Int. J. High Technology Ceramics **2** (1986) 143–155

Coaxial Resonator Method to Determine Dielectric Properties of High Dielectric Constant Microwave Ceramics

E. Kemppinen and S. Leppävuori

Microelectronics Laboratory, Department of Electrical Engineering,
University of Oulu, Finland

SUMMARY

The performance of a resonator incorporating a ceramic depends strongly on the dielectric properties of the ceramic. This paper discusses the measurement of these properties using the coaxial resonator method at high UHF frequencies. It is possible to calculate the dielectric constant with an accuracy better than 1%. If the conductivity of the metal coating on the resonator is known, it is also possible to calculate the loss tangent of the ceramic core.

The measured Q_0-values, which depended on losses in both the silver coating and the barium nonatitanate dielectric of the $\lambda/4$ resonators, were compared with the theoretically calculated maximum values, on the basis that the ceramic is lossless and the conductivity of the coating corresponded with that of bulk silver. Typically, 70–90% of the theoretical maximum values were attained. The loss tangent of the $Ba_2Ti_9O_{20}$ ceramic was between 1×10^{-4} and 2×10^{-4} at 1 GHz. The temperature coefficient of resonance frequency was 4 ppm °C^{-1}.

Measurements yield useful data suitable for practical design purposes. In addition, the dimensions of the resonators are small, ranging from a few millimetres to a few centimetres, depending on the frequency, dielectric constant, and desired Q_0-value.

The method can be used as a quality control technique for the ceramic manufacturing process.

1. INTRODUCTION

Low loss microwave ceramics are extremely useful for miniaturizing microwave and UHF components. The importance of these materials has increased recently with the greater demand for small portable telecommunications equipment employing UHF frequencies. The use in such applications makes it necessary to know the merits of the electrical performance of the materials employed. There are, however, difficulties in obtaining absolute values of dielectric properties at microwave and UHF frequencies, and particularly in comparing values measured in one laboratory with those from another.

There are two major reasons for this: (i) measurement problems themselves, and (ii) lack of control over ceramic composition and microstructure. The present paper addresses some of the difficulties but concentrates on a method, viz. the coaxial resonator method, which has a number of advantages over others, and which has been used by the authors in the context of microwave ceramics and device development projects.

Different methods have been used to determine the dielectric properties of low loss, high frequency ceramics. The important electrical parameters that are measured are the complex dielectric constant ($\varepsilon_r' + i\varepsilon_r''$), which yields the loss tangent ($\tan d = \varepsilon_r''/\varepsilon_r'$), and the variation of complex dielectric constant with temperature.

Commonly, three methods have been exploited, viz, the rod resonator method developed by Hakki and Coleman,[1] the waveguide method,[2] and the microstrip transmission line method.[3] In all three methods the ceramic samples are in the form of cylinders with aspect ratios such that they resonate with an external electromagnetic field. Knowing the geometry of the sample and the electromagnetic field configuration, it is possible to calculate the complex dielectric constant from the measured resonance curves. In practice, however, this is difficult, especially in the case of the second and third methods, because of the effects of radiation from the resonator.

The three methods referred to above are most suitable for frequencies above about 4 GHz since below this frequency the size of the dielectric resonator becomes impracticably large, so that it is difficult to manufacture them reliably. Internal defects and an uneven distribution in density may increase the losses even though changes in the real part of the dielectric constant may be small. For example, in the case of the rod resonator method, at a frequency of 1 GHz, the approximate dimensions of a barium nonatitanate dielectric cylindrical resonator would be 65 mm in diameter and 36 mm in height.

The advantage of the microstrip transmission line method is that it gives a true Q-factor for the resonator in the environment where it is normally to be used;[4] to some extent this also applies to the waveguide method. From the ceramist's standpoint it is important to have a reliable measurement method for both the development of microwave ceramics and for quality control in their manufacture.

Because of the problems related to the unwieldy size of the dielectric rod resonators at the upper end of the UHF range, above 500 MHz other types of resonators can be used in order to determine the electrical properties of the ceramics. In this paper the coaxial resonator method which employs a much smaller size sample than required by the above-mentioned methods is described. In addition, it is possible to obtain data that can be used for practical electronic circuit design in which the circuits contain coaxial or other types of TEM resonator.

