category (only the internal factors are considered)	Takewaka and Matsumoto [4]	$D_a = D_{w/b} D_c$ (cm^2/s) in which $D_{w/b} = 10^{-6.274 - 0.076(w/b) + 0.00113(w/b)^2}$ and $D_c = 1.2$, 1.0, 0.3, and 0.08 for high early strength Portland cement, ordinary Portland cement, blast furnace slag cement, and alumina cement, respectively.
	Papadakis et al. [25]	$D_a = 0.15 \frac{1 + \rho_c(b/w)}{1 + \rho_c(w/b) + \frac{\rho_c}{\rho_a}(a/b)} \left[\frac{\rho_c(w/b) - 0.85}{1 + \rho_c(w/b)} \right]^3 D_{H_2O}$ (cm^2/s) in which a/b is the aggregate-to-cement ratio, ρ_c and ρ_a are the mass densities of cement and aggregates respectively and D_{H2O} is the chloride diffusion coefficient in an infinite solution ($= 1.6 \times 10^{-5}$ cm^2/s for NaCl)
	Stewart and Rosowsky [26]	$D_a = 10^{-10 + 4.66(w/b)}$ (cm^2/s)
	Boulfiza et al. [12]	For concrete with no SCM or concrete with PFA $D_a = 10^{-3.9(w/b)^2 + 7.2(w/b) - 14.0}$ (m^2/s) For concrete containing GBFS or SF $D_a = 10^{-3.0(w/b)^2 + 5.4(w/b) - 13.7}$ (m^2/s)

SCM stands for supplementary cementing materials; PFA stands for pulverized-fuel ash (fly ash); GBFS stands for granulated blast-furnace slag; and SF stands for silica fume.

2.2. Modelling chloride penetration for cracked concrete

Considering a RC rectangular beam with flexural cracks induced by service load, as depicted in Fig. 1, in which w_k is the crack width (mm) induced by service load and $s_{r,max}$ is the maximum crack spacing (mm) [28]. For such a case, the diffusion mechanisms of chloride ingress into the cracked concrete can be modelled as follows.

Based on the partition of chloride diffusion through cracked concrete, as shown in Fig. 2, the diffusion coefficient of chloride penetration for cracked concrete, D_{cc} (m^2/s), is expressed as [11,13]

$$D_{cc} = \frac{AD_a + A_{cr} D_{cr}}{A + A_{cr}}, \tag{11}$$

where A is the area (mm^2) of uncracked concrete; D_{cr} is the value of the chloride diffusion coefficient (m^2/s) inside the crack; and A_{cr} is the area (mm^2) of the crack.

For the RC rectangular beam, Eq. (11) may be rewritten as

$$D_{cc} = \frac{b \cdot \left(s_{r,\,max} - w_k \right) D_a + b \cdot w_k D_{cr}}{b \cdot s_{r,\,max}} = D_a + \frac{w_k}{s_{r,\,max}} (D_{cr} - D_a). \tag{12}$$

Eq. (12) is the proposed apparent diffusion coefficient for load-induced cracked concrete, where D_a is given by Eqs. (8a) and (8b) and D_{cr} is obtained using the regression analysis of experimental data provided by Djerbi et al. [13], as shown in Fig. 3. The empirical expressions are given as follows:

$$D_{cr} = \begin{cases} (0.16 w_k - 3) \times 10^{-10} & 30\mu m \le w_k \le 100\mu m \\ 13 \times 10^{-10} & w_k > 100\mu m \end{cases} \left(\mathrm{m}^2/\mathrm{s} \right). \tag{13}$$

