

# Ultrasonic piezoceramic motor

## The computation of traveling-wave velocity on the stator surface and excitation by PWM modulation with higher harmonic suppression

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### Abstract

The article describes operating principle of traveling-wave piezoelectric ultrasonic motor. This motor makes use of the fact that when a surface traveling wave propagates along finite elastic body of the stator, the rotor is driven due to the friction between the stator and the rotor. The main part of this paper is focused on the computation of traveling-wave speed on the surface of piezoceramic transducer. Furthermore, the methods of the ultrasonic piezoceramic motors driving are presented. In detail there are described the possibilities of feeding by the nonharmonic signals, particularly by the pulse width modulation (PWM). The proportion of pulse width to the turn-off interval is selected with the stress on higher harmonic suppression. This way of ultrasonic piezoceramic drive controlling reduces reactive input power and increases efficiency. The realization of the control system by means of field programmable gate array (FPGA) and digital signal processing is introduced too.

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### 1. Introduction

Ultrasonic piezoceramic motor gives rotative motion on the output shaft and it can be constructed according to several principles. It spurred many proposals on the use of various modes of vibration, e.g. longitudinal, flexural or torsional, to obtain elliptic motion. The methods of obtaining elliptic motion can be roughly divided into those using a single vibration mode and those using two or more vibration modes. In our case those using two-vibration mode can be further classified into the traveling wave type. Driving the degenerate vibrational modes at different phases produces the rotational motion of the motor. Many different shapes including rods, rings, disks, bars and cylinders were tried with the modes that were suitable for this construction of ultrasonic

motors. This paper shows the computation of traveling-wave velocity on the surface of piezoceramic disk or ring motor.

This design of drive dictates the solution of power electronic circuit and control system. Two main well-known principles are seen in practice very often [1]. One of them is a motor using standing wave, other possibilities are drives with traveling wave. The article describes power electronics for exciting of a piezoceramic motor using traveling wave on the surface of the stator.

### 2. Principle of the propagating-wave type motor

The stator has a ring shape and consists of a piezoelectric driving transducer and an elastic part, fixed to the piezoelectric transducer. The rotor is composed of elastic moving part and a friction coat. In the piezoelectric transducer, the surface particles perform an elliptic motion due to the coupling of forced longitudinal and transverse elastic waves.

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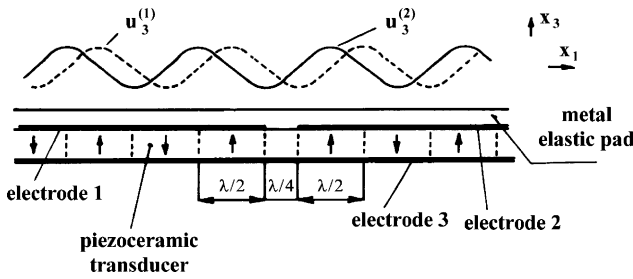


Fig. 1. Unrolled disc of piezoelectric ultrasonic slip motor.

This generated surface acoustic wave produces the contact between a stator and a rotor in sufficient number of line segments and drags the rotor [2].

For a better explanation of the principle we develop the ring of the stator to the line (Fig. 1). The stator structure consists of piezoceramic segments with length, half-wave length, of the elastic wave ( $\lambda/2$ ). The polarization direction of the segments is alternating, in Fig. 1 it is marked with arrows. Applying the dc voltage to the electrodes, the segments increase or decrease their thickness.

Exciting the electrodes 1, 3 and 2, 3 with harmonic voltages of a suitable frequency with phase shift  $\pi/2$  in time, two standing flexural deformations  $u_3^{(1)}$ ,  $u_3^{(2)}$  of the whole stator ring are generated with phase shift  $\lambda/4$  in space. They are a function of time and  $x_1$ , and we can describe them as:

$$u_3^{(1)} = A(x_3) \cos kx_1 \cos \omega t, \quad u_3^{(2)} = A(x_3) \sin kx_1 \sin \omega t. \quad (1)$$

The amplitude of the vibrations depends on  $x_3$ ,  $k$  is a wave number,  $\omega$  is an angular frequency of the exciting voltage and  $t$  is the time. Superimposing both displacements  $u_3^{(1)}$  and  $u_3^{(2)}$ , we obtain the final displacement  $u_3$ ,

$$u_3 = u_3^{(1)} + u_3^{(2)} = A(x_3) \cos k(x_1 - vt), \quad v = \omega/k, \quad (2)$$

where  $v$  is the velocity of traveling elastic wave in  $x_1$  direction. The propagating wave is generated by superimposing two standing waves, whose phases differ by  $\pi/2$  to each other both in time and in space.