2. SPECIMEN MANUFACTURE AND THEORY OF CERAMIC-FILLED COAXIAL RESONATORS

Ceramic resonator cores with a central cylindrical hole were cold-pressed from calcined ceramic powder in a steel mould and then sintered with an appropriate temperature/time profile to a peak temperature of 1350 °C. In the sintering process the shrinkage was around 20 %. Starting materials and the fabrication route are detailed elsewhere.[5]

Thick film silver paste was brushed on to the surface of the sintered resonator and fired-on at temperatures in the range 900–930 °C, depending on the ceramic and silver, to give a high conductivity coating. Two silver coatings were applied and separately fired-on to ensure that the thickness of the layer was more than ten times the skin depth, that is more than about 20 μm.

Because of the high dielectric constant of microwave ceramics, three basic TEM coaxial resonator types are possible. These are:

1. $\lambda/4$ resonator—where all the surfaces, with the exception of one end, are covered with metal;
2. $\lambda/2$ resonator—where all the surfaces are covered with metal but where, in the middle of the outer conductor, there is a small opening for coupling; and
3. $\lambda/2$ resonator—where all the surfaces, except for both the ends, are covered with metal.

In this paper emphasis is on the $\lambda/4$ type, since this is of practical importance, for example, in miniature UHF filter design. The second type was used for the accurate determination of the dielectric constant ε_r' of the ceramic core. The practical value of the third type of resonator may be less than the others, except for ceramics with a very high dielectric constant, since the measured Q-values are reduced by the imperfect reflections from both open-circuited ends. However, it has the advantage that there are no conduction losses at the ends. In the present study it was employed to demonstrate that it is possible to determine the dielectric constant fairly accurately using it.

2.1. Q-values for the dielectric $\lambda/4$ coaxial resonators

The unloaded Q_0-value of the resonator is given by

$$1/Q_0 = 1/Q_c + 1/Q_d + 1/Q_r \tag{1}$$

where Q_c is the Q-value due to the conductor losses, Q_d that due to the dielectric losses of the ceramic core, and Q_r that due to radiation losses. In this study the most important factors in determining Q_0 are Q_c and Q_d. The Q_r-factor is considered insignificant because the calculated dielectric constant (see Table 1) of the $\lambda/4$ resonator is nearly equal to that of the $\lambda/2$ resonator with short-circuited ends, implying that the fringing fields are negligible.[6]

TABLE 1

Dielectric Constants of $Ba_2Ti_9O_{20}$ as Determined by the Various Resonator Methods

Resonator type	*Frequency range/MHz*	*Dielectric constant*	*Standard deviation*
$\lambda/4$ resonator	900	36·86	0·14
$\lambda/2$ short-circuited ends	1 800	36·63	0·02
$\lambda/2$ open-circuited ends	1 800	37·12	0·25
Rod resonator	8 300	37·41	0·20

It is possible to calculate the Q_c-value for a coaxial $\lambda/4$ resonator of inner radius a, outer radius b, and length L from the definition of Q and well-known coaxial transmission line theory,[7] assuming a sinusoidal propagation in the z-direction, as follows:

$$Q_c = 2\omega W_e/P_c \tag{2}$$

$$W_e = \frac{\varepsilon_0\varepsilon_r'}{4}\int_0^L\int_0^{2\pi}\int_a^b \frac{V_0^2}{(\ln b/a)^2}\cdot\frac{1}{r}(2A\sin kz)^2\,\mathrm{d}r\,\mathrm{d}\phi\,\mathrm{d}z \tag{3}$$

$$P_c = \frac{R_m}{2}\int_0^L\oint_{a,b}\frac{Y_0^2V_0^2}{(\ln b/a)^2}\cdot\frac{1}{r}(2A\sin kz)^2\,\mathrm{d}r\,\mathrm{d}\phi\,\mathrm{d}z + \frac{R_m}{2}\int_0^{2\pi}\int_a^b\frac{Y_0^2V_0^2}{(\ln b/a)^2}\cdot\frac{1}{r}(2A)^2\,\mathrm{d}r\,\mathrm{d}\phi \tag{4}$$

