

# Crystallographic characterization of silicon nitride ceramics sintered with $Y_2O_3$ – $Al_2O_3$ or $E_2O_3$ – $Al_2O_3$ additions

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## Abstract

Silicon nitride ceramics were sintered using  $Y_2O_3$ – $Al_2O_3$  or  $E_2O_3$ – $Al_2O_3$  ( $E_2O_3$  denotes a mixed oxide of  $Y_2O_3$  and rare-earth oxides) as sintering additives. The intergranular phases formed after sintering was investigated using high-resolution X-ray diffraction (HRXRD). The use of synchrotron radiation enabled high angular resolution and a high signal to background ratio. Besides the appearance of  $\beta$ - $Si_3N_4$  phase the intergranular phases  $Y_3Al_5O_{12}$  (YAG) and  $Y_2SiO_5$  were identified in both samples. The refinement of the structural parameters by the Rietveld method indicated similar crystalline structure of  $\beta$ - $Si_3N_4$  for both systems used as sintering additive. On the other hand, the intergranular phases  $Y_3Al_5O_{12}$  and  $Y_2SiO_5$  shown a decrease of the lattice parameters, when  $E_2O_3$  was used as additive, indicating the formation of solid solutions of  $E_3Al_5O_{12}$  and  $E_2SiO_5$ , respectively.

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## 1. Introduction

Silicon nitride ceramics have been intensively investigated, due to its elevated thermo-mechanical properties such as creep, corrosion and oxidation resistance [1,2]. One of the difficulties found in the fabrication process is the sintering to attain high relative densities. Therefore, the use of additives to form a liquid phase and to promote densification by particle rearrangement and solution-precipitation processes is required. On the other hand, the additives form a secondary intergranular glassy phase after sintering, which decrease the high-temperature properties of these materials. The crystallization of these additives into refractory phases may reduce this problem [1–3].

The additive systems mostly employed to the sintering of  $Si_3N_4$  are  $AlN$ – $Y_2O_3$  and  $Al_2O_3$ – $SiO_2$ , for the formation of

$SiAlONs$  or solid solutions of  $Si_3N_4$  [1,4,5]. Additive systems of  $Y_2O_3$ – $SiO_2$  and  $Al_2O_3$ – $Y_2O_3$  are also used in several molar fractions and may be crystallized into phases such as:  $Y_2SiO_5$ ,  $Y_2SiO_7$ ,  $YSiO_2N$ ,  $Y_2Si_3N_4O_3$  (mellilite) or  $Y_3Al_5O_{12}$  (YAG – yttrium aluminum garnet) by an appropriate heat treatment after sintering. These crystalline phases exhibit good mechanical properties and a high crack propagation resistance, which contribute to increase the fracture toughness of the sintered ceramics [1,6].

It has been shown in previous works [7–10] that the mixed oxide,  $E_2O_3$ , composed by yttrium and rare-earth oxides, is a potential substitute for pure  $Y_2O_3$  to the sintering of  $Si_3N_4$  ceramics, resulting in materials with similar physical, chemical and mechanical properties. It has also been found that the mixed oxide is a solid solution composed mainly by  $Y_2O_3$  (44 wt.%),  $Yb_2O_3$  (17 wt.%),  $Er_2O_3$  (14 wt.%) and  $Dy_2O_3$  (11 wt.%) and other rare-earth oxides in fractions smaller than 3 wt.%. The mixed oxide crystallizes in a  $Y_2O_3$ -type structure [10].

Recently, the mechanical properties of  $Si_3N_4$ -ceramics sintered with  $Al_2O_3$ – $Y_2O_3$  or  $Al_2O_3$ – $E_2O_3$  additions have been investigated [9]. The results of the both ceramic materials

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prepared using these additives were quite similar in terms of densification, microstructure and mechanical properties, such as hardness, flexural strength and fracture toughness. For the  $\alpha$ -SiAlONs prepared using these additives, similar characteristics of microstructure and mechanical behavior were detected in the materials prepared with  $\text{Al}_2\text{O}_3$ – $\text{Y}_2\text{O}_3$  or  $\text{Al}_2\text{O}_3$ – $\text{E}_2\text{O}_3$  [8,10].