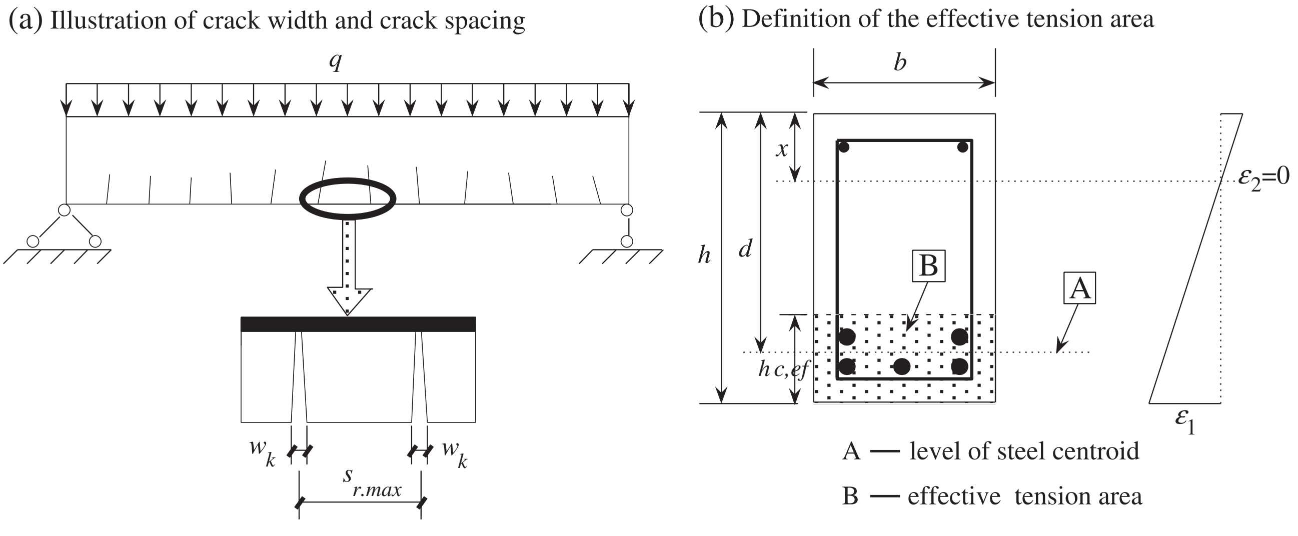

(a) Illustration of crack width and crack spacing
(b) Definition of the effective tension area

Fig. 1. Flexural cracking pattern in RC beam and definition of the effective tension area.

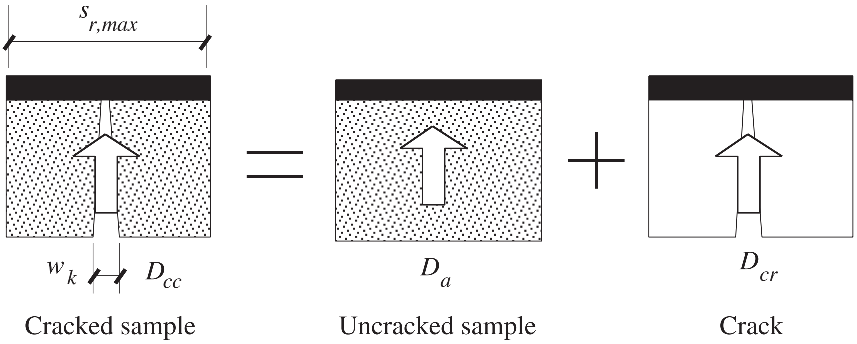


Fig. 2. Partition hypothesis of chloride diffusion through cracked concrete.

According to Eurocode 2 [28], $s_{r,max}$ and w_k are expressed by Eqs. (14) and (15), respectively.

$$s_{r,\,max} = 3.4c + 0.425k_1k_2\frac{\phi}{\rho_{p,eff}}, \tag{14}$$

$$w_k = s_{r,\,max}\frac{\sigma_s - k_t\dfrac{f_{ctm}}{\rho_{p,eff}}\left(1 + \alpha_e\rho_{p,eff}\right)}{E_s}, \tag{15}$$

in which c = cover to the reinforcement (mm); k_1 = coefficient which takes account of the bond properties of the bonded reinforcement ($k_1 = 0.8$ for high bond bars and $k_1 = 1.6$ for bars with an effectively plain surface); k_2 = coefficient which takes account of the distribution of strain ($k_2 = 0.5$ for bending and $k_2 = 1.0$ for pure tension); ϕ = bar diameter (mm); $\rho_{p,eff} = A_s/A_{c,eff}$; $A_{c,eff}$ = effective tension area (mm^2), i.e., $A_{c,eff}$ is the area of concrete surrounding the tension reinforcement of depth, $h_{c,ef}$ (mm), where $h_{c,ef}$ is the lesser of $2.5(h-d)$, $(h-x)/3$ or $h/2$ (see Fig. 1(b)); σ_s = the stress (MPa) in the tension reinforcement assuming a cracked section; k_t = the factor dependent on the duration of the load ($k_t = 0.6$ for short term loading and $k_t = 0.4$ for long term loading); α_E = the ratio E_s/E_{cm}; E_s = the elasticity of the steel modulus = 2.1×10^5 (MPa); E_{cm} = the secant modulus of elasticity (MPa); and f_{ctm} = mean value of axial tensile strength of concrete (MPa). The details of f_{ctm} and E_{cm} changed with the concrete strength can be found in Ref. [28].

