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Modelling Pressure Patterns and Failure of Ceramic Cake During Drying Within Plaster Mould

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Abstract: Hydraulic pressure created by mould's suction within a ceramic cake formed out of slipwater has been simulated over its length and time on the analogy of a modified thermal flow theory by diffusion-convection equation. The simulation results show that the pressure flow through the particulate medium of the cake, which also moves could lead to pressure accumulation with time in the layers close to the mould–cake interface and consequently develop internal stresses of tensile nature in the region. The results further indicate that the particulate cohesion of the cake would be disrupted at 0.1 cm position at 7.1 minutes as the resultant stress could exceed the green item's tensile strength. © 1997 Elsevier Science Limited and Techna S.r.l.

1 INTRODUCTION

In recent decades, a number of works have been reported on the solid–liquid separation problems in ceramics.[1–3] These attempt to put forward relationships between flow parameters and the structure of filter cake developed from slip or suspensions. One of the major concerns that has prompted these studies is the growth of defects in green items which subsequently contribute to the item failure. Direct studies on failure however have remained very limited. The importance of the problem is due to the fact that failure results in wastage of enormous cost and time in industry.

There is very limited literature available on line. One of the very few works proposed a quantitative theory on cracks and wrap of simply shaped semi-saturated clay items.[4] On the analogy of thermal stress theory it was shown that the stresses within the body as it dries up can be calculated if water flow is assumed to be similar to the heat flow process. The theory however lacked experimental confirmation. Another work was on the cracks in extruded green products of heavy clay.[5] The causes of failure were identified on experimental basis and the need to measure and model the failure phenomena was emphasized. The work thus lacked a theoretical framework. The subject therefore has restricted theoretical or experimental basis. When experimental evidences are limited, mathematical models may provide an alternative way for theoretical pursuit of the problem. The advantage of such an exercise is that a parameter can be simulated on computer under a set of boundary conditions and more importantly the variability in the process parameter can be studied numerically over space and time without resort to experimentation.

2 OBJECTIVE

In this paper, a simplified mathematical model has been developed that simulates hydraulic pressure within a ceramic cake formed within plaster mould. Pressure flow through pores in the cake has been considered analogical to a modified heat flow process. The simulated pressure has then been used to calculate the stress exerted on the pore walls of the cake. The stresses are then compared with the particular cohesion of the sample to evolve a simple failure criterion.

3 METHOD

3.1 Physical theory

We begin from a premise that a slipwater made with particles of a compressible material has just been transformed by mould's capillary suction into cake of finite length (see Fig. 1). This means the interstices or the pores within cake are completely filled with moisture. To begin with, the cake is thus completely saturated and still within plaster mould. It is therefore in a soft plastic condition. As the mould continues to draw out moisture the cake transforms into a compact. A premise of this kind is assumed because failure in form of cracks on green items, shaped out of slip-cast material often becomes imminent as the soft body leather hardens.

The partial differential equation (pde) that governs the hydraulic pressure or moisture flow field in a ceramic cake can be conceived to be of thermal diffusion type. The capillary action by the mould increases the pressure within the cake which is consequently released with the moisture into sink and the cake thus becomes a compact. By analogy with heat flow, the compaction or pressure release from the cake can be conceived to be a diffusion process. The necessary pde is given by

$$\delta p/\delta t = C_v \times \delta^2 p/\delta x^2 \qquad (1)$$

where p is the hydraulic pressure (KPa) within cake pores, x the cake length, t the time (s), C_v the consolidation or compaction coefficient ($cm^2 s^{-1}$) analogous to thermal diffusivity. It is defined as

$$C_V = k/y_w \times m_V$$

where k ($cm\,s^{-1}$) is the cake's permeability analogous to thermal conductivity, y_w ($gm\,cm^{-3}$) is the unit weight of water analogous to the density of the medium and m_v ($cm^2 gm^{-1}$) is the coefficient of volume decrease analogous to the heat capacity times. This coefficient is defined as the fractional change in the void volume per unit pressure released from cake. The idea of this diffusion equation and the coefficient has been drawn from soil science where soil workers use these in consolidation studies of soils under pressure.[6] Consolidation in soils is similar to compaction in ceramics. Recourse to soil science has been taken as these coefficient values are rarely available in ceramics.