The aim of this work was the investigation of the crystalline intergranular phases formed into  $\text{Si}_3\text{N}_4$  ceramics sintered with  $\text{Al}_2\text{O}_3$ – $\text{Y}_2\text{O}_3$  or  $\text{Al}_2\text{O}_3$ – $\text{E}_2\text{O}_3$  mixtures as additives. These analyses were carried out using high-resolution X-ray diffractometry with synchrotron light as source. This setup provides a high-resolution determination of lattice parameters and a high intensity of the X-rays into the samples. These characteristics allow the investigation of small variations in the crystal structures of  $\text{Si}_3\text{N}_4$  due to the incorporation of  $\text{Al}^{3+}$  or  $\text{O}^{2-}$  ions from the additives and also the detection of small amounts of intergranular phases.

## 2. Experimental procedure

### 2.1. Sample preparation

For the preparation of the powder batches,  $\alpha$ - $\text{Si}_3\text{N}_4$  (UBE Industries, E-10),  $\alpha$ - $\text{Al}_2\text{O}_3$  (Baikalox, 99.9%),  $\text{Y}_2\text{O}_3$  (H.C. Starck, 99.98%) and  $\text{E}_2\text{O}_3$  (DEMAR-FAENQUIL, 99.98%) powders were used. The batches were prepared by mixture of 86 vol.%  $\text{Si}_3\text{N}_4$  and 14 vol.% of the following additive mixtures:  $\text{Y}_2\text{O}_3$ – $\text{Al}_2\text{O}_3$  (SNYA sample) or  $\text{E}_2\text{O}_3$ – $\text{Al}_2\text{O}_3$  (SNEA sample). The fraction of the additives was calculated to form  $\text{Y}_3\text{Al}_5\text{O}_{12}$  as intergranular phase, namely, 40 mol% of  $\text{Y}_2\text{O}_3$  (or  $\text{E}_2\text{O}_3$ ) and 60 mol% of  $\text{Al}_2\text{O}_3$ . The compositions of the prepared powder batches in wt.% are listed in Table 1. The different weight-percentage of the additives used are due to the different densities of  $\text{Y}_2\text{O}_3$  ( $5.01 \text{ g cm}^{-3}$ ) and  $\text{E}_2\text{O}_3$  ( $6.51 \text{ g cm}^{-3}$ ), as determined by He-pycnometry.

The batches were prepared by ball-milling at 500 rpm for 6 h in isopropilic alcohol. The suspension was dried in a rotary evaporator and the put inside an oven at  $100^\circ\text{C}$  for 24 h. After drying, the powders were sieved for deagglomeration. Green bodies were prepared by uniaxial pressing in a steel die (50 MPa) and isostatic pressing under 300 MPa.

Sintering of the samples was done in a furnace with graphite resistance putting on a graphite crucible with a powder bed composed of  $\text{Si}_3\text{N}_4$  (70 wt.%) and BN (30 wt.%) powders. The samples were heated with a rate of  $10^\circ\text{C/min}$  up to  $1900^\circ\text{C}$  in 1.5 MPa nitrogen atmosphere. The soaking time was 2 h at  $1900^\circ\text{C}$ . In order to crystallize the intergranular glassy phase,

the samples were heat-treated at  $1400^\circ\text{C}$  for 24 h under 0.1 MPa nitrogen atmosphere [11].

The density of the samples was determined by the immersion method in distilled water, according to the Archimedes' principle.

### 2.2. Phases analysis

For each compound, three sintered samples were crushed into powders and homogenized, for high-resolution X-ray diffraction (HRXRD) analysis. The powders were sieved to select particles smaller than  $20 \mu\text{m}$ . The analyses were done at the Brazilian Synchrotron Light Laboratory (LNLS) at Campinas-Brazil, using the XRD1-D12A beamline. A Hubber diffractometer with multiple-axis was used. The measurements were conducted with a configuration of two coupled concentric circles ( $\omega$ – $2\theta$ ), using a monochromatic beam of 10 keV ( $\lambda = 1.2398 \text{ \AA}$ ) energy. The powders were carefully pulverized on a sample holder and it was rotated to improve the random orientation of the crystalline planes. The diffracted beam was collected using a (2 0 0) oriented Ge analyzer crystal and scintillation detector. The patterns were obtained at room temperature in q-space ( $q = 4\pi\lambda^{-1} \sin \theta = d^{-1}$ ) in order to optimize the number of points for each peak. The data were collected in the interval of  $0.50 \leq q \leq 6.50$ , with 0.01 steps using 1 s of counting time. After the data acquisition,  $q$  was converted to  $2\theta$ . For each compound 2 measurements were conducted, to guarantee reproducibility of the analysis; the standard deviation of the calculated lattice parameters has been smaller than  $0.007 \text{ \AA}$ , see Table 2.

