Ceramics International **21** (1995) 407–411

Modelling of Moisture Flow in a Cast Sample During Drying

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(Received 5 May 1994; accepted 7 January 1995)

Abstract: The diffusion–convection equation of mathematical physics has been used under simplified conditions to develop a model that simulates moisture flow within a cast sample of finite length which undergoes shrinkage under convective influence of the suction mold. The model reveals that under transient conditions, the moisture concentration within the cast oscillates with the increase in the convective velocity and once transient conditions subside, the rate of moisture removal tends to retard due to compaction as the convective velocity is increased.

1 INTRODUCTION

Mathematical models on ceramic slips and their solidification are numerous in literature. Most of them deal with the growth studies of slip into filter cakes[1–3] while some compute the moisture content also within slip, cast and mold at the solidification stage.[4,5] Recently a computer based model has been suggested to predict the typical casting behaviour of high solid slips.[6] But mathematical models on the moisture flow within casts with the simultaneous compaction of the latter are not known.

2 OBJECTIVE

Considerable shrinkage in the cast takes place upon fast withdrawal of moisture by suction mold from it and the resultant stress that develops could often lead to failure or cracks in the sample. An earlier work by the present author on the failure analysis of a refractory pot which is fabricated from deflocculated slips had revealed moisture as a significant factor likely to enhance the chance of pot failure.[7] This work presents a mathematical model which has been developed to simulate the moisture flow within cast under different regimes of mold's suction. The paper discusses the effects of various suction levels on the transformation of the cast into a leather hard one dimensional body.

3 METHOD

The partial differential equation (pde) that governs the moisture flow through the one dimensional cast into the mold under suction effect of the latter is that of the diffusion–convection type and is given by

$$\delta c/\delta t = D^* \delta^2 c/\delta x^2 + V^* \delta c/\delta x$$

where c is the moisture concentration (%), t the time (s) and x the space (cm). D is the diffusion coefficient of moisture through the cast and V the convective velocity with which the cast shrinks due to mold's suction which is external to the cast system. When this velocity is negligible we may treat $V = 0$. In such a case only diffusion due to concentration gradient of moisture controls the flow within the cast. Dietzel and Mostetzky[5] used the above pde in such a context in which they essentially studied the moisture flow in a three phase system from slipwater through the solid cast into mold when the boundary between the solid–liquid phases in the slip moves along with the cast till the whole of the slip transforms into a soft cake.

In the case where the cast shrinks in unit time a distance more than the moisture travels in same time, the term V becomes quite significant. This situation is often encountered in practice when the suction of the mold on the slip–cast system

Ceramics International. 0272-8842/95/$9.50

increases which leads to quick moisture removal together with the shrinkage of the cast sample.

Since our objective is to study the moisture flow within the cake simultaneously with its compaction, in our system V is assumed to be significant. We would solve the above pde subject to the conditions which are simplified as follows.

(1) Initial:

$$c(x,\ t = 0) = bx + a \text{ for } 0 < x < 1$$

where b and a are constants.

(2) Boundary:

(i) $c(x, t) = C_0$ for $x = 0$
(ii) $c(x, t) = C_1$ for $x = 1$ for all $t \geq 0$
(iii) $V = 0$ for $x = 0$
(iv) $V = 0$ for $0 \leq x \leq 1$ at $t = 0$

The boundary conditions (iii) and (iv) indicate that the convective velocity V with which the cast shrinks is initially zero throughout the cast and at the mold–cast interface at $x = 0$ it ceases at all times.

