

# The temperature effect on the resonant spectra of thin piezoelectric ceramic disk

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## Abstract

This study examines the influence of the temperature on the low-frequency resonant spectrum of piezoelectric ceramics based on lead zirconate titanate solid solution  $\text{Pb}(\text{Zr,Ti})\text{O}_3$ -based piezoelectric ceramics with heat variations ranging from 25 to 150 °C, concluding that temperature increases cause the resonant spectrum to shift to higher frequencies. Furthermore, the variation in fundamental series resonant frequency  $f_s$  (with temperature variation) is larger than the variation of fundamental parallel resonant frequency  $f_p$  (with temperature variation). Another result of increased temperature is the decrease in effective electromechanical coupling factor  $k_{\text{eff}}$ . Finally, the effect of temperature on the overtone mode is more obvious than that of the fundamental mode.

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## 1. Introduction

After their discovery in 1950, researchers have intensively studied lead zirconate titanate [ $\text{Pb}(\text{Zr,Ti})\text{O}_3$ ; PZT] ceramics because of their excellent piezoelectric properties [1–4]. PZT piezoelectric ceramics are widely used in electronics, such as resonator, frequency control devices, filters, transducers, and sensors. Previous to their use in electronic devices, the characteristics of piezoelectric ceramics were usually measured at room temperature through resonance measurement, as outlined in the IEEE Standard on Piezoelectricity [5]. In many contemporary applications, however, piezoelectric ceramics are used in devices that may be operated at temperatures either above or below room temperature. Subsequently, it is necessary to study the behavior of piezoelectric ceramics over a wide range of

possible operating temperatures to predict the responses of the devices.

For devices such as resonators, resonant frequency and temperature stability are important parameters. Although the temperatures coefficients of resonant frequencies are known to be dependent on the material properties of resonator material as well as on the resonator dimensions and vibration mode [6], the temperature effects on the electromechanical properties are less well-understood. However, some data obtained from resonance measurements [7,8] are available on the temperature variation of low field properties such as piezoelectric coefficients.

The piezoelectric ceramic disk is the most convenient shape for easy duplication for use in the production of radial resonators that have a frequency range of 30 kHz to above 5 MHz [1]. In addition to the fundamental mode, overtone modes also exist in radial vibration, although their characteristics, as well as the influence of temperature on the radial overtone vibration mode, are rarely discussed. Thus, this study examines the influence of temperature on the resonant spectrum of piezoelectric ceramic disks having a high diameter to thickness ratio and frequency range from 100 kHz to 1 MHz in this paper.

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## 2. Experimental process

This study used hard type piezoelectric ceramic thin disks supplied by Eleceram Co., Ltd., Taiwan. Ceramic compositions were in the vicinity of the MPB of PZT in the tetragonal range, and were doped with minor  $\text{MnO}_2$  and  $\text{Sb}_2\text{O}_3$ . Samples had a diameter of 15 mm, a thickness of 0.9 mm, and a diameter to thickness ratio of 16.67.

Polishing, cleaning and coating the flat surfaces of the samples with silver electrode facilitate Ohmic contact for measurement of electric properties. Next, we poled samples under a high electric field (3.0 kV/mm) at a temperature of 100 °C for 1 h in a silicone oil bath. After polarization, we surveyed piezoelectric properties with an HP4194A Impedance/Gain-Phase Analyzer based on the IEEE standards [1], and then determined the samples' resonant spectrum at a frequency range from 100 kHz to 1 MHz, recording all results with a PC-based data acquisition system. Immersing samples into a tank filled with silicon oil revealed the temperature dependence of the resonant spectrum. We controlled the temperature of the silicon oil with a resolution of 0.1 °C using a temperature controller consisting of a thermocouple sensor and a controllable metallic heater. Because the silicon oil used in this study has a boiling point of 185 °C, for safety reasons, the highest temperature attained in this test was 150 °C.

## 3. Results and discussions

Fig. 1 shows the resonant spectrum of the unloaded piezoelectric ceramic thin disk at a frequency range from 100 kHz to 1 MHz at 25 °C. Four vibration modes are in this frequency range. The first is fundamental radial vibration mode, and then three others are radial overtone modes with inharmonic frequency separation.