The simulations of the diffractograms were performed using the Powdercell software [12]. The structural parameters as lattice parameters and atomic positions of the crystal structures were refined by the Rietveld method using the FullProf computer program [13].

### 2.3. Microstructure

The microstructure of the sintered samples was observed by scanning electron microscopy (SEM LEO-1450VP, England) equipment with secondary electron detector. The samples were cut and polished. The surface of the samples was plasma-etched.

The grain size distribution and the aspect ratio were determined by the method proposed by Wötting et al. [14], which is based on the measurements of the length and width of 10% of the largest grains observed in the cross-section analyzed.

Table 1  
Composition of the samples

Composition	(In wt.%)			
	$\text{Si}_3\text{N}_4$	$\text{Al}_2\text{O}_3$	$\text{Y}_2\text{O}_3$	$\text{E}_2\text{O}_3$
SNYA (86 vol% $\text{Si}_3\text{N}_4$ + 14 vol% $\text{Y}_2\text{O}_3$ – $\text{Al}_2\text{O}_3$ )	81.4	8.0	10.6	–
SNEA (86 vol% $\text{Si}_3\text{N}_4$ + 14 vol% $\text{E}_2\text{O}_3$ – $\text{Al}_2\text{O}_3$ )	78.8	7.6	–	13.6

Table 2  
Lattice parameters and volume of  $\text{Y}_2\text{SiO}_5$  and  $\text{E}_2\text{SiO}_5$

	$\text{Y}_2\text{SiO}_5$ (SNYA)	$\text{E}_2\text{SiO}_5$ (SNCA)
$a$ (Å)	12.439(7)	12.408(8)
$b$ (Å)	6.751(5)	6.733(4)
$c$ (Å)	10.414(7)	10.395(7)
$\beta$ (°)	102.65(5)	102.63(5)
$V$ (Å <sup>3</sup> )	853.3(9)	847.3(9)

The number in the parenthesis denotes standard deviation.

### 3. Results and discussion

#### 3.1. Materials characterization

The relative density of the silicon nitride ceramics prepared using  $Y_2O_3-Al_2O_3$  as additive (SNYA) and  $E_2O_3-Al_2O_3$  as additive (SNEA) after sintering was found to be near 99.5% of the theoretical density for both samples. Furthermore, both materials exhibited similar mechanical properties, as hardness of 13 GPa, flexural strength around 680 MPa and fracture toughness of 6.7 MPa m<sup>1/2</sup> [9].

#### 3.2. Phase analysis

The X-ray diffraction patterns of SNYA and SNEA samples after sintering are shown in Fig. 1(a) and (b), respectively. In both diffractograms, narrow and well-defined peaks are present, indicating crystalline phases. The predominant matrix phase is constituted by  $\beta-Si_3N_4$ , as can be verified by comparison with the simulated diffraction pattern of pure  $\beta-Si_3N_4$ , shown in Fig. 1(c). Further low intensity diffraction peaks were attributed to the crystalline intergranular phases.

The refinement of the lattice parameters of the  $\beta-Si_3N_4$  phase was done using the Rietveld method, based on the literature data: hexagonal system, space group  $P6_3$ ,  $a = 7.595(1)\text{\AA}$  and  $c = 2.9023(6)\text{\AA}$ , and the atomic positions according to Grün [15]. The refined lattice parameters for the

$\beta-Si_3N_4$  phase were:  $a = 7.6093(5)\text{\AA}$  and  $c = 2.9134(3)\text{\AA}$  for the SNYA sample and  $a = 7.6063(7)\text{\AA}$  and  $c = 2.9124(5)\text{\AA}$  for the SNEA sample. These values, slightly higher than that reported in the literature indicate a possible dissolution of small amount of  $Al^{3+}$  and/or  $O^{2-}$  of the additives into the  $Si_3N_4$  structure. The refined atomic positions for both samples SNYA and SNEA were very close to the reported values. Thus, the structure of  $\beta-Si_3N_4$  for both samples studied, SNYA and SNEA, were similar, indicating that the different additives used, e.g. respectively  $Y_2O_3-Al_2O_3$  and  $E_2O_3-Al_2O_3$  lead to similar materials.

The intergranular phases were identified by expanding the y-axis of the X-ray diffraction patterns, as can be seen in the Fig. 2.

