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Short communication

Optimization of sample number for Weibull function of brittle materials strength

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Abstract

The estimation of the parameters of the Weibull function was investigated with regard to reliability. The sample number, confident level, and relative error of estimated parameters are three key factors and should be involved for evaluating the strength of brittle materials. The results indicated that the scatter of scale parameter was much less than that of shape parameter. The relationship of sample number, confident level, and relative error of shape parameter was established. The lower limit of sample number should be more than 20 in order to decrease the scatter of shape parameter.

Keywords: C. Strength; Brittle materials; Weibull function

1. Introduction

Weibull distribution function has been widely used for evaluting fracture strength data of brittle materials and is given by [1–3]:

$$F(t) = 1 - \exp\left[-(\frac{t-\gamma}{\eta})^m\right](t \geqslant \gamma, m, \eta > 0) \quad (1)$$

where $F(t)$ is the probability of failure for fast fracture of component subjected to an applied stress (t). m, shape parameter, is the most important measurement of materials which characterizes the scatter of the materials properties. Large values of m indicate uniformity, while small m represents large scatter. η is the scale parameter or characteristic strength because the maximum density of the Weibull distribution is located at the position of $t = \eta$. γ is the location parameter or threshold stress below which the failure probability of materials is zero. The curve is moved only along the abscissa with the variation of γ. γ is small in magnitude relative to the average strength of the materials and is usually considered as negligible in the study [4–6]. Accordingly, the three-parameter Weibull function is simplified as two-parameter Weibull function. The density function of two-parameter Weibull function is expressed as follows:

$$f(t) = \frac{m}{\eta}\left(\frac{t}{\eta}\right)^{m-1}\exp\left[-\left(\frac{t}{\eta}\right)^m\right] \quad (2)$$

Up to now, many investigations have been conducted on estimating the parameters of Weibull function. However, these works only focused on the point estimation of parameters. In fact, the estimation of parameters is concerned with the reliability of materials. In order to evaluate the strength of brittle materials correctly and comprehensively, the sample number, confident level, and relative error of estimated parameters are three very important factors. In the present work, the relationship of these three factors was investigated and the sample number was optimized.

2. Experimental procedure

Flexural strength of glass material was measured by three-point-bending method The size of the sample was 3.0×4.0×40 mm, and the span was 30 mm. The loading rate of the crosshead was 0.5 mm min^{-1}.The flexural strength (σ_f) was calculated by:

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Table 1
Flexural strength of a kind of glass material

No.	Strength(MPa)	No.	Strength (MPa)	No.	Strength (MPa)	No.	Strength (MPa)
1	54.7	11	65.4	21	66.3	31	59.0
2	54.9	12	63.5	22	70.7	32	56.4
3	61.1	13	67.5	23	70.4	33	61.4
4	66.6	14	47.7	24	64.2	34	63.2
5	57.5	15	55.5	25	69.6	35	67.6
6	72.6	16	50.2	26	66.8	36	66.6
7	52.5	17	62.4	27	68.4	37	62.7
8	60.0	18	74.4	28	55.3	38	65.6
9	54.6	19	65.4	29	53.8	39	67.2
10	53.9	20	52.4	30	68.0	40	52.9

$$\sigma_f = 3PL/2BH^2 \quad (3)$$

where P is the fracture load (N), L is the span (mm), B and H are the thickness and height of the sample, respectively.

3. Results and discussion

Table 1 listed the strength data of a kind of glass, which were measured by three-point flexural method. For estimation of parameters, maximum likelihood estimation (MLE) was used because it has been considered as the most effective method, in which the Newton–Raphson iterative technique was employed to solve the likelihood Eqs. [5,6]. However, the as-estimated value of the shape parameter(m) is biased and should be modified:

$$m * mB(n) \quad (4)$$

where m^* is the unbiased estimated value, $B(n)$ is the modification factor which can be found in Reliability Table [7].

To investigate the effect of sample number on the shape parameter and scale parameter, the total sample (40) was divided into several groups which contained different number, i.e., 5, 8 10, 13, 20, and 40. The results are illustrated in Figs. 1 and 2. Obviously, the estimated values were closely related to the sample number. The fewer the sample number, the more large the scatter of estimated values. Furthermore, the scatter of scale parameter was much less than that of shape parameter. In order to optimize the sample number, the relative error of shape parameter (Em) was defined as:

$$Em = /\Delta m/2m^* \quad (5)$$

where Δm is the absolute value of m, and m^* is the estimator of m.

Then, the probability formula of shape parameter is given by the following:

$$P\{\Delta m^* \leqslant Em\} = 1 - \alpha \quad (6)$$

where 1-α is the confidence level.

According to the reliability theory [8,9], the upper limit (m_U) and lower limit (m_L) of m are:

$$m_U = B(n)m/Z_{a/2} \quad (7)$$

$$m_L = B(n)m/Z_{1-a/2} \quad (8)$$

where $Z_{\alpha/2}$ and $Z_{1-\alpha/2}$ are the percentiles of the statistics m^*/m distribution function, respectively. They are calculated by Monte-Carlo method and available in Reliability Table [7].