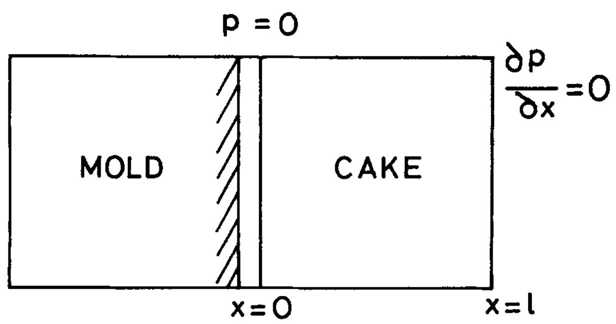

Fig. 1. Mould–cake diagram.

The above diffusion equation suggests that the cake pressure is released upon moisture withdrawal through a static particulate medium. If the particulate medium such as the cake through which moisture flow takes place has very fine particles which also move upon transport of moisture through the pores towards the mould and finally clog the cake–mould interface at $x=0$, a convective term is added to the diffusion equation and our required pde becomes a diffusion–convection equation as

$$\delta p/\delta t = C_v \times \delta^2 p/\delta x^2 + V \times \delta p/\delta x \qquad (2)$$

where V ($cm\,s^{-1}$) is the particle velocity. Fine particles' migration by moisture transference in the ceramic compacts and thereby skin formation at the surface are known phenomena.[7,8] The diffusion–convection principle has been used by authors in recent times to model moisture concentration in a cast sample grown from slip-cast material.[9] In this work the same principle has been used to simulate hydraulic pressure within a cake sample formed by the same method.

In our study eqn (2) is solved under conditions stated below as:

(1) Initial:

$$p(x, t=0) = a(x)^b \text{ for } 0 \le x \le 1$$

where a and b are empirical constants.

(2) Boundary:

$$\left.\begin{array}{lll} \text{(i)} & p(x,t)=0 & \text{for } x=0 \\ \text{(ii)} & \delta p/\delta t = 0 & \text{for } x=1 \\ \text{(iii)} & V=0 & \text{for } x=0 \end{array}\right\} \text{for all } t \ge 0$$

$$\text{(iv)} \quad V=0 \quad \text{for } 0 < x \le 1 \text{ at } t=0$$

The boundary conditions suggest that the finer particles do not move at the initial instant $t=0$ and at the mould–cake interface at $x=0$, the movement of the grains always ceases which means the skin effect always remains restricted at the interface. The hydraulic pressure remains always zero at $x=0$ since the pressure is instantaneously released at the interface while the $x=1$ end of the cake remains impermeable as shown by the zero pressure gradient. Equation (2) assumes that compaction takes place along the cake length only. This means that the pressure flow along the other two dimensions of the cake are neglected. The particle movement also takes place in uniaxial manner. The methods to arrive at solutions of the above pde which would satisfy the required initial and boundary conditions have been those of Laplace transformation and Contour integration discussed in the literature.[10,11]

Under initial and the boundary conditions the analytical solution of our transient pde 2 for the empirical constant $b=1$ is

$$p(x,t) = a \times e^{V(l-x)/2C_V} \times \mathrm{Sinh}\,Vx/2C_V/V \times (\mathrm{Sinh}\,Vl/2C_V - \mathrm{Cosh}\,Vl/C_V)/2C_V + ax + aVt \quad (3)$$

This process assumes importance in real-life fabrication systems[12] as particles from the cake may enter into the pores of the gypsum mould which terminates the flow process. This prompts us to propose a failure criterion.

3.2 Failure criterion

The mould's capillary action is a cause of a negative pressure also called suction within the mould. But the effect of capillary action would create a hydraulic pressure which is actually a raised hydrostatic pressure of the cake that is immediately released with the moisture. In the process of its release, the moisture movement through the cake pores carries along with it the finer particles which are assumed to ultimately clog the mould–cake interface. Consequently the flow lines are impeded and the excess hydrostatic pressure within cake cannot be instantly released due to obstruction at the interface. If the obstruction at the interface increases, the hydraulic pressure within cake results in the development of internal stresses which are exerted on the pore walls. In other words the suction effect coupled with particle migration produces an excess hydrostatic pressure of tensile nature within pores which is exerted on the walls and subsequently transmitted through the thickness of the pore wall if the particular cohesion is not strong enough to sustain the tension. The pore walls within cake thus have to withstand an internal tension in order to sustain cake's particular structure.