Two intergranular crystalline phases have been identified:  $Y_3Al_5O_{12}$  (YAG) and  $Y_2SiO_5$ . The  $Y_2SiO_5$  phase, not expected, could be formed by the reaction of  $SiO_2$ , inherently present on the surface of the  $Si_3N_4$  particles, and  $Y_2O_3$ . It should be mentioned that all diffraction peaks, belonging to  $\beta-Si_3N_4$ , as to the intergranular phases appear in the both materials prepared, indicating that the same phases have been formed, independently of the additive system employed,  $Al_2O_3-Y_2O_3$  (SNYA) or  $Al_2O_3-E_2O_3$  (SNEA).

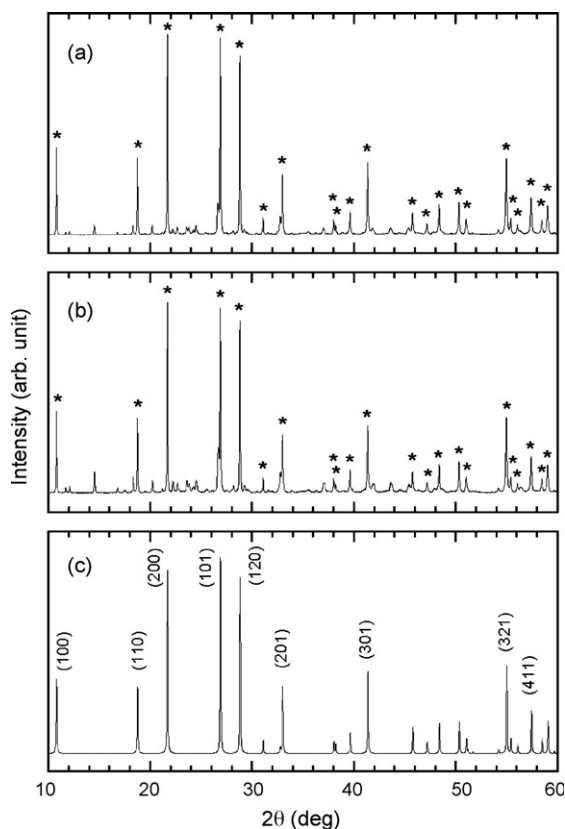


Fig. 1. X-ray diffractograms of (a) SNYA and (b) SNEA samples. In (c) a simulation of  $\beta-Si_3N_4$  phase is shown. The symbol (\*) indicates the peaks belonging to  $\beta-Si_3N_4$  phase.

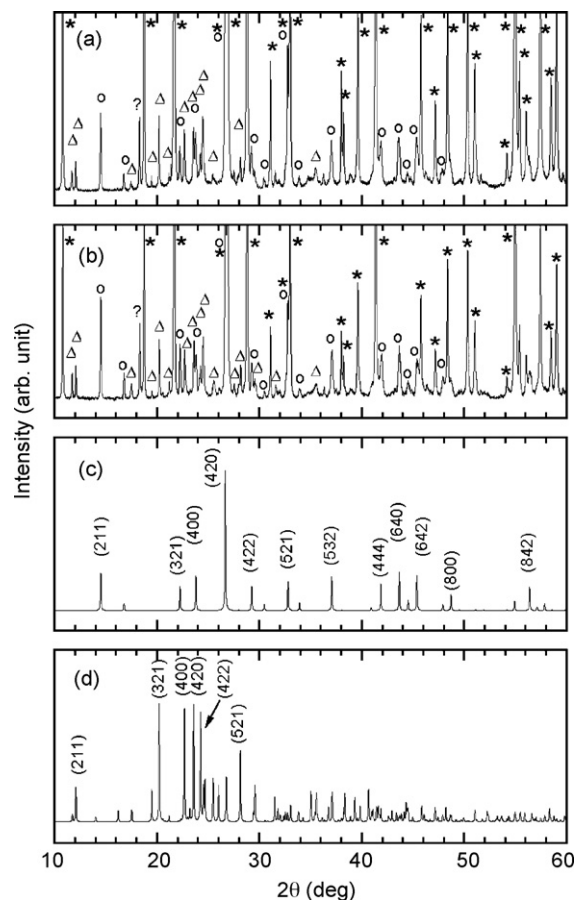


Fig. 2. Expanded X-ray diffractograms of (a) SNYA and (b) SNEA samples. In (c) and (d) simulation of  $Y_3Al_5O_{12}$  and  $Y_2SiO_5$  phases are shown, respectively. The peaks were identified by the symbols (\*):  $\beta-Si_3N_4$ , (o):  $Y_3Al_5O_{12}$  and ( $\Delta$ ):  $Y_2SiO_5$ .