### 3. Equations of motion of surface acoustic waves propagating along piezoceramic material

The surface acoustic waves satisfy the system of equations of motion [3]

$$\rho \frac{\partial^2 u_j}{\partial t^2} - c_{ijkl}^E \frac{\partial^2 u_k}{\partial x_i \partial x_l} - e_{kij} \frac{\partial^2 \varphi}{\partial x_k \partial x_i} = 0 \quad (3)$$

and the Maxwell equation  $\nabla \cdot D = 0$  in quasi-static approximation

$$e_{ikl} \frac{\partial^2 u_k}{\partial x_i \partial x_l} - \varepsilon_{ik}^S \frac{\partial^2 \varphi}{\partial x_i \partial x_k} = 0. \quad (4)$$

We consistently use the Einstein's summation rule to simplify the notation. The density of the transducer,  $\rho$ , is a material constant,  $c_{ijkl}^E$  are elastic stiffnesses for a constant electric field,  $e_{ijk}$  are the piezoelectric constants,  $\varepsilon_{ik}^S$  are the permittivities for a constant strain,  $u_j$  are the components of displacement in  $x_1$  direction and  $\varphi(t)$  is the time-varying scalar potential.

From (3) and (4) we take into consideration only those solutions, which describe the plane waves and characterise the movement of surface particles. The surface is not subject to the action of external elastic stresses and the relating boundary conditions are:

$$T_{31} = T_{32} = T_{33} = 0, \quad \text{due to } x_3 = 0. \quad (5)$$

Let us assume that the acoustic wave propagates in  $x_1$  direction and is a linear combination of partial waves, which are expressed as follows:

$$u_i = \alpha_i e^{ikbx_3} e^{jk(x_1 - vt)}, \quad \text{due to } i = 1-3, \quad (6)$$

$$\varphi = \alpha_4 e^{ikbx_3} e^{jk(x_1 - vt)}, \quad \text{due to } x_3 < 0.$$

The product  $kb$  denotes the coefficient of attenuation, which characterises the change of surface acoustic wave amplitude along  $x_3$  axis.

Partial waves, expressed by (6), must satisfy the equations of motion (3) and the electrical Eq. (4). A substitution from (6) into differential Eqs. (4) and (5) leads to a system of homogenous equations

$$(\Gamma_{ab} - \delta_{ab} \rho v^2) \alpha_a = 0 \quad \text{due to } a, b = 1-4, \quad (7)$$

where  $\delta_{ab}$  is the Kronecker symbol,  $\delta_{44} = 0$ .

Due to a high crystal symmetry of the piezoceramic transducer the coefficients have a simple form. The Eq. (7) yields a nontrivial solution for such a combination of coefficients  $b$  and velocity  $v$ , when the determinant of the system vanishes:

$$|\Gamma_{ab} - \delta_{ab} \rho v^2| = 0. \quad (8)$$

From (8) we obtain

$$(\Gamma_{22} - \rho v^2) \begin{vmatrix} \Gamma_{11} - \rho v^2 & \Gamma_{13} & \Gamma_{14} \\ \Gamma_{13} & \Gamma_{33} - \rho v^2 & \Gamma_{34} \\ \Gamma_{14} & \Gamma_{34} & \Gamma_{44} \end{vmatrix} = 0. \quad (9)$$

The solution, expressed by condition

$$\Gamma_{22} - \rho v^2 = c_{66}^F + c_{44}^E - \rho v^2 = 0, \quad (10)$$

characterises bulk shear waves in the plane parallel to the transducer surface and does not influence assumed deformations of the surface. That is why we take into consideration only the partial elastic waves  $u_1$  and  $u_3$ , taking  $u_2 = 0$ . The solution of the system is given as:

$$\begin{vmatrix} \Gamma_{11} - \rho v^2 & \Gamma_{13} & \Gamma_{14} \\ \Gamma_{13} & \Gamma_{33} - \rho v^2 & \Gamma_{34} \\ \Gamma_{14} & \Gamma_{34} & \Gamma_{44} \end{vmatrix} = 0. \quad (11)$$

Because  $\Gamma_{11}$ ,  $\Gamma_{33}$  and  $\Gamma_{44}$  are functions of  $b^2$ , the system of equations of motion is fulfilled by six partial waves for six coefficients  $b$ , which satisfy the defined phase velocity (11). We can express it as an equation of sixth degree of variable  $b$  in the form:

$$p_6 b^6 + p_4 b^4 + p_2 b^2 + p_0 = 0, \quad (12)$$

where  $p_n$  are real numbers. We are especially interested in that range of phase propagation velocity  $v$ , where (12) has three pairs of complex conjugated roots. We use only three roots in the bottom part of the complex plane, which satisfy the solution of surface acoustic wave,  $u_i = 0$  due to  $x_3 \rightarrow -\infty$ .