The above pde assumes that only one dimensional or normal transport of moisture takes place. This shows a transient or time dependent behaviour until a steady state is reached when $\delta c/\delta t$ becomes zero. We first consider the transient behaviour. By use of Laplace transformation and contour integration we get a solution of the above pde that satisfies the initial and boundary conditions. The solution is

$$\begin{gathered} c(x,\ t) = \{C_1 - (bl + a)\} * e^{(l-x)V/2D} * \\ \sinh xV/2D/\sinh lV/2D + (C_0 - a)*e^{-xV/2D} * \\ \sinh(l-x)V/2D/\sinh lV/2D - b * e^{V(l-x)/2D} * \\ (x\sinh lV/2D * \cosh xV/2D - l\cosh lv/2D * \\ \sinh xV/2D)/(\sinh lV/2D)^2 - V * b * e^{V(l-x)/2D} * t * \\ \sinh xV/2D * \sinh lV/2D/(\sinh lV/2D)^2 + \\ bx + a + b * V * t \end{gathered}$$

The above solution has been used to predict the moisture states at different space points within cast at an interval of 15 min from the initial time point $t = 0$ to 120 min.

We have a unidimensional soft cast length of l with a moisture concentration given by a straight line of type $c = bx + a$ initially. As has been proposed, boundaries at $x = 0$ and l ends have constant concentration C_0 and C_1, respectively, at all times (see Fig. 1).

The test data utilized for the model are from Dietzel and Mostetzky.[5] These investigators studied cast growth from slip water that contained klingenbergerton clay particles, 97 wt% of which belonged to a grain size less than 0·5 μm. At the

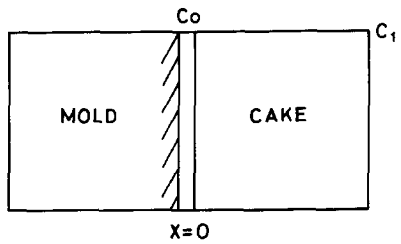

Fig. 1. Mold-cast diagram.

time instant t = 200 min, the slip grew into a cast of length 0·7 cm whose internal moisture content was determined This revealed a straight line profile. In this study the present author has treated the conditions at the time instant t = 200 min, as observed by Dietzel and Mostetzky, as the initial conditions at $t = 0$ when no convective force is assumed to act on the system. The internal moisture content determined at t = 200 min has been use to obtain estimates of the slope b = 18·96 and the intercept a = 35·43 of the straight line. The diffusion coefficient D = 9·75 * 10^{-4} cm^2/s of moisture through the cast as computed by Dietzel and Mostetzky at t = 200 min under temperature 21·4°C and viscosity 1·74 Pa.s has been used as the diffusion coefficient of moisture through the cast of length l = 0·7 cm. The concentrations at the two ends of the cast are C_0 = 35·5% and C_1 = 47·5%, respectively.

The convective velocity V = 2·99 * 10^{-4} cm/s has been hypothetically considered to run the model. The average grain size δ = 0·5 μm of the clay particles is assumed.

4 RESULTS AND DISCUSSION

Out of the two parameters D and V the latter influences the immmobilization of moisture. Under mold's suction the clay particles of the soft cast with greater mass compared to that of moisture molecules will acquire a higher momentum and tend to shrink at the mold–cast interface at x = 0 faster than the moisture molecules. It is therefore of interest to examine the response of the moisture concentration to the changes in V, the convection term. For this we define a dimensionless quantity called peclet number

$$Pe = V * \delta / D$$

which describes moisture flow through the cast of average grain size δ. In general, if Pe is high, convection dominates while if it is low, diffusion dominates the flow.

We use three different values for V of 2·99 * 10^{-4}, 2·99 * 10^{-3} and 2·99 * 10^{-2} cm/s while D and δ remain fixed.

4.1 Transient case

Under a transient case, a numerical simulation is conducted for a period of 2 h in order to predict the space–time distribution of moisture within cast at intervals of 15 min. The change in moisture concentration is shown in Figs 2–4. Initially the moisture concentration within the cast is represented by a straight line with a positive slope which indicates presence of higher concentration of moisture in the interior of the cast compared to that in the layers near $x = 0$ which is the interface between the cast and the mold. By Fick's law then moisture would flow from higher concentration to lower concentration which is represented by the first term of our pde. The second term would accelerate the flow and the combined effect of these two terms would be that as time progresses the concentration would change and consequently the slope would become negative.