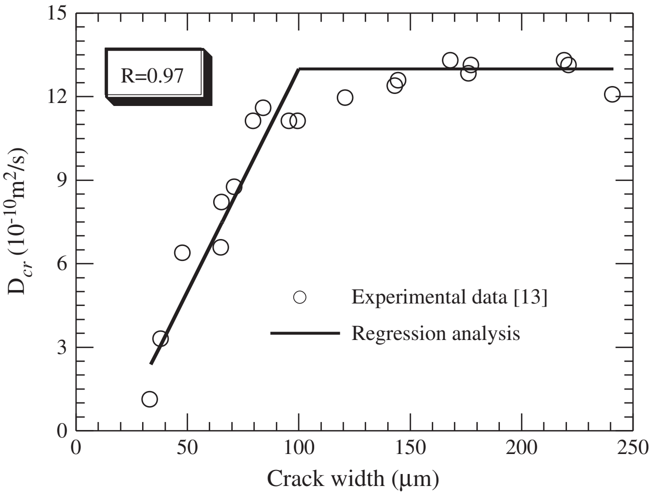


Fig. 3. Effect of crack width on diffusion coefficient inside the crack.

Replacing D_a by D_{cc} in Eq. (10), the free chloride content at depth x at time t, $C_f(x,t)$, for cracked concrete can be obtained as

$$C_f(x\ ,\ t) = C_s^f\left[1 - erf\left(\frac{x}{2\sqrt{D_{cc}\cdot t}}\right)\right]. \tag{16}$$

2.3. Surface content of free chloride

A large number of experimental data for surface chloride concentration due to the application of de-icing salts for individual bridges or groups of bridges have been published in the literature and statistical distributions for the surface content have also been suggested. Table 2 shows a summarization [18] of surface concentration of total (acid-soluble) chloride (kg/m^3) for structures exposed to de-icing salts.

In order to obtain the corresponding distributions of free (water-soluble) chloride concentration, the following relationship between water- and acid-soluble chloride concentrations proposed by Mohammed and Hamada [21] is used, which is developed from the experimental data of ordinary Portland concrete in 15- to 50-year-old wharves in Japan.

$$C_t = 1.23C_f. \tag{17}$$

Since the relation is linear, the corresponding statistical distributions of water-soluble chloride concentrations can be easily determined.

The data described by Hoffman and Weyers [30] comprising samples taken from 321 concrete bridge decks in the USA appears to be the most comprehensive to date and its mean of 3.5 kg/m^3 is in broad agreement with the other data [15]. Thus, based on the experimental data provided by Hoffman and Weyers [30] and Eq. (17), the surface concentration of free (water-soluble) chloride, C_s^f, for concrete structures exposed to de-icing salts is modelled by a lognormal distribution with the mean and coefficient of variation of 2.85 kg/m^3 and 0.5, respectively.

2.4. Threshold chloride concentration

A review of the summarizations of chloride threshold levels published in the literature [e.g., 31,32] shows that the chloride threshold value necessary to initiate corrosion of steel bars in concrete is mostly provided

Table 2
Surface concentration of total chloride (kg/m^3) for structures exposed to de-icing salts.

Researchers	Range	Mean	Coefficient of variation	Distribution
Wallbank [29]	0.25–15	6.5	0.7	Lognormal
Hoffman and Weyers [30]	1.2–8.2	3.5	0.5	Lognormal
Enright and Frangopol [14]	2.45–9.8	5.63	0.1	Lognormal
Kirkpatrick et al. [16]	0.15–5.25	2.51	0.68	Gamma

in terms of total chlorides or acid-soluble Cl^- (% by weight of cement/binder). Nevertheless, since it has been documented by many researchers, that it is the free chlorides present in the pore solution at the steel–concrete interface (i.e., chlorides not physically or chemically bound by the cement hydrates) that affect the passive oxide layer that protects the steel reinforcement from any corrosion activity [e. g., 33–35,43], the chloride threshold value in terms of free chlorides or water-soluble Cl^- (% by weight of cement) will be used in this study.