Let us assume that the cake is composed of spherical particles with an average particle size δ. Let D be the average cylindrical pore diameter within cake. The cake is thus conceived as a system of interstices which is equivalent to a collection of cylindrical pores. The pore circumference area and the pore wall area on average within cake are then written as $\pi D^2/4$ and $\pi D\delta$ respectively. Let E be the internal stress or tension on the pore wall exerted by the excess hydrostatic pressure due to mould's suction. Then in order that the pore will remain undisrupted, the forces due to excess pressure and the tension which act on the pore wall must balance with each other.

The force due to excess pressure is equal to the product of the pressure and circumference area of the pore. This is equal to $\pi D^2 p(x,t)/4$. The force due to tension on the pore wall is the product of the surface area of the wall and the tension. This is equal to $\pi D\delta E(x,t)$. These two forces are balanced and the internal tension from these forces can be written as

$$E(x,t) = p(x,t) \times D/4\delta$$

The tension is thus a function of thickness and time as it varies with hydraulic or excess pressure which varies with position and time. If D and δ are assumed to be constant on average, the tension becomes directly proportional to the hydraulic pressure. Rise in pressure would thus increase the tension on the pore wall and our criterion for failure will be that if this resultant tension exceeds the tensile strength of soft cake, the particulate structure will be disrupted and cracks would appear on the green item.

4 TEST DATA

To test the model, the hydraulic pressure data in this work has been collected from a literature[2] source. It provides graphical profiles of the normalized pressures as a function of fractional length of the filter cake for a variety of materials. The present authors have used the pressure distribution for specimen labelled B which is a moderately compressible material. The normalized hydraulic pressure is a ratio between the hydraulic pressure and the pressure drop Δp_c across cake represented graphically in the source as a function of cake's fractional length. The pressure drop across cake is defined as the difference between pressure applied and the pressure at the mould–cake interface. The normalized hydraulic pressure data were collected from graph at chosen points along fractional length to fit an empirical relationship between normalized pressure and fractional length as

$$p/\Delta p_c = a \times (x/l)^b \quad (4)$$

If the cake length $l=1$ cm, multiplication of the right-hand side in (4) by pressure drop Δp_c will provide us the desired hydraulic pressure as a function of length as below

$$p = a \times \Delta p_c(x)^b \tag{5}$$

In Fig. 2, the curve labelled B represents the empirical relation between hydraulic pressure and cake length for the compressible cake labelled B in the source figure.[2] The values of a and b were empirically derived as 1 and 0.91 respectively. Since value of b is close to 1 we treat $b=1$ which enables us to make use of eqn (3) which is an analytical solution of the pde 2. The constant pressure drop $\Delta p_c \approx 160$ KPa in our study has been assumed and it lies within 100–200 KPa, the reported range of the mould's suction pressure.[2] The straight line labelled A in Fig. 2 provides us with the desired initial hydraulic pressure distribution within the cake for $b=1$ on the basis of which the subsequent pressure was simulated at different positions and time.

The particle velocity $V=2.99\times10^{-3}\,\mathrm{cm\,s^{-1}}$ is hypothetical and has been utilized earlier by the present author.[9] The consolidation coefficient $C_v = 2.315 \times 10^{-2}\,\mathrm{cm^2\,s^{-1}}$ pertains to general class of compressible material and is drawn from a soil mechanics literature.[13] The tensile strength of compressible specimen in soft plastic condition e.g. clay is reported to be 40.8 KPa in the literature.[14] This value has been assumed as the tensile strength of our compressible cake.