The characteristic equation of resonant frequencies for radial vibration modes is [6]

$$\eta J_0(\eta) = (1 - \sigma) J_1(\eta) \quad (1)$$

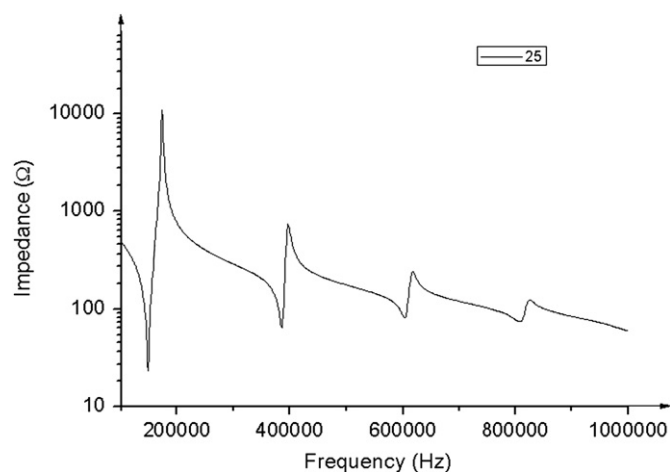


Fig. 1. The resonant spectrum of the unloaded piezoelectric ceramic thin disk in the frequency range from 100 kHz to 1 MHz at 25 °C.

where  $J_0$  is the Bessel function of the first kind and zero order,  $J_1$  is the Bessel function of the first kind and first order,  $\eta$  is the frequency constant and  $\sigma$  is the Poisson's ratio.

For resonance condition (obtained for minimum resonators impedance) for homogeneously poled ceramic resonators, is

$$\eta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \dots \quad (2)$$

The series resonant frequency for the  $n$ th vibration mode of the piezoelectric ceramic thin disk in the radial vibration mode can be expressed as follows [3]:

$$f_{(sn)} = \frac{\eta_n}{2\pi r} \sqrt{\frac{1}{s_{11}^E \rho (1 - \sigma^2)}} \quad (3)$$

When considering piezoelectric ceramic thin disks in radial vibration mode, the ratio between the fundamental series resonance frequency and the overtones' series resonance frequencies is inexpressible as integers. Instead, the values can be articulated as  $\eta_n/\eta_1$ . Fig. 2 shows the relation between mode orders to the frequency constant  $\eta_n$  and frequency constant ratio  $\eta_n/\eta_1$ . The frequency constant  $\eta_n$  and frequency constant ratio  $\eta_n/\eta_1$  both in conjunction with the mode order, but the slope of the frequency constant ratio  $\eta_n/\eta_1$  curve is less than that of the frequency constant  $\eta_n$  curve. This means that the separation between the types of vibration modes decreases with the increase of mode order.

Fig. 3 shows the variation of fundamental radial vibration mode for piezoelectric ceramic thin disks with temperature. Both the fundamental series resonant frequency  $f_s$  and the fundamental parallel resonant frequency  $f_p$  increase in conjunction with temperature, as shown in Fig. 4. Fundamental series resonant frequency  $f_s$  and fundamental parallel resonant frequency  $f_p$  relationships with temperature can be fitted by the second order polynomial, respectively, as follows:

$$f_s = 147.04 + 0.02126(T) + 1.3714 \times 10^{-4}(T^2) \quad (4)$$

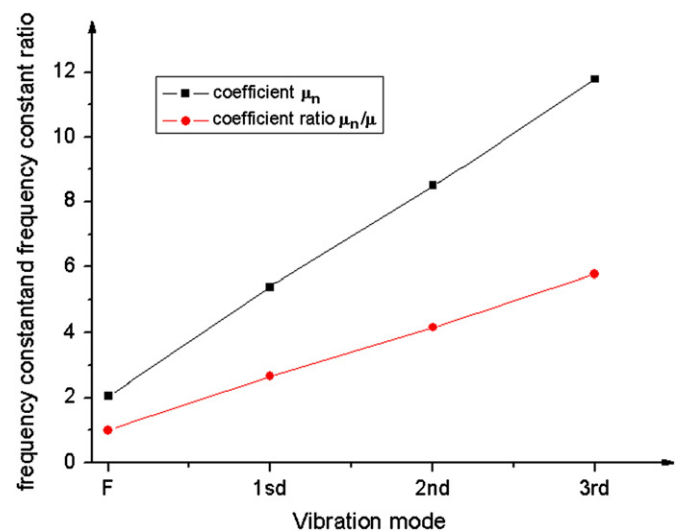


Fig. 2. The relationship between mode orders to the frequency  $\eta_n$  and the constant ratio  $\eta_n/\eta_1$ .