The structural parameters of the  $Y_3Al_5O_{12}$  phase were refined by the Rietveld method, based on the literature data: cubic structure, space group  $1a3d$ ,  $a = 12.000$  Å, and the atomic positions according to Euler and Bruce [16]. The lattice parameters determined by the refinement were:  $a = 12.023(3)$  Å for SNYA and  $a = 12.004(3)$  Å for SNEA. The lattice parameter of this phase shown a slight variation, depending on the additive system used. The decrease of  $0.02$  Å in the lattice parameter of the YAG phase in the SNEA sample in comparison to that of SNYA sample indicates the formation of a solid solution,  $E_3Al_5O_{12}$ . This interpretation is supported by the difference found in the smaller lattice parameter of the solid solution  $E_2O_3$  ( $a = 10.588(1)$  Å) when compared to that of the pure  $Y_2O_3$  ( $a = 10.6080(1)$  Å) [10]. The refinement of the oxygen atomic positions of  $Y_3Al_5O_{12}$  in SNYA and SNEA samples were almost unchanged, presenting values very close to the reported ones.

The appearance of  $Y_2SiO_5$  phase in both samples was confirmed by the comparison to the JCPDS-ICDD files (36–1476) [17]. There are known two polymorphs for  $RE_2SiO_5$  phase ( $RE = Y$  or rare-earth atom) [18]: the so-called  $X_1$  phase (monoclinic, space group  $P2_1/c$ ), stable at room temperature for  $RE = La$  to  $Gd$  and  $X_2$  phase (monoclinic, space group  $C2/c$ ), stable at room temperature for  $RE = Dy$  to  $Lu$ . They are also known as high-temperature phase ( $X_1$  phase) and low-temperature phase ( $X_2$  phase). The  $Y_2SiO_5$  phase appearing in both samples SNYA and SNEA was identified to exhibit the  $X_2$ -type structure.

The crystal structure of the  $X_1$  phase has been recently investigated by Wang et al. [18]. On the other hand, very few data about the  $X_2$  phase are available in the literature. The lattice parameters for the  $Y_2SiO_5$  phase were reported in JCPDS-ICDD file (36–1476) [17]:  $a = 12.50$  Å,  $b = 6.728$  Å,  $c = 10.42$  Å and  $\beta = 102.68^\circ$ . The initial atomic positions for the refinement of the  $Y_2SiO_5$  phase were based on the data of  $Lu_2SiO_5$  phase reported by Gustafsson [19], since no data were found for  $Y_2SiO_5$  and also based on a similar monoclinic symmetry for both compounds. Thus, the refined lattice parameters and unit cell volume ( $Z = 8$ ) for  $Y_2SiO_5$  are shown in Table 2. Note that the lattice parameters for  $E_2SiO_5$  phase in the SNEA sample are smaller than that of SNYA sample. This is an indication of the formation of a solid solution,  $E_2SiO_5$ , where Y atomic sites are partially substituted by rare-earth atoms, such as Yb, Er, Dy, etc. The

atomic positions refined for  $Y_2SiO_5$  are shown in the Table 3. The atomic positions refined for  $E_2SiO_5$  were found to be similar to that of  $Y_2SiO_5$ .

The refinement taking account three phases show good final reliability factors, calculated as defined in the literature [20]:  $R_p = 13.8\%$ ,  $R_{wp} = 18.0\%$  and  $R_{exp} = 6.59\%$  for SNYA sample and  $R_p = 13.8\%$ ,  $R_{wp} = 18.0\%$  and  $R_{exp} = 6.59\%$  for SNEA sample. The Bragg reliability factors calculated for each phase were:  $R_B = 4.42\%$  ( $\beta-Si_3N_4$ ),  $9.37\%$  ( $Y_3Al_5O_{12}$ ) and  $18.6\%$  ( $Y_2SiO_5$ ) for SNYA sample and  $R_B = 5.55\%$  ( $\beta-Si_3N_4$ ),  $10.2\%$  ( $E_3Al_5O_{12}$ ) and  $18.4\%$  ( $E_3Al_5O_{12}$ ) for SNEA sample. The phase fractions calculated in vol.% for each sample were:  $\beta-Si_3N_4$  (87.6%),  $Y_3Al_5O_{12}$  (7.0%) and  $Y_2SiO_5$  (5.4%) for SNYA sample and  $\beta-Si_3N_4$  (81.3%),  $E_3Al_5O_{12}$  (10.1%) and  $E_2SiO_5$  (8.6%) for SNEA sample.