where W_e is the time-average electric field energy in the core volume and P_c represents the losses caused by the resistivity of the silver coating on the curved surfaces and the one end. A is a normalization factor, k the propagation constant, R_m the surface resistance, Y_0 the free space admittance and V_0 the voltage difference between the conductors. In the above equations ε_0 is the permittivity of vacuum, ω the angular frequency and r, ϕ and z cylindrical co-ordinates with the z-axis along the axis of the cylinder and the origin at the coated end. A little manipulation yields the result[8]

$$Q_c \frac{\delta}{\lambda} = \frac{1}{4 + \dfrac{2L(1 + b/a)}{b \ln b/a}} \tag{5}$$

where δ is the skin depth ($\delta = (\pi f \sigma \mu_0)^{-1/2}$), σ the conductivity of the silver coating, μ_0 the magnetic permeability of vacuum and λ the wavelength in the dielectric.

Figure 1 illustrates a set of curves calculated from eqn (5) for Q_c at 900 MHz for various dielectric constants and cross sections. In eqn (5) the conductivity of bulk silver ($\sigma = 61$ MS m^{-1}) has been used in calculating

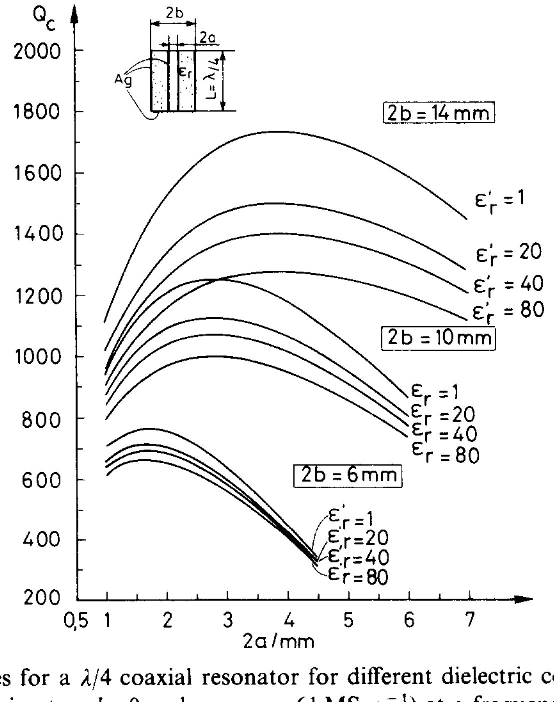

Fig. 1. Q_c-values for a $\lambda/4$ coaxial resonator for different dielectric constants and cross sections (assuming tan $d = 0$ and $\sigma = \sigma_{Ag} = 61$ MS m^{-1}) at a frequency of 900 MHZ.

the skin depth. The Q_c-value is at a maximum when the ratio b/a is about 3·6; this can be verified by fixing b and by setting the derivative of eqn (5) with respect to a to zero.

If the dielectric core were totally lossless and with no radiation from the open end, these Q_c-values would be the highest achievable by the $\lambda/4$ type coaxial resonators, according to eqn (1).

2.2. Determination of the dielectric constant

The determination of the dielectric constant ε_r' from the resonance frequency and the length of the core is straightforward. It is also very accurate in the case of $\lambda/2$ resonators because of the exact boundary conditions at the short-circuited ends. The following formulae are valid:

$$\lambda/4 \text{ case: } \varepsilon_r' = (c/4Lf_r)^2 \tag{6}$$

$$\lambda/2 \text{ case: } \varepsilon_r' = (c/2Lf_r)^2 \tag{7}$$

where c is the velocity of light in vacuum, f_r the resonance frequency, and L the length of the core. The dielectric constant can also be calculated from the harmonic frequencies using the same principle.

2.3. Determination of loss tangent and skin depth

Because there are two unknowns, loss tangent and skin depth, in eqn (1) for the Q_0-value, to determine them two simultaneous equations must be solved. In the present study this was achieved from measurements on two resonators of equal cross sections but of different lengths.