Some threshold chloride concentrations to depassivate the reinforcing steel as reported in the literature [32,36–44] are listed in Table 3. As can be observed from Table 3, there is little agreement among the values obtained. One reason for the lack of accordance is due to the chloride threshold depending on several factors such as, cement type, supplementary cementing materials, C_3A content of cement, concrete mix proportions, *w*/*b* ratio, nature of reinforcing steel, proportion of water to soluble chlorides, age of concrete, carbonation, curing conditions, temperature, relative humidity and source of chloride penetration among others. Other reasons such as the definition of the threshold value itself [32] and detection methods [45] also contributed to the scatter of the reported values.

Field experience and research [46] have shown that on existing structures subjected to chloride ions, a threshold concentration of about 0.026% (by weight of concrete) is sufficient to break down the passive film and subject the reinforcing steel to corrosion. This approximately equates to 0.17% by weight of cement.

Because of the wide scatter of the observed values, and also as a consequence of the lack of consistent statistical data, a uniform distribution for the threshold chloride concentration seems relevant [18]. In this study, the statistical parameters of the threshold chloride concentration in terms of free chlorides (% by weight of cement), $C_{cr,f}$, are suggested as: uniform distribution with the range of [0.10–0.25].

3. Time to corrosion initiation: probabilistic analysis

In this study, it is assumed that reinforcement corrosion starts when the concentration of free chloride ions in concrete at the surface of reinforcing bars reaches the threshold chloride concentration. As described earlier, since there is significant uncertainty associated with the threshold chloride concentration and the concentration of free chloride ions at the surface of reinforcing bars, it may be more appropriate to use a probabilistic approach to predict the time to corrosion initiation.

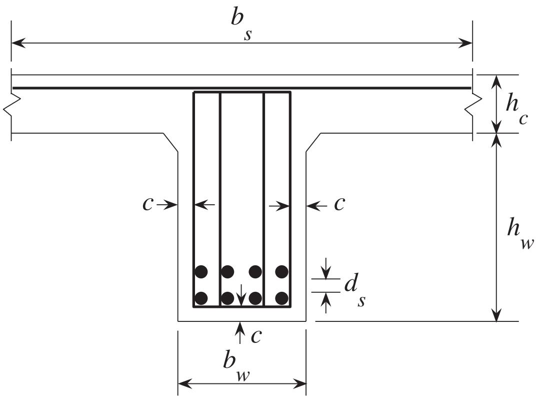


Fig. 4. Example of RC bridge beam.

In view of a probabilistic approach, the concentration of free chloride ions at the time *t* at the surface of reinforcing bars, $C_f(s,t)$ and threshold chloride concentration in terms of free chlorides, $C_{cr,f}$, can be expressed as exterior deterioration load effect (*S*) and interior resistance (*R*), respectively. Therefore, the performance function for corrosion initiation can be given by

$$G(t) = C_{cr,f} - C_f(s,t). \tag{18}$$

Therefore, the probability of corrosion initiation at time *t*, $P_i(t)$, and the time to corrosion initiation, T_i, can be formulated as follows.

$$P_i(t) = P[G(t) \le 0] = P\left[C_{cr,f} \le C_f(s,t)\right], \tag{19}$$

Table 3
Some threshold chloride concentration as reported in the literature.

