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Vibration analysis for piezoceramic ring

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Abstract

The vibration property of piezoceramic ring is investigated by theoretical analysis, numerical simulation, and experimental measurement in this paper. Based on the piezoelectric constitutive and kinematical equation, the expression for resonant frequency of piezoceramic ring is derived by setting the boundary of vibrating ring as free. The expressions are calculated with numerical method especially for vibration modes in radial and wall-thickness direction. The relationship between resonant frequencies of two modes and the radius (or thickness) is obtained. The results calculated are compared with measurements by Impedance Analyzer, and the errors between them are analyzed in the paper.

Keywords: Piezoelectric ring; Vibration modes; Radius and thickness; Analysis

1. Introduction

As is known to all, piezoelectric effect is widely applied in ultrasonic, acoustic and electroacoustic devices such as piezoelectric microphones, pickups buzzers, ultrasonic thickness gauges, flaw detectors and B-type ultrasound imaging probes, underwater sound velocimeters, cylinder transmitting transducers and piezoelectric ring hydrophones, etc. The key elements used to transform electric to acoustic signal reciprocally are piezoelectric vibrators, where the plate, beam, tube, ring, sphere and unimorph (or bimorph) wafers are common and among which the piezoelectric tubes or rings have widespread application in ultrasonic and water acoustic detection. This is because they have higher sensitivity, simpler structure, and distributed directivity in radial direction.

Vibration property of piezoelectric vibrators affect operating frequency of device directly and thus has been the research focus [1–4]. Recent analysis of piezoelectric tubes or rings mainly focuses on radial vibration, while there is little on thickness direction. Chi Hung Huang et al. [5] have done detailed analysis of piezoelectric rings' vibration modes and deduced the frequency equations of vibration for radial, tangential and axial directions by solving the displacement of piezoelectric rings. However, the analysis about the radius and wall-thickness directions was not done. Shuyu [6] applied equivalent circuit diagram to analyze the radial vibration of piezoceramic–metal lantern ring transducer which did not involve thickness vibration. By analyzing the stress of piezoelectric ring, applying piezoelectric equations and kinematic equations, the mathematical models of resonant frequency for each mode is deduced and the theoretical results are verified by experiments.

2. Frequency equations of piezoelectric ring

A piezoelectric ring in cylindrical coordinate, polarized in radial direction where the electrodes are covered on the inner and outer surfaces of the ring, is shown in Fig. 1. The height of the ring is h and the inner and outer radii are a and b, respectively. Then the average radius of the piezoelectric ring is $r=(a+b)/2$ and the radius thickness is $t=b-a$.

As the piezoelectric ring is symmetrical about the z-axis, the electric field is applied along the r-axis direction. Therefore, the vibration along the z-axis direction can be ignored. The components of stress, strain and displacement are axially symmetric as z-axis. The boundary effect in electric field can be also ignored. So piezoceramic constitutive equations and kinematic equation of the piezoelectric ring

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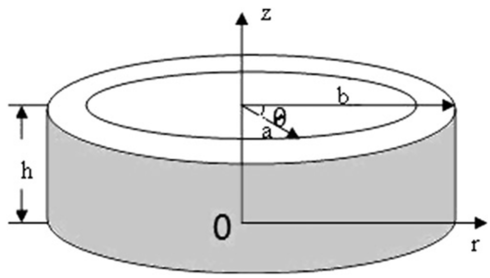

Fig. 1. Piezoelectric ring in cylindrical coordinate.

in cylindrical coordinate are given by [7]:

$$\begin{cases} S_\theta = s_{11}^E T_\theta + s_{13}^E T_r + d_{31} E_r \\ S_z = s_{12}^E T_\theta + s_{13}^E T_r + d_{31} E_r \\ S_r = s_{13}^E T_\theta + s_{33}^E T_r + d_{33} E_r \\ D_r = d_{31} T_\theta + d_{33} T_r + \varepsilon_{33}^T E_r \end{cases} \tag{1}$$

$$\begin{cases} \rho \dfrac{\partial^2 \xi_r}{\partial t^2} = \dfrac{\partial T_r}{\partial r} + \dfrac{T_r - T_\theta}{r} \\ \rho \dfrac{\partial^2 \xi_\theta}{\partial t^2} = \dfrac{1}{r} \dfrac{\partial T_\theta}{\partial \theta} \end{cases} \tag{2}$$