If the lengths of the resonators are L and $3L$, then the resonance frequency of the first resonator resonating in the $\lambda/4$ mode would be expected to be the same as that of the second resonator resonating at the $3\lambda/4$ mode. It can be shown from the basic definition, eqn (2), that the theoretical Q_c-value of the $3\lambda/4$ coaxial resonator is the same as that indicated by eqn (5) except for a factor '3' in the numerator. Because the loss tangent and skin depth must be equal at the same frequency, the following two equations can be written by combining eqns (1) and (5):

$$\frac{1}{Q_0} = \frac{\delta}{\lambda}(4 + LB) + \tan d \tag{8}$$

$$\frac{1}{Q_0'} = \frac{\delta}{\lambda}(4 + 3LB) + \tan d \tag{9}$$

where Q_0 and Q_0' are the measured Q_0-values of the two resonators of lengths L and $3L$, respectively, and

$$B = 2\frac{1 + b/a}{b \ln b/a}$$

is the shape factor depending only on the cross-sectional geometry. From these equations the following solutions can be obtained:

$$\delta = \frac{3\lambda}{8}\left(\frac{1}{Q_0} - \frac{1}{Q_0'}\right) \qquad (10)$$

$$\tan d = \frac{1}{Q_0} - \frac{3}{8}\left(\frac{1}{Q_0} - \frac{1}{Q_0'}\right)(4 + LB) \qquad (11)$$

When using this method it is essential that the quality of the ceramic and the metal paste are the same for both resonators. If this is not so the loss tangent or the skin depth in both eqns (8) and (9) cannot be regarded as equal, which invalidates the procedure. The length of the resonators can be adjusted by lapping so that the test frequencies are equal.

3. RESULTS

The unloaded Q_0-values were measured with a Hewlett Packard 8410 network analyser. The measuring jig and the shape of a typical resonance curve are illustrated in Fig. 2. The coupling between the two probes coupled to the electric field was better than 45 dB without the resonator on the jig.

In making the measurement of the Q_0-values, the normal -3 dB method was used. Because of the appreciable coupling ($S_{21} = -20\,\text{dB} \cdots -25\,\text{dB}$, where S_{21} is the forward transmission scattering parameter and $|S_{21}|^2 = P_L/P_A$, where P_L is the power at the load and P_A is the maximum available power from the generator) the final Q_0-values were calculated as follows:[9]

$$Q_0 = \frac{f_r}{f_2 - f_1} \cdot \frac{1 + K}{K} \qquad (12)$$

where the correction factor K is

$$K = 10^{(-S_{21}/20)} - 1 \qquad (13)$$

where $S_{21} < 0$, f_r is the resonance frequency, and f_1 and f_2 are the lower and upper -3 dBs frequencies, respectively.

In Fig. 3a the typical results of the measured Q_0-values at 900 MHz are shown, together with the theoretical maximum values calculated from eqn

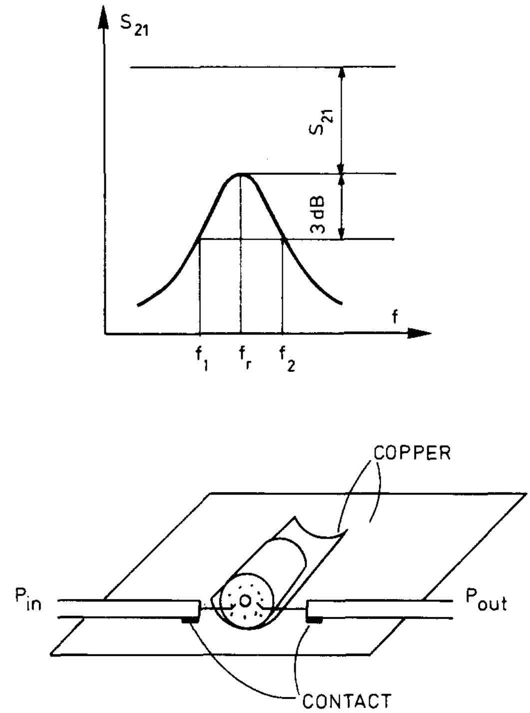

Fig. 2. The coaxial resonator measuring system and a typical resonance curve.