### 3.3. Microstructure

The SEM micrographs of the sintered samples, SNYA and SNEA, are shown in Fig. 3.

The microstructures of both samples are very similar and are composed of elongated  $\beta-Si_3N_4$  grains (dark region) with a grain size of about  $2 \mu m$  with an aspect ratio of 6, surrounded

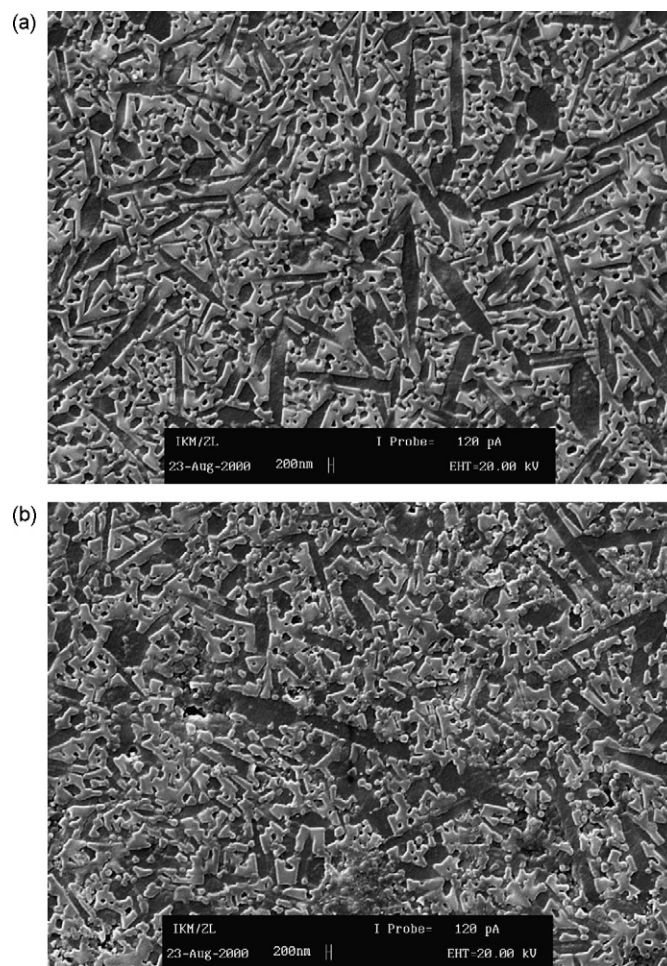


Fig. 3. SEM micrographs of the sintered samples: (a) SNYA and (b) SNEA after heat-treatment at  $1400^\circ C$  for 24 h in  $0.1$  MPa of  $N_2$ .

Table 3  
Fractional coordinates for  $Y_2SiO_5$

Atom	Wyckoff site	x	y	z
Y(1)	8f	0.501(7)	0.799(6)	0.483(7)
Y(2)	8f	0.288(4)	0.947(9)	0.671(5)
Si	8f	0.27(1)	0.43(2)	0.59(1)
O(1)	8f	0.21(2)	−0.04(5)	0.57(3)
O(2)	8f	0.02(2)	0.56(3)	0.69(2)
O(3)	8f	0.29(2)	0.24(4)	0.71(3)
O(4)	8f	0.45(2)	0.59(3)	0.70(2)
O(5)	8f	0.13(2)	0.92(4)	0.38(2)

The number in the parenthesis denotes standard deviation.

by the intergranular phase (clear region). In the clear region it is not possible to identify the multiple phases probably due to low contrast exhibited by close composition of the intergranular phases.

#### 4. Conclusions

Silicon nitride ceramics were sintered using  $Y_2O_3$ – $Al_2O_3$  (SNYA) or  $E_2O_3$ – $Al_2O_3$  (SNEA) as additive. In both samples the same phases were identified:  $\beta$ - $Si_3N_4$ ,  $Y_3Al_5O_{12}$  and  $Y_2SiO_5$ . The crystal structures of  $\beta$ - $Si_3N_4$  formed using both additives exhibit were similar lattice parameters. In the sample SNEA it was detected the formation of solid solutions of  $E_3Al_5O_{12}$  and  $E_2SiO_5$ , isostructural to respectively  $Y_3Al_5O_{12}$  and  $Y_2SiO_5$ .

The similarity in the physical properties for both SNYA and SNEA samples show that the silicon nitride ceramics could be produced using  $E_2O_3$ – $Al_2O_3$  as an alternative sintering additive.

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