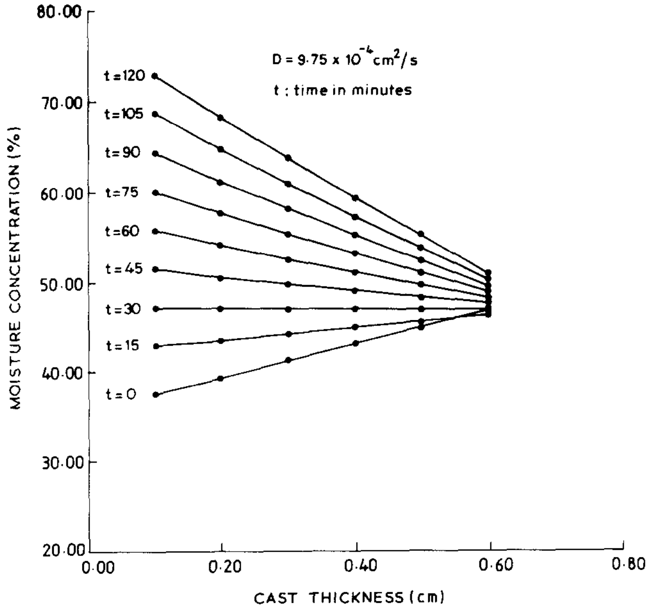

Fig. 2. Space–time moisture profile within cast sample for $V = 2{\cdot}99 * 10^{-4}$ cm/s and $Pe = 1{\cdot}5333 * 10^{-5}$.

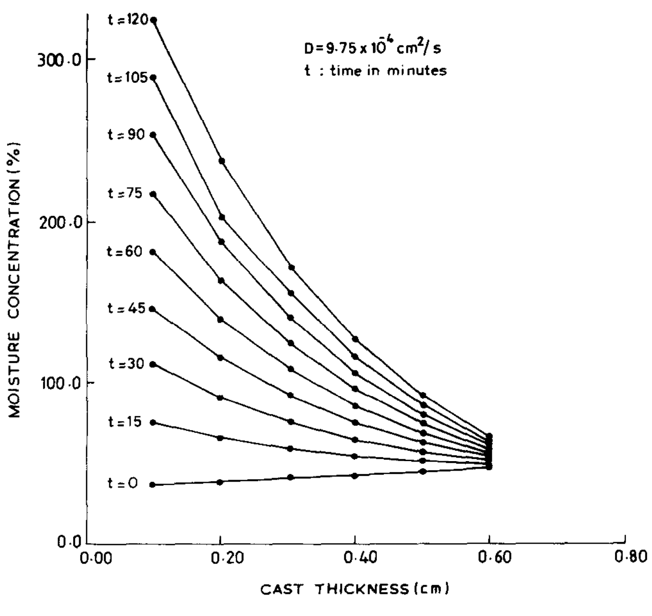

Fig. 3. Space–time moisture profile within cast sample for $V = 2{\cdot}99 * 10^{-3}$ cm/s and $Pe = 15{\cdot}333 * 10^{-5}$.

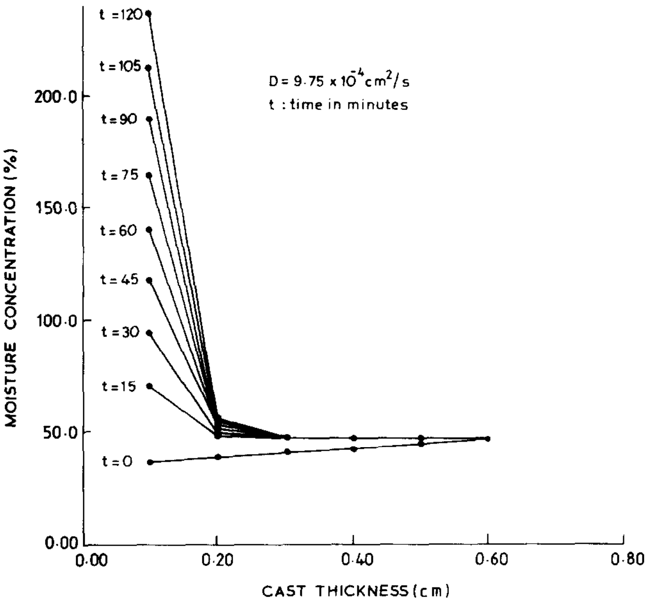

Fig. 4. Space–time moisture profile within cast sample for $V = 2{\cdot}99 * 10^{-2}$ cm/s and $Pe = 153{\cdot}33 * 10^{-5}$.