(5) ($\sigma = \sigma_{Ag} = 61\,\mathrm{MS\,m^{-1}}$, and $\tan d = 0$). The measured values were in the range 70–90 % of the theoretical, the closeness of the theoretical being dependent on the quality of the ceramic and silver coating pastes. The Q_c-value, according to eqn (5), is proportional to the square root of the conductivity. For that reason the resulting Q_0-values are different for the different pastes that were used in the study. Figure 3b illustrates the measured Q_0-values at 900 MHz for four commercially available pastes, as well as the calculated curve from eqn (5) under the same conditions as shown in Fig. 3a.

On measuring the Q_0-values at around 900 MHz of seven different groups of resonators, each group containing between eight and 15 resonators, all of which had been manufactured in a similar manner, it was found that the standard deviation of the Q_0-values in each group was from 3 to 10 % of the mean value.

When the desired Q_0-value of a $\lambda/4$ coaxial resonator with a high ε_r' at, say, 900 MHz is about 1000, and the loss tangent of the ceramic is approximately 1×10^{-4}, it is self-evident from eqns (1) and (5) that the conductor losses dominate, and special attention has to be paid to the quality and the conductivity of the covering paste. On the other hand, as can be concluded

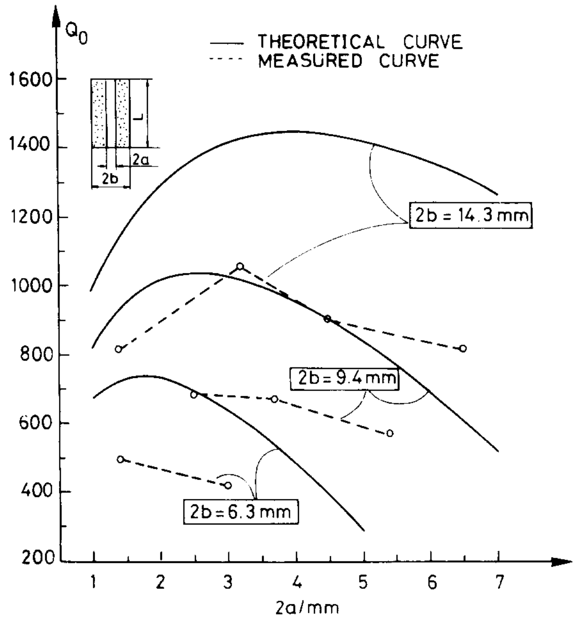

Fig. 3a. Measured and calculated Q_0-values from eqn (5) at 900 MHz ($\tan d = 0$, $\sigma = \sigma_{Ag} = 61\ \mathrm{MS\,m^{-1}}$) for barium nonatitanate coaxial resonators of different cross sections. The covering paste was 'Dupont 8032' randomly selected for initial trials.

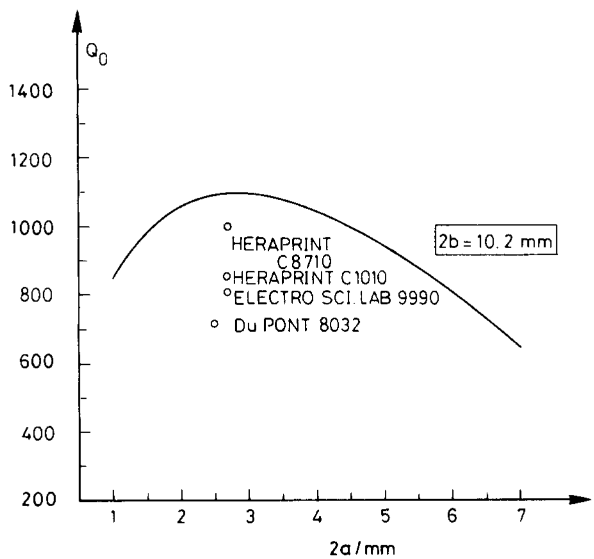

Fig. 3b. Comparison between Q_0-values for different commercially available silver pastes at 900 MHz. The calculated curve from eqn (5) ($\tan d = 0$, $\sigma = \sigma_{Ag} = 61\ \mathrm{MS\,m^{-1}}$) for barium nonatitanate is also shown.

from eqn (1), the higher the desired Q_0-value, the greater is the significance of losses arising in the ceramic core.