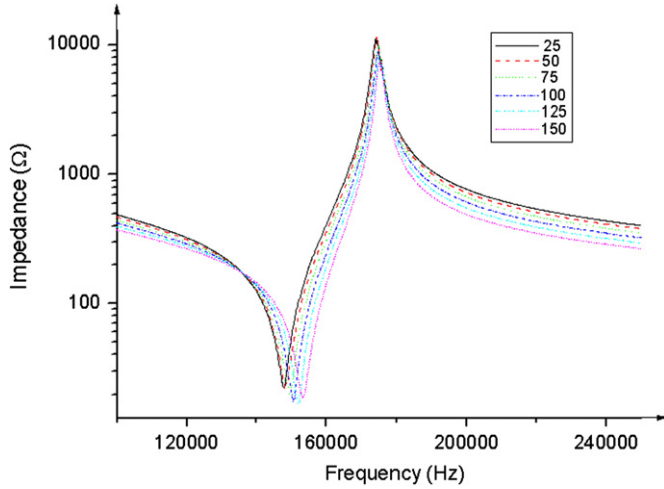


Fig. 3. The variation of fundamental radial vibration mode with temperature.

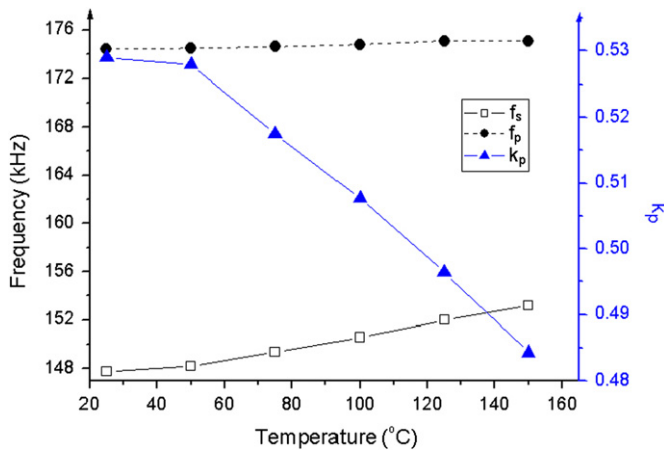


Fig. 4. The variation of fundamental series resonant frequency  $f_s$ , fundamental parallel resonant frequency  $f_p$  and the effective electromechanical coupling factor  $k_{\text{eff}}$  with temperature.

$$f_p = 174.25 + 0.00479(T) + 8.57143 \times 10^{-6}(T^2) \quad (5)$$

Fig. 4 also shows the variation of the effective electromechanical coupling factor with the temperature in the measured temperature range. The effective electromechanical coupling factor decreases with increasing temperature, and the result is consistent with that obtained by Sabat et al. [7] and Moure et al. [8], where the relation between the effective electromechanical coupling factor and measured temperature can be fitted by the second order polynomial

$$k_{\text{eff}} = 0.53348 - 8.16286 \times 10^{-5}(T) - 1.67429 \times 10^{-6}(T^2) \quad (6)$$

The linear temperature coefficient of resonant frequency (TCF) and the linear temperature coefficient of the effective electromechanical coupling factor (TCk) are defined as [9]

$$\text{TCF} = \frac{1}{f_r} \frac{\partial f_r}{\partial T} \quad (7)$$

$$\text{TCk} = \frac{1}{k_{\text{eff}}} \frac{\partial k_{\text{eff}}}{\partial T} \quad (8)$$

The temperature coefficient of fundamental resonant frequency is positive and in the order of  $10^{-4}/^\circ\text{C}$ . However, the temperature coefficient of the effective electro-mechanical coupling factor (TCk) is negative and in the order of  $10^{-5}/^\circ\text{C}$ .