The average pore diameter of cake was worked out as $D=0.203\,\mu$m. The average particle size that gives us pore thickness was $\delta=2.5\mu$m which is the arithmetic mean of the modal particle size 0–5 μm for a compressible cake.[15] Particles were assumed to be spherical in shape. Spherical particles of uniform size, can pack in a number of regular arrangements and the void:solid ratio in each arrangement is available in the literature.[16] The arithmetic mean of the theoretical void:solid ratios of different arrangements was worked out and multiplied with the solid volume of a spherical particle with $\delta=2.5\,\mu$m to get the average void associated with the particle. The porosity value of the cake made from the specimen B was utilized from the literature[2] and this was divided by the average void associated with the particle. This gave the average number of voids within cake. The voids were also assumed to be spherical. This number of voids when multiplied by the volume of a spherical void equals the cake porosity. Since the void number and the porosity are known, void diameter can be worked out from this relation. This provided us a simple measure of average pore diameter.

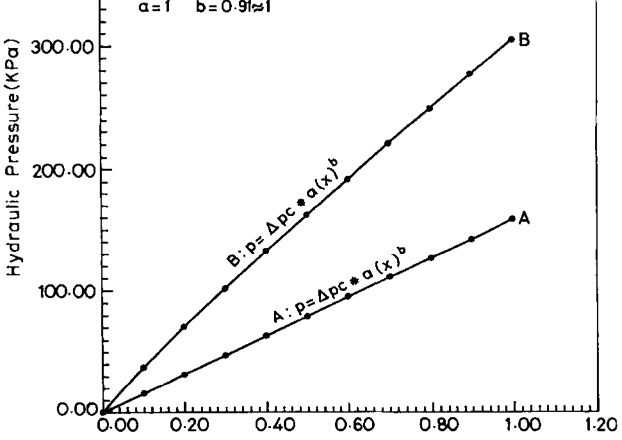

Fig. 2. Initial hydraulic pressure profile.

5 RESULTS AND DISCUSSION

We would find that in our pde 2, of the two terms C_v and V the first term dominates the flow. Under mould's suction pressure the finer particles will travel along with moisture through the pores to the $x=0$ interface. We define a dimensionless quantity called Peclet number as

$$\mathrm{Pe} = V \times \delta / C_v$$

which is a measure of the amount of pressure released through the cake of average particle size δ by convection to that by conduction or diffusion. The physical significance of this number is that if Pe is high, convection dominates the pressure flow and if it is low the diffusion dominates. For our system it works out to be 3.23×10^{-6} and therefore diffusion dominates the flow process.

In Fig. 2 the straight line labelled A displays the hydraulic pressure distribution at $t=0$ within cake. The straight line representation of hydraulic pressure at the initial stage indicates presence of higher pressure in the cake interior compare to that in the interfacial layers near $x=0$. By diffusion law, pressure would flow from the higher to lower concentration as governed by the first term of our pde 2. Since change in pressure is accompanied with moisture flow which sets finer particles in motion, the effect of the particle migration on the release of pressure is accounted by the second term in the pde 2. The combined effect of these two terms is that as time progresses pressure would change with position and time. On the basis of the transient solution (3) of our pde 2 an analytical simulation is conducted at the interval of 1 min to predict the hydraulic pressure and the resultant tension at chosen points within cake.

Figure 3 shows for $t>0$, the pressure near the $x=1$ end of the cake tends to reduce while that near the interface at $x=0$ rises with time. Compared to the initial hydraulic pressure as shown in Fig. 2, the slope of the hydraulic pressure with length therefore inverts as time changes. Figure 3 reveals that pressure profile at all chosen points within cake however increases with time. The increase in the layers close to the interface $x=0$ is prominent and extends up to 0.6 cm within cake beyond which the changes in pressure with cake length become slow at all times as bulk of the pressure gets withdrawn towards the interface due to the mould's suction. Increase in the pressure can be attributed to the particle movement which tends to obstruct the flow into the mould as particles move through the pores and finally get clogged to the mould–cake interface. Also the moisture transport continues simultaneously from the cake interior towards the interface due to diffusion process. The hydraulic pressure therefore remains always higher in the interface region compared to the cake's interior layers. These observations indicate that the cake tends to dry up from its $x=1$ end which is impermeable.