The dielectric constant of the ceramic core was calculated using eqn (6) for the $\lambda/4$ resonator and using eqn (7) for both the $\lambda/2$ type resonators. These values are compared with those obtained by the rod resonator method[1] in Table 1. In making the measurements, a coupling between the resonator and the probes (S_{21}) that was as loose as possible was used in order to prevent the probes from having any effect on the resonance frequency.

It can be seen that in the case of the $\lambda/4$ resonator and the short-circuited $\lambda/2$ resonator, the dielectric constants ε_r' are nearly equal. The slightly higher dielectric constant in the case of the $\lambda/4$ resonator is most probably explained by the weak fringing fields at the open end: the effect is most marked in the case of the open circuited $\lambda/2$ resonator because of the action of the fringing fields at both ends.

It should be noted that the dielectric constant ε_r' is affected by the final density of the resonator; the more porous the sample the lower the dielectric constant. For this reason the rod resonator method gives higher dielectric constants than the coaxial method because a higher density can be achieved for the simple cylindrical specimen than for the coaxial resonator specimens. When measuring the dielectric constant ε_r', the accuracy of the short-circuited $\lambda/2$ coaxial resonator measuring method can be made better than 1%, providing specimen geometry is carefully controlled, particularly parallelism of the ends.

In the L and $3L$ measurements to determine the loss tangent of the ceramic core and the skin depth of the covering silver paste, two resonators with Q_0-values, as indicated in Table 2, were utilised. The calculation of the unknown factors is based on eqns (10) and (11) as outlined above.

The calculated skin depth of 2·2 μm is 0·2 μm greater than that calculated for bulk silver. It can therefore be deduced that the conductivity of the paste is 83% that of silver, and by using it (and preparing it in the same way)

TABLE 2

Determination of Loss Tangent and Skin Depth Using Two Barium Nonatitanate Coaxial Resonators of Different Lengths (the shorter resonator resonates at $\lambda/4$-mode and the longer one at $3\lambda/4$-mode: the silver paste was 'Herraprint C8710')

Resonance mode	*2a/mm*	*2b/mm*	*L/mm*	*f/MHz*	Q_0	Q_0'	*δ/μm*	*tan d*
$\lambda/4$	2·7	10·2	11·5	1 058	912		2·2	1.4×10^{-4}
$3\lambda/4$	2·7	10·2	34·6	1 055		1 031		

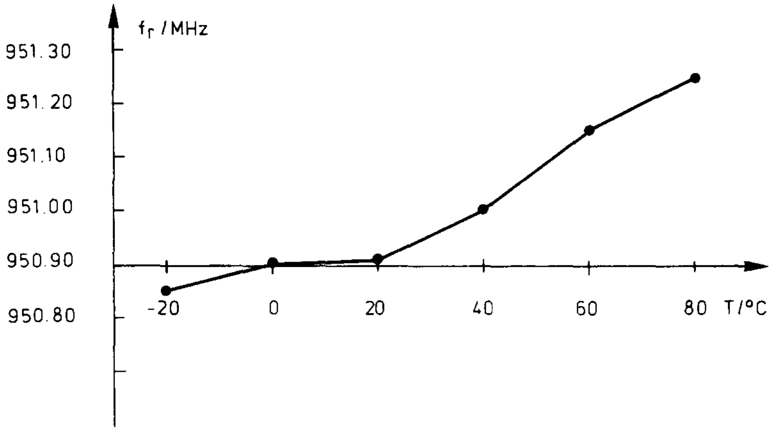

Fig. 4. The variation of resonance frequency of a barium nonatitanate coaxial $\lambda/4$ resonator with temperature.

approximately 90% of the theoretical Q_0-value can be achieved, provided the ceramic is totally lossless. Figure 1 shows the maximum achievable Q_0-values (i.e. for pure Ag coating on lossless ceramic) for different geometries.