Eq. (3) shows that the series resonant frequency of the piezoelectric ceramic disks is inverses proportionally to the radius of ceramic disk  $r$ , the compliance  $s_{11}^E$ , the density  $\rho$  and the Poisson ratio  $\sigma$ . Therefore, the temperature variation of series resonant frequency depends on the temperature variation within these factors. Expressing the temperature variation for the radius of ceramic disk  $r$  is the thermal expansion coefficient, which is negative for poled piezoelectric ceramics, and in the order of ppm/ $^\circ\text{C}$  [10,11]. This means that an increase in temperature reduces the radius  $r$  of ceramic disk. Furthermore, the density  $\rho$  and the Poisson ratio  $\sigma$  are nearly constant in the measured temperature range, the compliance  $s_{11}^E$  decreases with increasing temperature, and its temperature coefficient is in the order of  $10^{-4}/^\circ\text{C}$  [12], which is the same temperature coefficient as the fundamental resonant frequency. Thus, decreases of the radius  $r$  of ceramic disk and compliance  $s_{11}^E$  with the temperature result in increases of the series resonant frequency.

Fig. 5 shows the influence of the whole resonant spectrum versus the temperature. Increasing the temperature causes the whole resonant spectrum to shift to a higher frequency. In addition, both the series resonant frequency  $f_{\text{sn}}$  and the parallel resonant frequency  $f_{\text{pn}}$  of each overtone increase in conjunction with the temperature, as shown in Fig. 6.

The connection between series resonant frequency  $f_{\text{sn}}$  and the parallel resonant frequency  $f_{\text{pn}}$  of each overtone mode with the temperature can be fit by the second order polynomial, as follows:

The first overtone mode:

$$f_{s1} = 385.44 + 0.07126(T) + 1.71419 \times 10^{-4}(T^2) \quad (9)$$

$$f_{p1} = 303.96 + 0.08389(T) + 4.57143 \times 10^{-5}(T^2) \quad (10)$$

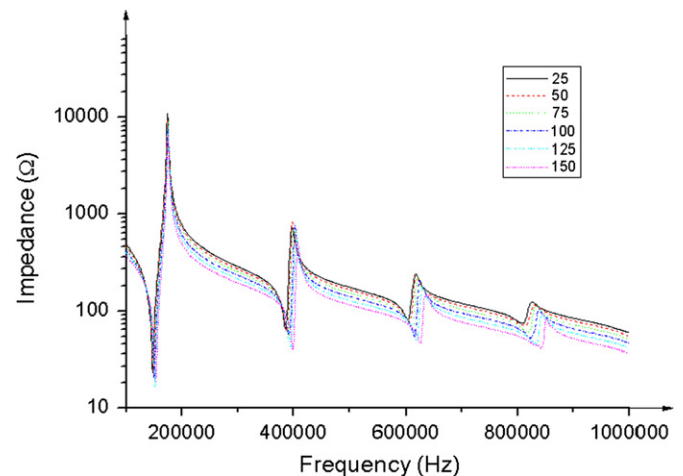


Fig. 5. The variation of whole resonant spectrum with temperature.