The phase propagating velocity of the surface wave and all three corresponding coefficients  $b(n)$  must satisfy not only the Eq. (8), but also the boundary conditions (5) and the condition of continuity of electrical displacement  $D_3$  on the transducer surface. Let us substitute for  $T_{ij}$

$$T_{ij} = c_{ijkl}^E \frac{\partial u_k}{\partial x_l} - e_{kij} \frac{\partial \varphi}{\partial x_k} = 0 \quad (13)$$

into (5) and assume, that the displacement components of surface particles of the transducer  $u_i$  and potential  $\varphi$  consist of linear combination of three partial waves,

$$u_i = \sum_{n=1}^3 C_n \alpha_i^{(n)} e^{jkb^{(n)}x_3} e^{jk(x_1-vt)}, \quad \text{due to } i = 1, 3, \quad (14)$$

$$\varphi = \sum_{n=1}^3 C_n \alpha_4^{(n)} e^{jkb^{(n)}x_3} e^{jk(x_1-vt)}. \quad (15)$$

The boundary conditions can then be expressed as a system of three homogenous equations:

$$q_{jn} C_n = 0, \quad q_{jn} = c_{3jkl}^E \alpha_k^{(n)} b_l^{(n)} + e_{l3j} \alpha_4^{(n)} b_l^{(n)}, \quad (16)$$

where  $b_1^{(n)} = 1$ ,  $b_2^{(n)} = 0$ ,  $b_3^{(n)} = b^{(n)}$  and  $\alpha_k^{(n)}$  and  $\alpha_4^{(n)}$  are components of Eigen-vectors corresponding to  $n$  values of coefficients  $b^{(n)}$ .

#### 4. The computation of traveling-wave velocity on the surface of piezoceramic drives

As follows from the analysis of (11), the roots of (12) are complex conjugated if the phase propagating velocity is smaller than the propagating velocity of bulk shear waves, it means if

$$v^2 < \frac{c_{44}}{\rho}. \quad (16)$$

For any selected propagating velocity  $v$ , which satisfies the condition (16), we calculate from (11) the coefficients  $p_n$  of (12). Using a suitable method of computation the roots  $b^{(n)}$

are derived. Further we compute from the equations:

$$\frac{\alpha_1^{(n)}}{\alpha_4^{(n)}} = \frac{\Gamma_{13}\Gamma_{34} - \Gamma_{14}\Gamma_{33}}{\Gamma_{11}\Gamma_{33} - \Gamma_{13}^2}, \quad \frac{\alpha_3^{(n)}}{\alpha_4^{(n)}} = \frac{\Gamma_{13}\Gamma_{14} - \Gamma_{11}\Gamma_{34}}{\Gamma_{11}\Gamma_{33} - \Gamma_{13}^2}. \quad (17)$$

for every root  $b^{(n)}$ , being in the bottom half of complex plane, the ratio of coefficients  $\alpha_i^{(n)}/\alpha_4^{(n)}$ . The propagating velocity and coefficients  $b^{(n)}$  fulfill the system of equations of boundary conditions (15), if the determinant of the system is equal to zero,

$$|q_{jn}| = 0. \quad (18)$$

By the calculation let us find such a velocity  $v$ , for which the determinant  $|q_{jn}|$  has the smallest value. For this velocity the ratio of amplitudes  $C_1/C_3$  and  $C_2/C_3$  can be determined as:

$$\frac{C_1}{C_3} = \frac{q_{12}q_{23} - q_{13}q_{22}}{q_{11}q_{22} - q_{12}q_{21}}, \quad \frac{C_2}{C_3} = \frac{q_{13}q_{21} - q_{11}q_{23}}{q_{11}q_{22} - q_{12}q_{21}}. \quad (19)$$

#### 5. Design of piezoelectric ultrasonic slip motor

For the rotor diameter 50 mm the frequency of exciting voltage was selected to be 40 kHz. The applied piezoelectric ceramic PKM 15A has following parameters:  $c_{44} = 3.0 \times 10^{10} \text{ N m}^{-2}$ ,  $\rho = 7900 \text{ kg m}^{-3}$  and  $\sqrt{c_{44}/\rho} = 1940 \text{ m s}^{-1}$ .