Researchers	Conditions	Water-soluble Cl^- (% by weight of cement)
West and Hime [36]	Structures in a temperate climate	0.25
de Rincon et al. [37]	OPC, FA, SF (exposure to air)	0.4
Alonso et al. [38]	OPC,FA($w/b = 0.5$)	1.0–4.0
Kayyali and Haque [39]	Cl^- added as admixture	
	Medium strength concrete (M)	1.15
	High strength concrete (H)	0.85
	High strength concrete + supplement (HS)	0.80
	High strength concrete + supplement + fly ash (HSF)	0.45
	External chloride ingress M, H, HS, HSF	0.2
Hussain et al. [40]	Cement with different C_3A content ($w/b = 0.6$)	
	C_3A content = 2.43%	0.14
	C_3A content = 7.59%	0.17
	C_3A content = 14.00%	0.22
Hussain et al. [41]	Cement with different C_3A content ($w/b = 0.55$)	0.22–0.29
Alonso et al. [32]	Cl^- added in the mixing water OPC ($w/b = 0.5$)	0.39–1.16
Oh et al. [42]	Cl^- added as admixture	
	OPC ($w/b = 0.55$)	0.12
	OPC ($w/b = 0.45$)	0.11
	OPC ($w/b = 0.35$)	0.12
	OPC + 15% PFA ($w/b = 0.45$)	0.11
	OPC + 30% PFA ($w/b = 0.45$)	0.07
	OPC + 30%GBFS ($w/b = 0.45$)	0.13
ACI [43]	Prestressed concrete	0.06
	Reinforced concrete exposed to chloride in service	0.15
	Reinforced concrete that will be dry or protected rom moisture in service	1.00
	All other reinforced construction	0.30
Mohammed and Hamada [44]	OPC exposure to air ($w/b = 0.5$)	0.4–0.8

Table 4
Design details.

Flexural reinforcements (2 layers)		Stirrups	Spacing of the stirrups (mm)	Steel yield strength (MPa)	Concrete grade	Spacing between steel layers d_s (mm)	Cover c (mm)
Bottom	Upper						
4Ø32	4Ø32	4Ø10	150	500	C40	32	70

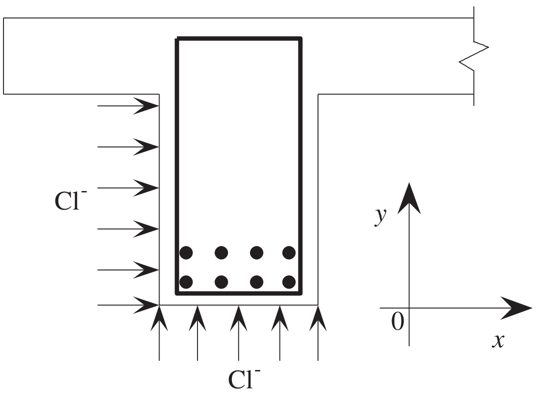


Fig. 5. Damage scenario for chloride ion penetration into bridge beam.

and

$$T_i = [P_i(t) \geq P_{i\,max}]. \tag{20}$$

where $P_{i\max}$ is the intended maximum probability of corrosion initiation.

In this study, the computation of the probability is based on the use of the Monte Carlo Simulation (MCS) technique [47], which involves sampling at random to simulate artificially a large number of experiments and to observe the result.

4. Illustrative example

Consider a RC bridge beam as shown in Fig. 4. The bridge consists of three 9.0 m simply supported spans, and of five beams equally spaced 2.6 m (b_s) apart. The design dimensions are $h_c = 200$ mm, $h_w = 600$ mm and $b_w = 400$ mm. The design details of flexural reinforcement, stirrups, concrete grade and concrete cover are listed in Table 4.

The damage scenario, i.e., runoff, for chloride ion penetration into bridge beam shown in Fig. 5 as has been described by Enright and Frangopol [14] is considered here. The statistical parameters for corrosion variables considered in this example are shown in Table 5, in which the cement content of 400 kg/m^3 is assumed. Here, the five basic random variables are assumed as mutually independent since there is no direct relationship among them.

Table 5
Statistical parameters for random variables.

Parameter	Mean or range	Coefficient of variation	Distribution
Surface content of free chloride, C_s^f (kg/m^3)	2.85	0.5	Lognormal
Threshold chloride concentration, $C_{cr,f}$ (kg/m^3)	[0.4–1.0]	0.247	Uniform
Live load (kN/m^2)	8	0.5	Gumbel (Sarveswaran and Roberts [48])
Concrete strength (MPa)	48	0.13	Normal (Bartlett and MacGregor [49])
Concrete cover (mm)	70	0.14	Normal (Weyers et al. [50])