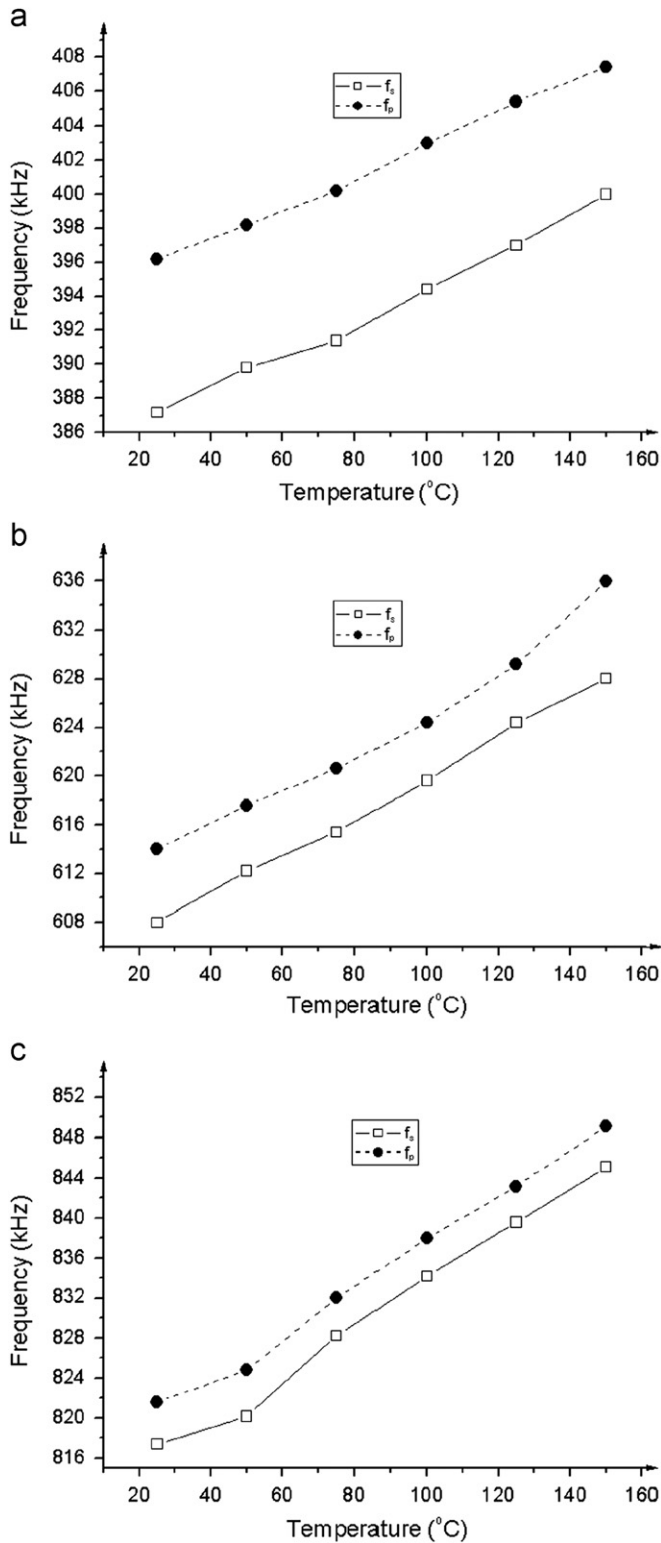


Fig. 6. (a) The variation of the first overtone series resonant frequency  $f_{s1}$  and its parallel resonant frequency  $f_{p1}$  with temperature. (b) The variation of the second overtone series resonant frequency  $f_{s2}$  and its parallel resonant frequency  $f_{p2}$  with temperature. (c) The variation of the third overtone series resonant frequency  $f_{s3}$  and its parallel resonant frequency  $f_{p3}$  with temperature.

The second overtone mode:

$$f_{s2} = 604.42 + 0.14391(T) + 9.71429 \times 10^{-5}(T^2) \quad (11)$$



Fig. 7. The temperature coefficient of resonant frequency (TCF) of the piezoelectric ceramic disk versus mode order.

$$f_{p2} = 612.64 + 0.05383(T) + 6.62857 \times 10^{-4}(T^2) \quad (12)$$

The third overtone mode:

$$f_{s3} = 811.03 + 0.21616(T) + 8.85714 \times 10^{-5}(T^2) \quad (13)$$

$$f_{p3} = 815.83 + 0.19959(T) + 1.57143 \times 10^{-5}(T^2) \quad (14)$$

The temperature coefficient of resonant frequency (TCF) of the piezoelectric ceramic disk increases in conjunction with the mode order, as shown in Fig. 7.

#### 4. Conclusion

Increasing the temperature from 25 to 150 °C raises the fundamental series resonant frequency  $f_s$  and the fundamental parallel resonant frequency  $f_p$  of the radial vibration mode, but the effective electromechanical coupling factor is decreased with the rising of temperature. The temperature coefficient of the fundamental resonant frequency (TCF) is positive and in the order of  $10^{-4}/^{\circ}\text{C}$ , although the temperature coefficient of the effective electromechanical coupling factor (TCK) is negative and in the order of  $10^{-5}/^{\circ}\text{C}$ . Therefore, decreasing the radius  $r$  of ceramic disk and the compliance  $s_{11}^E$  with temperature causes increases in the series resonant frequency.

Increases of temperature also cause the whole resonant spectrum from 100 kHz to 1 MHz to shift to higher frequency. The relationship between series resonant frequency  $f_{sn}$  and parallel resonant frequency  $f_{pn}$  of each overtone mode with the temperature can be fitted by the second-order polynomial. Finally, the temperature coefficient of the resonant frequency (TCF) of the piezoelectric ceramic disks increases in conjunction with the mode order.

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