Where ξ_r, ξ_θ are displacement components; T_r, $T_\theta$and T_z are stresses; S_r, S_θ and S_z are the normal strains in r, θ and z direction.

$$S_r = \frac{\partial \xi_r}{\partial r}\ ,\ S_\theta = \frac{\xi_r}{r} \tag{3}$$

Eq. (1) can be written as:

$$T_r = A_1 S_r + A_2 S_\theta + A_3 E_r \tag{4}$$

$$T_\theta = B_1 S_r + B_2 S_\theta + B_3 E_r \tag{5}$$

Where

$$\begin{cases} A_1 = \dfrac{s_{11}^E}{s_{11}^E s_{33}^E - s_{13}^E s_{13}^E} & B_1 = \dfrac{-s_{13}^E}{s_{11}^E s_{33}^E - s_{13}^E s_{13}^E} \\ A_2 = \dfrac{-s_{13}^E}{s_{11}^E s_{33}^E - s_{13}^E s_{13}^E} & B_2 = \dfrac{s_{33}^E}{s_{11}^E s_{33}^E - s_{13}^E s_{13}^E} \\ A_3 = \dfrac{s_{13}^E d_{31} - s_{11}^E d_{33}}{s_{11}^E s_{33}^E - s_{13}^E s_{13}^E} & B_3 = \dfrac{s_{13}^E d_{33} - s_{33}^E d_{31}}{s_{11}^E s_{33}^E - s_{13}^E s_{13}^E} \end{cases} \tag{6}$$

Now that the steady-state vibration of the ring has the same vibration frequency with impressed harmonic incentive electric field, we may get:

$$\xi_r = \xi \exp(j\omega t) \tag{7}$$

Substitute Eqs (3)–(6) into the first equation in (2), we can get:

$$\frac{d^2\xi}{dr^2} + \frac{1}{r}\frac{d\xi}{dr} - \frac{n^2}{r^2}\ \xi + k^2 \xi + \frac{A_3 - B_3}{A_1 r} E_r = 0 \tag{8}$$

Where

$$n^2 = \frac{B_2}{A_1},\ k^2 = \frac{\omega^2 \rho}{A_1} \tag{9}$$

The general solution of (8) is as follows:

$$\xi = C_1 J_n(kr) + C_2 Y_n(kr) - C E_r \frac{S_{0,n}(kr)}{k} \tag{10}$$

where

$$C = \frac{A_3 - B_3}{A_1} \tag{11}$$

Substitute (10) into (3) and then into (8), we get:

$$\begin{aligned} T_r = & \left[A_1 k J_{n-1}(kr) + \frac{A_2 - A_1 n}{r} J_n(kr) \right] C_1 \\ & + \left[A_1 k Y_{n-1}(kr) + \frac{A_2 - A_1 n}{r} Y_n(kr) \right] C_2 \\ & - A_1 C E_r \frac{S'_{0,n}(kr)}{k} - \frac{A_2}{r} C E_r \frac{S_{0,n}(kr)}{k} + A_3 E_r \end{aligned} \tag{12}$$

where

$$S'_{0,n}(kr) = \frac{1}{2} k \left[(n-1) S_{-1,n-1}(kr) - (n+1) S_{-1,n+1}(kr) \right]. \tag{13}$$