Figure 2 shows for $t > 0$, the moisture concentration near the $x = l$ end of the cast gradually decreases and that near the mold–cast interface at $x = 0$ increases. Moisture builds up considerably up to 0·3 cm due to its migration towards the interface while a slow rise continues beyond that point within the cake as time progresses. The slow change at points beyond $x = 0{\cdot}3$ cm is caused by the rise in the moisture content near the interface which resists the subsequent flow. The accumulation is also due to compaction of cast particles at the $x = 0$ interface which retards the flow of moisture into mold. Under the suction effect of mold the particles of the loosely agglomerated cast acquire a velocity and as they approach $x = 0$ the velocity tends to zero, particles undergo compaction and come to rest.

After 2 h when considerable time has elapsed, concentration at $x = 0{\cdot}1$ cm in the neighbourhood of the interface almost doubles within the cast compared to the initial moisture level, while the concentration at $x = 0{\cdot}6$ near the other end l is considerably reduced. This indicates progressive removal of moisture and the fact that the layers near the $x = l$ end are likely to dry up faster. These features are observed for lowest $Pe = 1{\cdot}5333 * 10^{-5}$.

Figure 3 shows as Pe is increased to $15{\cdot}333 * 10^{-5}$ due to increase in V, a higher amount of moisture accumulation takes place near the cast–mold $x = 0$ interface. In this case the moisture movement is accompanied by a greater convective or shrinkage velocity and therefore concentration rises rapidly at most of the space points within cast. The concentration undergoes significant change with time. The change at the space points closer to interface is remarkable and extends up to 0·5 cm of the cast beyond which the time changes become slow as bulk moisture gets drawn towards the interface. A greater convective velocity also implies stronger compaction of the cast particles at the $x = 0$ interface. The higher accumulation near the interface is also due to the obstruction of flow as a result of greater compaction in the cast at the interface besides the bulk moisture transport from its interior. Figure 3 reveals that with time, the accumulation of moisture at various points within cast becomes higher compared to the previous case in Fig. 2 as the convective velocity is increased.

When V is still increased, Pe increases to $153{\cdot}33 * 10^{-5}$. Figure 4 reveals that the concentration drops quickly to a certain level of 47% from 0·3 cm onwards and becomes static throughout the x dimension of the cast. Due to increased convective velocity, the bulk moisture is immediately withdrawn and accumulated near the interface $x = 0$. There is no change in the concentration beyond $x = 0{\cdot}3$ cm until the end of the simulation. The only notable change is $x = 0{\cdot}1$ cm in the proximity of interface. This is the result of rise in concentration of moisture removed rapidly from the interior of the cast which accumulates as the flow gets impeded by the particular compaction at the cast–mold interface at $x = 0$. However, the accumulation of moisture with time at different space points within cast in this case is lower as in the Fig. 4 display compared to the previous case in Fig. 3 despite the fact that the convective velocity is now much higher.

The above discussion indicates that under transient conditions, the spatial moisture content would oscillate within cast as the convective velocity or the shrinkage is increased. The question now arises what happens when a long time has elapsed and the flow system has attained what is called a steady state. The steady state discussion now follows.

4.2 Steady state

After a long time, concentration within cast would vary only with thickness and would become independent of time. This is the equilibrium or steady state, i.e. $\delta c/\delta t = 0$. At this state the transient or time dependent conditions cease. Once this state is attained, moisture levels will not change with time, i.e. the system becomes static or quasi-static in a strict sense.