Using the material constants the phase velocity of propagating surface wave  $v = 1480 \text{ m s}^{-1}$  was calculated. The computed ratio of displacement amplitudes in  $x_3$  and in  $x_1$  direction is 1.65 [4].

#### 6. Ultrasonic piezoceramic motor driven by sinusoidal signal

The present state of art is drifting by the two harmonic signals with  $\pi/2$  phase shift and constant frequency. The setting of frequency is identical with electric reverb of the whole exciting circuit including inductance, then the rotation velocity depends on an amplitude of driving voltage only. The schematic principle of converter for motor is shown in Fig. 2.

There is the question. Will the piezoceramic drive be moved by nonharmonic signals?

For the purposes we created workplace, which is configurational from complex programmable logic devices with programming language VHDL (Very High Speed Integrated Circuits using Hardware Description Language) and Digital Signal Processing Analog Devices 21061 (DSP AD). By the workplace we can put selected input signals on the piezoelectric drives. This way of testing is easily alterable.

The basic driving signal requirements are:

- the 1st harmonic components of driving signal must be constant;

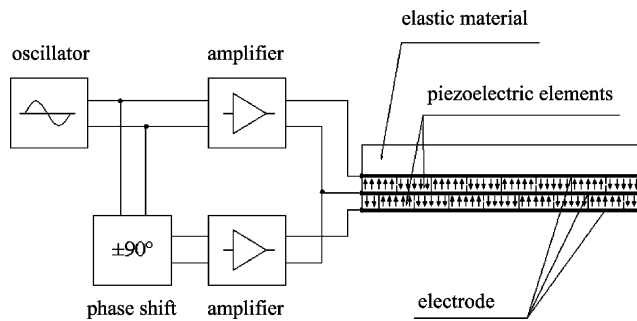


Fig. 2. Principle of sinusoidal drives for ultrasonic piezoceramic motor.

- the driving signal suitably approximates sinusoidal waveform;
- the higher harmonic components are much lower;
- the mean value must be easily controllable due for setting of rotation speed;
- the signal is functionally and easily generated if changes of switching impulsions are equidistant in time.

## 7. PWM sinusoidal generator with minimal content of higher harmonics

The electromagnetic drives (induction motors) are driven by the pulse width modulated (PWM)-signals. Our lab has good experiences with this method and for that reason we tried to use the same way of controlling on the piezoceramic drives [5].

Our aim is to substitute driving harmonic signal for unipolar or bipolar signals. The switching signals have to be simple generated by digital logical circuits.

The unipolar signals were designed with 12, 24 and 36 samples.

- (1) [011110100001]
- (2) [01011111010101000000101]
- (3) [0010111111111010011010000000001011]

We can see that signals are symmetric in half-period (Fig. 3).

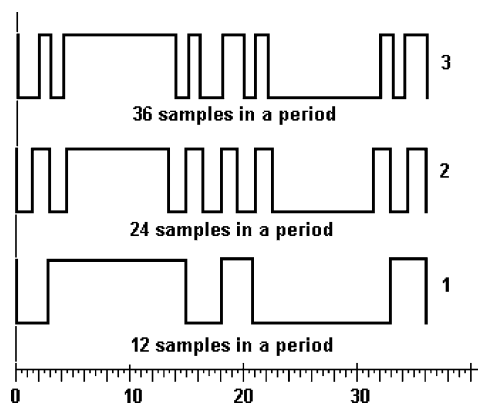


Fig. 3. Unipolar PWM signals with the same period but with different number of samples.

Table 1

Relative harmonic component amplitudes of signals with regard to the fundamental lemniscates

Harmonic compound	Amplitude (%)			
	Signal number			
	(1)	(2)	(3)	Rectangular
1st	100.00	100.00	100.00	100.00
3rd	0	0	0	33.33
5th	20.01	10.36	5.89	20.00
7th	14.31	23.28	4.66	14.29
9th	0	0	0	11.11
11th	9.12	28.63	15.79	9.09

Table 1 presents the relative amplitudes of individual harmonic components. Each waveform (1–3) is set in the Complex Programmable Logic Device (CPLD) as the list of figure in one period.