While the piezoelectric ring is making free vibration, the boundary conditions are $T_r|_{r=a}=0$ and $T_r|_{r=b}=0$. Substitute them into Eq. (12), we can get:

$$\begin{aligned} & \begin{bmatrix} A_1 k J_{n-1}(ka) + \dfrac{A_2 - A_1 n}{a} J_n(ka) & A_1 k Y_{n-1}(ka) + \dfrac{A_2 - A_1 n}{a} Y_n(ka) \\ A_1 k J_{n-1}(kb) + \dfrac{A_2 - A_1 n}{b} J_n(kb) & A_1 k Y_{n-1}(kb) + \dfrac{A_2 - A_1 n}{b} Y_n(kb) \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} \\ & = \begin{bmatrix} A_1 C E_r \dfrac{S'_{0,n}(ka)}{k} + \dfrac{A_2}{a} C E_r \dfrac{S_{0,n}(ka)}{k} - A_3 E_r \\ A_1 C E_r \dfrac{S'_{0,n}(kb)}{k} + \dfrac{A_2}{b} C E_r \dfrac{S_{0,n}(kb)}{k} - A_3 E_r \end{bmatrix} \end{aligned} \tag{14}$$

When the determinant of coefficient of the left-hand side of the equation equals zero, the resonant frequency equation of the ring can be obtained as follows:

$$\begin{vmatrix} A_1 k J_{n-1}(ka) + \dfrac{A_2 - A_1 n}{a} J_n(ka) A_1 k Y_{n-1}(ka) + \dfrac{A_2 - A_1 n}{a} Y_n(ka) \\ A_1 k J_{n-1}(kb) + \dfrac{A_2 - A_1 n}{b} J_n(kb) A_1 k Y_{n-1}(kb) + \dfrac{A_2 - A_1 n}{b} Y_n(kb) \end{vmatrix} = 0 \tag{15}$$

3. The calculation of frequency equation

Eq. (15) is a transcendental equation, which can be solved by graphical method.

$$y_1 = \left[A_1 k J_{n-1}(ka) + \frac{A_2 - A_1 n}{a} J_n(ka) \right] \left[A_1 k Y_{n-1}(kb) + \frac{A_2 - A_1 n}{b} Y_n(kb) \right] \tag{16}$$

$$y_2 = \left[A_1 k Y_{n-1}(ka) + \frac{A_2 - A_1 n}{a} Y_n(ka) \right] \left[A_1 k J_{n-1}(kb) + \frac{A_2 - A_1 n}{b} J_n(kb) \right] \tag{17}$$

Then Eq. (15) will become

$$y_1 - y_2 = 0 \rightarrow y_1 = y_2 \tag{18}$$

Matlab software is used to calculate the two curves y_1 and y_2, whose intersection point is just the solution of Eq. (15).

By taking the dimension of a PZT-5 piezoelectric ring with inner radius of 7.5 mm, outer radius of 12.5 mm and height of 3 mm as an example, graphical method is used to get the resonant frequency of the ring and the frequency range is chosen from 0 MHz to 1 MHz to solve Eq. (18). The result obtained are shown in Fig. 2.

We can see from Fig. 2 that there are four intersection points, which means that there are four vibration modes between 0 and 1 MHz and those points correspond to four resonant frequencies of 49 kHz, 291 kHz, 571.5 kHz and 860 kHz. The parameters of Fig. 2 are shown in Table 1.

Aiming at mode one and mode two, changing the radius, thickness and height of the ring the relationship the resonant frequencies of the two modes change along with the ring's size is obtained, which is shown respectively in Fig. 3 and Fig. 4. It can be seen from Fig. 3 that the resonant frequency has insignificant changes along with the thickness and the height of the ring. However, the resonant frequency decreases with the radius rapidly as the radius increases. This means that mode one is a radial vibration mode with resonant frequency mainly lies with the average radius. From Fig. 4 we may see that under mode two the resonant frequency decreases rapidly as the thickness increases while there is little change in frequency along the height and the radius. It means that mode two is the vibration mode in thickness direction whose resonant frequency mainly lies in the ring's thickness.