The temperature coefficients of the resonance frequency of the coaxial resonators were measured in the temperature range -20 °C to $+80$ °C. It should be noted that because of the silver coating the measured values are characteristic of the entire resonator system, which is significant from the point of view of practical application. The variation in resonance frequency with temperature of a typical barium nonatitanate coaxial $\lambda/4$ resonator is shown in Fig. 4.

The temperature coefficient of the pure barium nonatitanate coaxial $\lambda/4$ resonator is 4 ppm $°C^{-1}$ which is a very good value for many practical applications. The value can be increased by carrying out suitable doping.[5]

4. CONCLUSION

The objective of the study was to assess the coaxial resonator method for the evaluation of dielectric properties of low loss ceramics at microwave frequencies. This paper describes a practical, and, in many cases, a sufficiently accurate method for evaluating the dielectric properties of low loss ceramics. It is especially suitable at high UHF and low microwave frequencies, where the sample size for methods employing cylindrical dielectric resonators becomes impracticably large. The drawback of the coaxial method is that in eqns (8) and (9) there are two unknown factors, the loss tangent of the ceramic and skin depth of the coating. Both of these depend on the overall fabrication process and when one of these is known, the other can be calculated. At 1 GHz the skin depth of the silver coating was calculated to be 2·2 μm, indicating the conductivity of the paste to be approximately 83% that of pure silver. By the same principle the loss tangent of the core was evaluated to be $1{\cdot}4 \times 10^{-4}$. Using the best of the silver pastes on a resonator with an outer diameter of 10·2 mm and an inner diameter of 2·7 mm, a Q_0-factor of 1000 was attained, which is approximately 90% of the calculated maximum value.

For the determination of the dielectric constant ε_r' of the ceramic the coaxial resonator method is very satisfactory. For ceramics having high dielectric constants the $\lambda/4$ coaxial resonators can be used with reasonable accuracy but a small increase in ε_r' may be obtained when eqn (6) is used because of the effect of the open end. The temperature coefficient of resonance frequency was found to be 4 ppm $°C^{-1}$. The values obtained by this method can be used immediately as practical design data.

ACKNOWLEDGEMENTS

The work on this project was undertaken at the Microelectronics Laboratory at the University of Oulu, Finland, and was financed by The Finnish Academy of Sciences.

REFERENCES

1. Hakki, B. W. and Coleman, P. O., A dielectric resonator method of measuring inductive capacities in the millimetre range, *IRE Trans. MTT*, July 1960, 402–10.
2. Plourde, J. K. and Chung-li Ren, Application of dielectric resonators in microwave components, *IEEE Trans. MTT*, **MTT-27**(8) (August 1981) 754–70.
3. Khanna, A. and Garault, Y., Determination of loaded, unloaded and external quality factors of a dielectric resonator coupled to a microstrip line, *IEEE Trans. MTT*, **MTT-31**(3) (March 1983) 261–4.
4. Kemppinen, E. and Leppävuori, S., An X-band doppler module made by thick film techniques. *Proc. 5th European Hybrid Microelectronics Conference*, Stresa, Italy, 1985.
5. Jaakola, T., Möttönen, J., Uusimäki, A., Rautioaho, R. and Leppävuori, S., Comparison of sintering behaviour and resulting microstructure of pre-heated and reaction sintered Nd-doped $Ba_2Ti_9O_{20}$ ceramics for use in microwave resonators, Report S86, 1985, Department of Electrical Engineering, University of Oulu, Finland.
6. Matthaei, G., Young, L. and Jones, E. M. T., *Microwave Filters, Impedance-Matching Networks, and Coupling Structures*, Artech House Inc., Dedham, Massachusetts, 1980.

7. Collin, R. E., *Foundations for Microwave Engineering*, McGraw-Hill Kogakusha Ltd, New York, 1966.
8. *The Microwave Engineers' Handbook and Buyers' Guide*, Horizon House Inc., Massachusetts, 1964.
9. Zverev, A. I., *Handbook of Filter Synthesis*, John Wiley & Sons Inc., New York, 1967.

Received 3 March 1986; accepted 27 March 1986