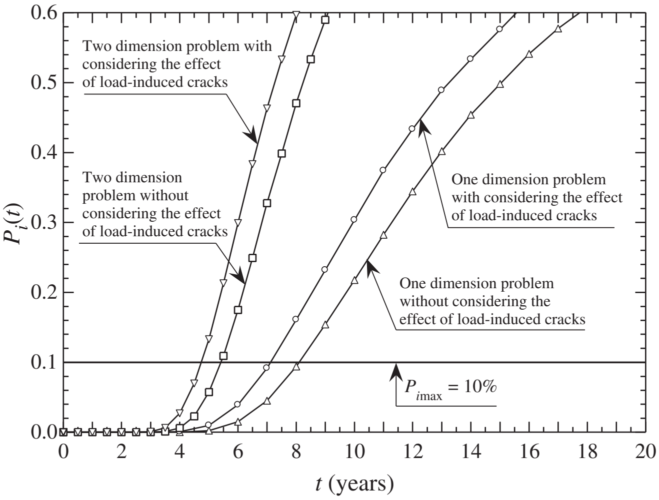
Four cases for predicting $C_f(s, t)$ of longitudinal bars (stirrups) are considered as follows.

Case 1/Case 2 One dimension (1-D) problem for the longitudinal bars $y = c$ (for the stirrups $y = c - 10$) without/with considering the effect of cracks.

Case 3/Case 4 Two dimension (2-D) problem for the longitudinal bars $x = y = c$ (for the stirrups $x = y = c - 10$) without/with considering the effect of cracks.

Results of the probabilistic analysis, i.e., the cumulative probability of corrosion initiation vs. the time of exposure for Cases 1–4, are presented in Fig. 6. The probability of corrosion initiation for the cases of considering the effect of the load-induced cracks is higher than the corresponding cases of without considering the effect of the load-

(a) Stirrups



(b) Longitudinal bars

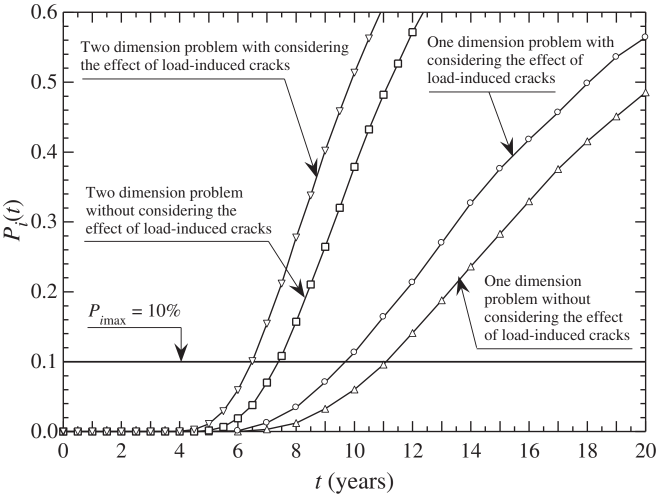


Fig. 6. Probability of corrosion initiation of steel reinforcement in the bridge beam.

Table 6
Time to corrosion initiation of the steel reinforcement in the beam.

$P_{i\max}=10\%$ (CEB-FIP [51])	T_i (years)		Decrease of T_i when considering the effect of load-induced cracks	T_i (years)		Decrease of T_i when considering the effect of load-induced cracks
	One dimension problem without considering the effect of load-induced cracks	One dimension problem with considering the effect of load-induced cracks		Two dimension problem without considering the effect of load-induced cracks	Two dimension problem with considering the effect of load-induced cracks	
Stirrups	8.12	7.14	12.07%	5.42	4.75	12.36%
Longitudinal bars	11.04	9.7	12.14%	7.38	6.48	12.20%

induced cracks. If the target probability of failure $P_{i\max}=10\%$ [51], the time to corrosion initiation can be determined and is shown in Table 6. As can be observed from Table 6, there is about 12% decrease of the time when considering the effect of the load-induced cracks for the illustrative example. That is, the result clearly demonstrates that prediction of the time to corrosion initiation without considering the effect of the load-induced cracks on the chloride ingress into concrete leads to the overestimation of the time to corrosion initiation and the underestimation of the probability of corrosion initiation.

5. Conclusions

From the investigation of the present study, the following conclusions can be made:

(1) A practical model for predicting the diffusion coefficient of chloride ingress into load-induced cracked concrete is proposed.
(2) Probabilistic analysis to estimate the time to corrosion initiation with/without considering the effect of the load-induced cracks on the chloride ingress into concrete has been carried out. Results of the analysis demonstrate the importance of considering the effect of the load-induced cracks for correct prediction of corrosion initiation in RC bridge beams exposed to chlorides.

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