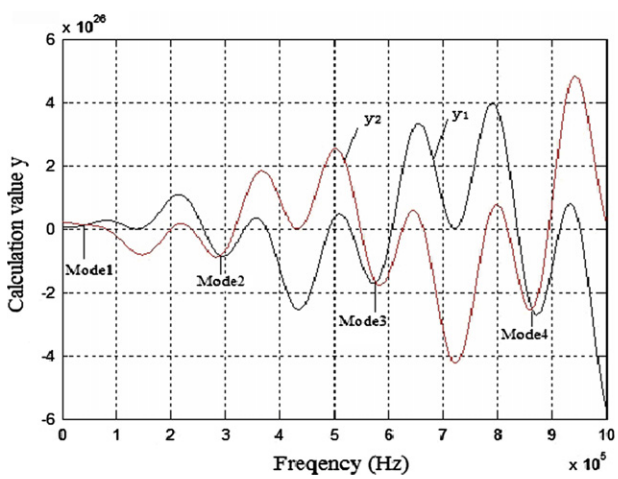

Fig. 2. Frequency curves of piezoelectric ring modes.

Table 1
Parameters of PZT-5.

Piezoelectric coefficient d_{33} (C/N)	374×10^{-12}
Piezoelectric coefficient d_{31} (C/N)	-171×10^{-12}
Elastic coefficient S_{11}^{E} (m^2/N)	16.4×10^{-12}
Elastic coefficient S_{12}^{E}(m^2/N)	-5.74×10^{-12}
Elastic coefficient S_{13}^{E} (m^2/N)	-7.22×10^{-12}
Elastic coefficient S_{33}^{E} (m^2/N)	18.8×10^{-12}
Density ρ (kg/m^3)	7.75×10^{3}

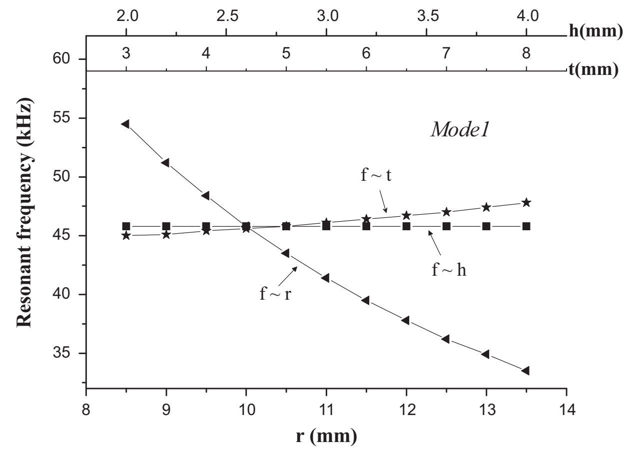

Fig. 3. Frequency–structure curve of Mode 1.

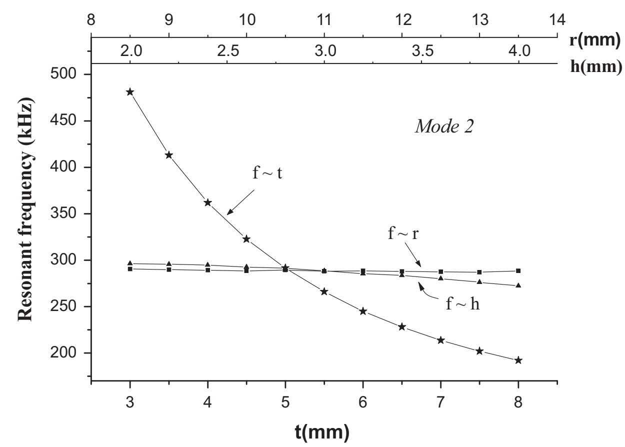

Fig. 4. Frequency–structure curve of Mode 2.

4. The comparison between theory and experiment

The PZT-5 type piezoceramic rings produced by Baoding Hongsheng Acoustic Equipment Factory are chosen as experimental samples. The electrodes are covered on the inner and outer surfaces of ring and the ring is polarized in the radial direction. The dimension of the ring is shown in Table 2.