Since internal tension within an average pore is directly related to the hydraulic pressure, the pattern of tension over time at chosen points within cake is similar to that of hydraulic pressure. Figure 4 describes the tension profiles. As pressure increases, tension also increases with length and time. Due to simultaneous movement of moisture and finer particles towards interface, resistance to the moisture flow grows which accumulates pressure in the layers close to the interface $x=0$ and as such the tension at $x=0.1$ cm position as displayed in Fig. 4 in the interfacial region within cake is always maximum. At $t=7$ min, the tension at $x=0.1$ position is 40.28 KPa which is just short of 40.80 KPa which is the tensile strength of the cake. Figure 5 displays the increase in tension at $x=0.1$ position with time. At time 7.1 min the tension at $x=0.1$ cm exceeds the tensile strength of the cake. A crack would therefore appear at this location as the tension exceeds the tensile strength of the cake item, which disrupts its particular cohesion.

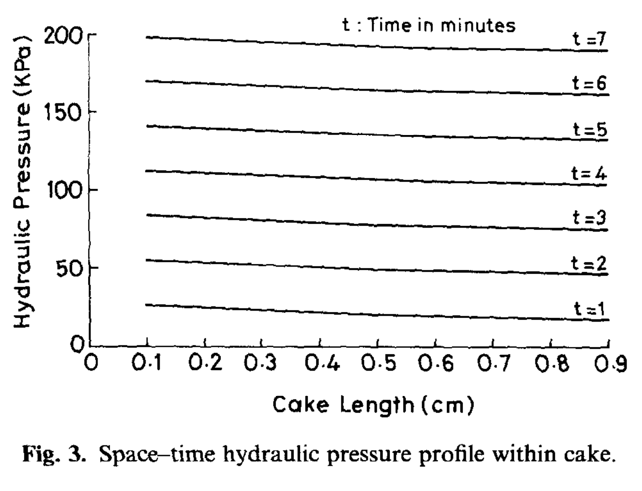

Fig. 3. Space–time hydraulic pressure profile within cake.

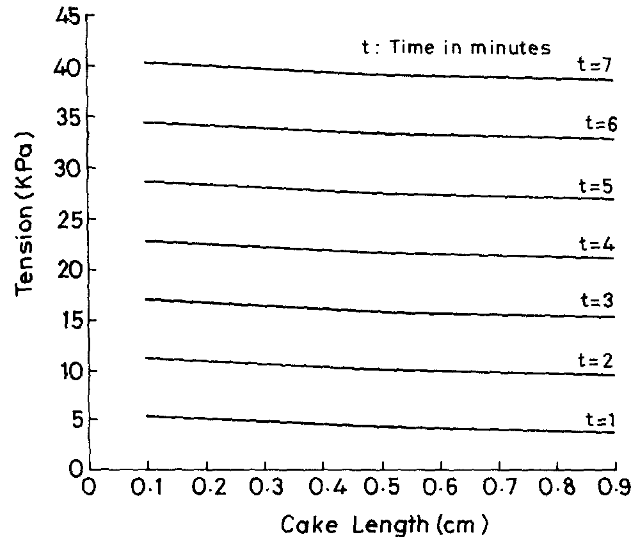

Fig. 4. Space–time tension profile within cake.

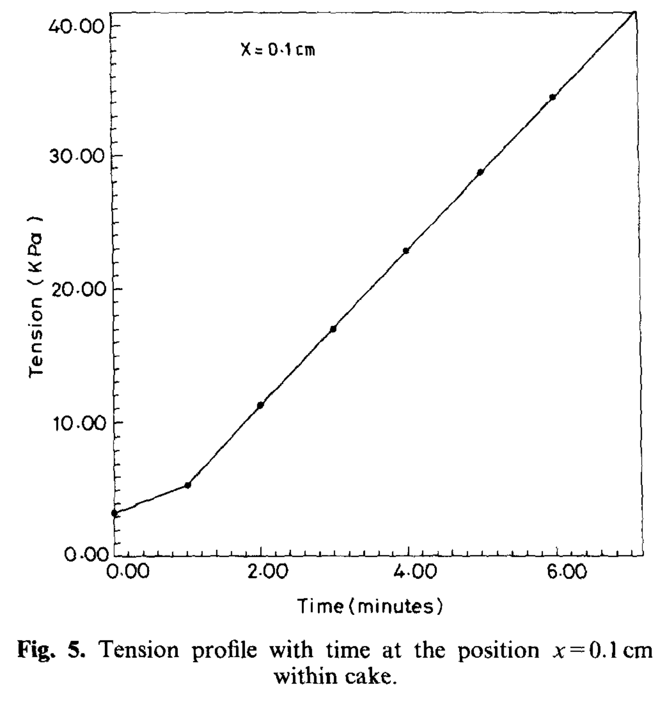

Fig. 5. Tension profile with time at the position $x=0.1$ cm within cake.