The relative error of m is given by:

$$2\Delta m = B(n)m\left(1/Z_{a/2} - 1/Z_{1-a/2}\right) \quad (9)$$

$$Em = \frac{1}{2}\left(\frac{1}{Z_{a/2}} - \frac{1}{Z_{1-a/2}}\right) \quad (10)$$

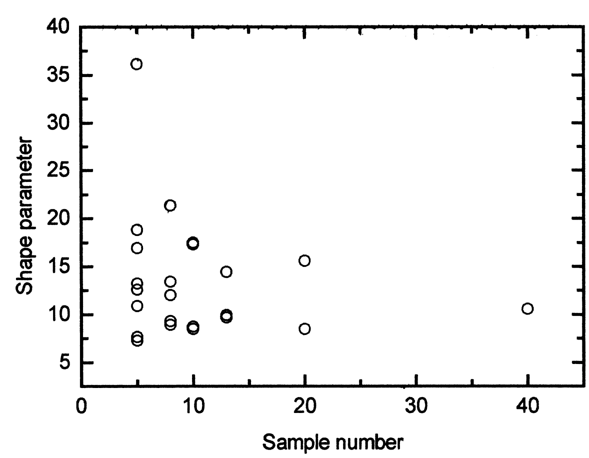

Fig. 1. Relationship between shape parameter and sample number.

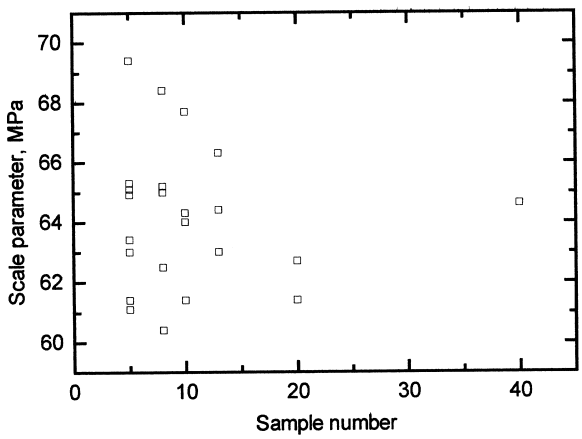

Fig. 2. Relationship between scale parameter and sample number.

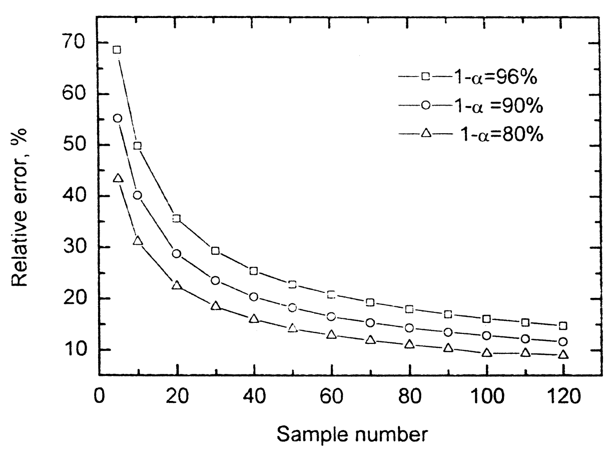

Fig. 3. Relationship of sample number, confident level, and relative error of shape parameter.

Eq. (10) described the relationship of the sample number, confident level, and relative error because percentiles ($Z_{\alpha/2}$ and $Z_{1-\alpha/2}$) of the statistics m^*/m distribution function were only dependent on the sample number and confident level but independent of flexural strength. Consequently, the optimum number of the sample (n) can be obtained from Eq. (10) for the given confident level and relative error which are chosen according to the requirement. In Fig. 3, we could find that the relative error of shape parameter is rapidly decreased with the increase of sample number if the sample number is small. However, the curve became gradual if the sample number is more than 20. The relative error of shape parameter was just slightly decreased with the increase of sample number if the sample number was more than 40. In order to obtain less scatter of shape parameter, it is essential that the sample number should be more than 20.

4. Conclusions

For the estimation of parameters of Weibull function, the sample number, confident level, and relative error of estimated parameters are three key factors and should be involved for evaluating the strength of brittle materials.

The scatter of scale parameter was much less than that of shape parameter. The relationship of sample number, confident level, and relative error of shape parameter was established. The lower limit of sample number should be more than 20 in order to decrease the scatter of shape parameter.

References

[1] W. Weibull, J. Appl. Mech. 18 (1951) 293.
[2] F.I. Baratta, ASTM STP 844 (1984) 194.
[3] D. Lewis, J. Am. Ceram. Soc. 29 (1976) 507.
[4] G. Fu, Z. Jin, Ceram. Soc. Bull. China 4 (1987) 28.
[5] K. Trustrum, J. Mater. Sci. 14 (1979) 1080.
[6] P. Kittle, J. Mater. Sci. 17 (1982) 922.
[7] S. Dai, H. Fei, L. Wang, Reliability Table, Defense Press of China, Beijing, 1984. p. 54.
[8] R.M. Nancy, Methods for Statistical Analysis of Reliability and Life Data. John Wiley & Sons, 1974. p. 126.
[9] S. Dai, H. Fei, L. Wang, Reliability Experiments and Statistical Analysis, Defense Press of China, Beijing, 1984. p. 95.