Obviously, we will not consider the initial moisture condition in this case and the time independent solution that will satisfy the constant boundary conditions of C_0 and C_1 at $x = 0$ and l, respectively, is given by

$$c(x) = (C_1 e^{Vl/D} - C_0/e^{Vl/D} - 1) * (e^{Vx/D} - 1) * e^{-Vx/D} + C_0 e^{-Vx/D}$$

Under steady state for prescribed values of D and V, a straight line moisture profile is seen with the thickness in Fig. 5. In this figure the numerical simulation suggests that for the lowest value of $Pe = 1{\cdot}5333 * 10^{-5}$, the change in concentration is steady. As Pe increases to $15{\cdot}333 * 10^{-5}$, change becomes parabolic and finally for the highest value of $Pe = 153{\cdot}33 * 10^{-5}$ a clear plateau in moisture level is observed along a fairly long dimension of the cast before the level tends to fall gradually.

The steady state plots thus indicate that the rate of removal of moisture gets retarded within cast with the increase in the convective or shrinkage velocity. This implies that a microstructure which is composed of a complicated network of fine and minute capillary and pores that is further dependent on the geometry of the particles would emerge within cast as a result of its shrinkage. A stronger compaction would result in the formation of narrow channels of pore and capillaries which could obstruct and stagnate the moisture flow after a considerable time. Stresses or pore pressure may develop from the stagnant fluid. Generally two types of stresses would act in the sample. One is the stress within moisture and the other is the total stress in the cast system. The difference of these stresses called effective stress is known to deform the cast if its yield conditions are exceeded.

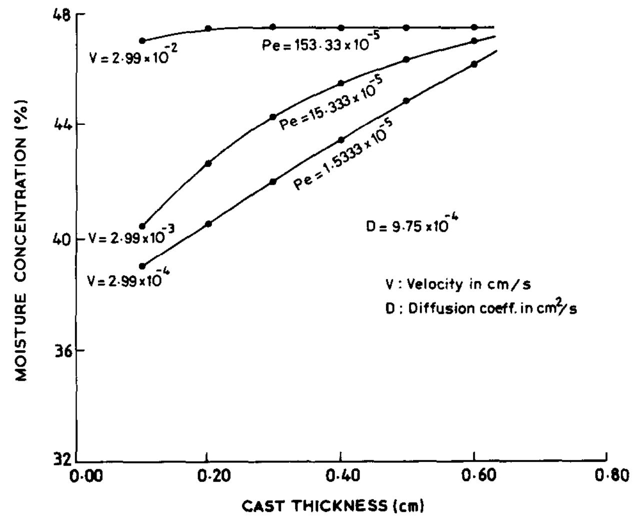

Fig. 5. Spatial distribution of moisture within cast sample under steady state.

5 CONCLUSIONS

The present study examines mathematically the interaction between clay and moisture within a cast sample under simplified conditions from the point of view of failure. With the increase in compaction, the flow system oscillates when conditions are time dependent but as the system stabilises, rate of stagnation of moisture within the sample increases which makes the chance of failure likely. A mathematical model which can simulate physical mechanism for failure of the sample now needs to be developed.

6 LIMITATIONS

It is only a 1-D flow model, diffusion can take place along y and z axes also. It is therefore a model for very simply shaped casts. The concentrations at the boundaries remain permanently constant which in practice are likely to change. The velocity V is assumed to be uniform over all cake thicknesses. But in practice it may vary with thickness as distance from the interface increases. It is considered equal to zero at $x = 0$ for all times. But as the cast layers near the $x = 0$ end shrink and become compact the velocity V may become zero at space points $0<x<1$ due to resistance offered to the cast particles by the compact portion of the cake in contact with the plaster mold. Hence velocity may vanish well before the particle layers of the cast would reach the $x = 0$ interface. So, V in practice may not be essentially uniform over all thicknesses. Hence it is a stagnant boundary model.

ACKNOWLEDGEMENTS

The author thanks Dr B. K. Sarkar, Director, Central Glass & Ceramic Research Institute, Calcutta for the kind permission to publish the work. The author also thanks Dr B. Mukherjee, Asst Director for valuable suggestions and fruitful discussion.

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