6 CONCLUSIONS

A mathematical model along with a simple failure criterion for green items in the soft plastic condition, formed out of slipwater in mould has been developed under simplified assumption to explain the physical mechanism of failure as item leather hardens. The internal tension within cake item disrupts its cohesion as the tension exceeds the item strength. The model when tested with some published data from ceramics and allied sciences theoretically replicates the patterns of hydraulic pressure at different time and positions within cake.

7 LIMITATION AND FUTURE STUDIES

It is a 1D model for simply shaped cakes and now needs to be extended to 2–3D cakes with variable shapes so that the propagation of crack can be monitored along planes in a 3D cake space. A constant consolidation coefficient has been assumed whereas in practice this coefficient being a function of permeability is likely to vary with position and time. The average pore diameter D need not remain constant over length in a compressible cake. The migration velocity V may become zero at any position $0 < x < 1$ well before the particles reach the interface. This means the skin effect in practice would not restrict to $x = 0$ as assumed in the model and may extend into the cake. It is therefore a stagnant boundary model. Variations in C_V, D and V over position and also time need further theoritical and experimental studies so that these real life complexities can be included in the future models.

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REFERENCES

1. HENCH, L. L. & ULRICH, D. R. (ed.), *Ultrastructure Processing of Ceramics, Glasses and Composites.* John Wiley & Sons, New York, 1984, pp. 439–491.
2. TIILER, F. M. & TSAI, C.-D., Theory of filtration of ceramics: I. Slip casting. *J. Am. Ceram. Soc.*, **69**(12) (1986) 882–887.
3. HAERLE, A. G. & HABER, R. A., Ultrasonic real-time monitoring of cake structure during slip casting. *J. Am. Ceram. Soc.*, **78**(3) (1995) 819–823.
4. ONODA JR, G. Y. & HENCH, L. L. (ed.), *Ceramic Processing Before Firing.* Wiley–Interscience, New York, 1978, pp. 261–276.
5. BANKS, P. J., Cracking of extruted clay products during drying. *Key Engng. Mater.*, **53-55** (1991) 433–438.
6. TERZAGHI, K., *Theoretical Soil Mechanics.* John Wiley & Sons, New York, 1943, pp. 270–272.
7. HOMPTON, J. H. D., SAVAGE, S. B. & DREW, R. A. L., Experimental analysis of fine-particle migration during ceramic filtration processes. *J. Am. Ceram. Soc.*, **75**(10) (1992) 2726–2732.
8. GRIMSHAW, R. W., *The Chemistry and Physics of Clays and other Ceramic Materials.* Ernest Benn Ltd, London, 1980, pp. 521–558.
9. BANERJEE, G., Modelling of moisture flow in a cast sample during drying. *Ceram. Int.*, **21**(6) (1995) 407–411.
10. CARSLAW, H. S. & JAEGER, J. C., *Conduction of Heat in Solids.* Oxford University Press, London, 1959, pp. 297–326.
11. KEANE, A. & SENIOR, S. A. (ed.), *Mathematical Methods.* Science Press, Sydney, 1961, pp. 201–242.
12. BANERJEE, G. & MUKHERJEE, B., A failure analysis of a ceramic body. *SCIMA*, **21**(2–3) (1992) 73–79.
13. DAS, B., *Advance Soil Mechanics.* McGraw Hill Book Company, New York, 1985, pp. 253–338.
14. PACKARD, R. Q., Moisture stress in unfired ceramic clay bodies. *J. Am. Ceram. Soc.*, **50**(5) (1967) 223–229.
15. DIETZEL, A. & MOSTETSKY, H., Mechanism of dewatering of a ceramic slip by plaster mould—I. Experimental investigation of diffusion theory of slip-casting process. *Ber. DKG*, **33**(1) (1956) 7–18.
16. DALLAVALLE, J. M., *Micromeritics.* Pitman Publishing Corporation, New York, 1948, pp. 123–148.