We can see the principle of PWM generator with the fixed setting of waveform in Fig. 4. In this instance the PWM signals are recorded into the Field Programmable Gate Array (FPGA). Simulation of the generator was accomplished by the MAX + PLUS II system and then set into circuit Lattice ispLSI 1016 (in-system programmability, Large Scale Integration). The PWM signals with  $\pi/2$  phase shift are out on the  $X_1$  and  $X_2$  pins [6,7].

The FPGA circuit creates PWM sinusoidal driving signal with minimal contents of harmonic components. The creation of the PWM lemniscates puts high requirements on the switching quality of the power transistors. For example we use the piezomotor with 75 kHz reverb frequency in our lab. The maximal number of samples in one period is determined by the frequency  $f_{XTAL}$  of the crystal oscillator build in Altera board and by the reverb frequency of the piezomotor  $f_R$ ,

$$\text{Number of samples} = \frac{f_{XTAL}}{f_R} = \frac{25175 \times 10^6}{75 \times 10^3} = 3356 \approx 336. \quad (20)$$

It follows that fundamental driving period of the motor is possible to divide into 336 samples maximally. We can use relatively simple FPGA circuit in this case. Optimal design of the three waveforms with 336 samples in period utilises 60 logic cells. The period length is 13.3  $\mu$ s, therefore fast switching power MOSFET transistors are used. Switching time of the transistors must be less than 4 ns.

Bipolar signal takes 0 and  $\pm 1$  level and it is described with the following digital dates:

$$(4) [0 + 1 + 1 + 1 + 100 - 1 - 1 - 1 - 10]$$

For application of bipolar driving signal it is necessary to switch the contacts of the piezomotor in reversed polarity.

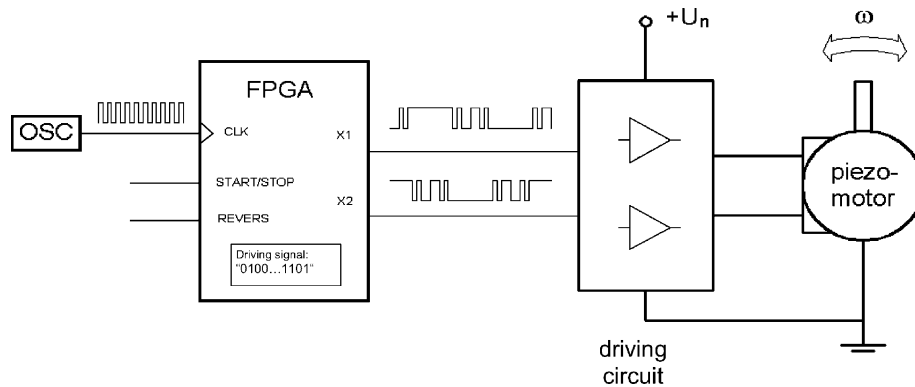


Fig. 4. Principle of PWM generator controlling with the fixed setting of waveform.

## 8. Generator of universal unipolar waveform

The options of programmable field FPGA are limited and for that reason we prepare new design of waveform controlling. The FPGA circuit is a logic unit only and parameters of waveforms are sent from digital signal processor DSP–AD. The reading dates from DSP are possible in several ways. We suppose parallel communication. This generator takes advantages of the digital signal processor 21061 produced by analog devices.

## 9. Conclusion

The theoretical evaluation of the electromechanical conversion process, optimisation of design and assessment of the possibility of a more powerful ultrasonic motor are being undertaken. The suggested calculation of the surface traveling-wave velocity and of the ratio of displacement amplitudes can be performed for any piezoelectric material with known material constants.

The piezoelectric ultrasonic slip motor with the piezoceramic transducer, using the elastic wave propagating along the stator surface, has relatively high efficiency of about 50%. The merits are compact size, simple design and easy production process. It is suitable for output powers of 10 W. The size of the transducer depends on a required power. The revolutions are depended on transducer diameter and frequency of the driving voltage. The nominal revolutions are  $600 \text{ min}^{-1}$  typically.

The simple unipolar signals (Table 1) were harmonic analysed and verified on the piezomotor. It seems that the harmonic suppression is sufficient. The two versions of control circuits were designed (see above) and tested with the piezomotor. The use of FPGA circuit is necessary for the reason of high working frequency in any case. The DSP calculates and sets up the coefficients only. The driving waveform is generated by the FPGA.

The piezoceramic actuators represent a capacitance load in the first instance and towards driving signals they behave as

a circuit of derived function. The application of high-quality fast switching power MOSFET transistors and power switching regulator reduces the deformation of driving signals.

## Acknowledgements

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