Impedance analyzer is used to measure the resonant frequency of each sample in mode one and mode two and then the curves which show the relationship of the resonant frequency along the radius and thickness directions are obtained. After substituting the parameters of the ring into Eq. (18), we get the corresponding resonant frequencies of the samples in the two modes. The results from calculation and measurement are presented in Fig. 5 and Fig. 6.

Fig. 5 shows that the theoretical and experimental results of the rings' resonant frequencies in radial vibration

Table 2
The structure size of ring samples.

Sample no.	Outer, r (mm)	Inner, r (mm)	Thickness (mm)	Height (mm)
1	25	9	8	3
2	25	11	7	3
3	25	13	6	3
4	25	15	5	3
5	25	17	4	3
6	25	18	3.5	3
7	25	19	3	3
8	25	21	2	3
9	25	23	1	3

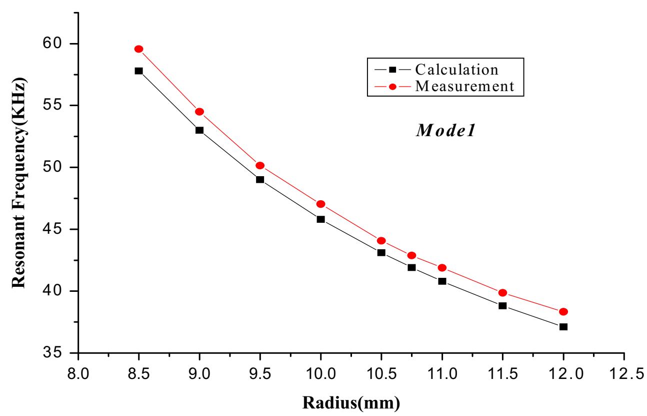

Fig. 5. Frequency–radius curve of Mode 1.

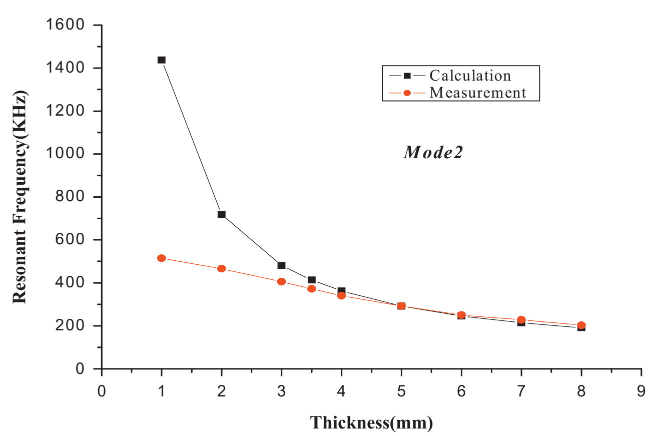

Fig. 6. Frequency–radius curve of Mode 2.

are in good agreement. The deviations of frequency are less than 1.8 kHz and the maximum deviation is 3.2%. It can also be seen that the theoretical results are all less than the measurements, which means that there are systematic errors of about 1–2 kHz in theoretical results.

From Fig. 6 it can be seen that theoretical results agree well with measurements for thickness of the ring between 3 and 8 mm. When the thickness is less than 3 mm, the deviation becomes very wide, which is because the thickness is close to the height and the vibrations in thickness and height direction are coupled and thus it is against the hypothesis in theoretical deduction.

5. Conclusions

Piezoelectric theory and mechanical vibration theory are used to deduce the vibration frequency equation of a piezoelectric ring. By solving the equation numerically, different vibration modes of the ring can be obtained, in which the vibration in radial and thickness directions are picked up for analysis. The relationship between the resonant frequency along the radial and thickness directions in the two modes is obtained by calculation and he results are compared with experimental measurements. The comparison shows that the theoretical results of resonant frequencies of radial vibration are in good agreement with measurements. The frequency deviation is less than 1.8 kHz and the maximum deviation is 3.2%. Theoretical results agree with measurements well within the thickness of 3–8 mm; but when the thickness is less than 3 mm, the deviation becomes wide. Thus the deduced frequency equation is inapplicable within the range.

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