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On the Paradox Between Crack Bridging and Crack Interaction In Quasi-Brittle Materials

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Abstract

Discontinuous cracks observed in quasi-brittle materials subjected to fracture mechanics testing may be idealised by two mechanical models, i.e. an interaction model between the main crack and collinear small cracks, and a crack-wake bridging model. A study on these two models is presented in this paper and it shows that the instability of the crack system is significantly dependent on the model used. While the interaction model leads to unstable crack propagation, the crack bridging model predicts stable crack growth. It is concluded that the crack bridging model is more consistent with experimental observations on fracture in quasi-brittle materials.

Introduction

Crack-bridging due to ligament and grain pull-out has been recognized as an important toughening mechanism in quasi-brittle materials such as the non-transformable ceramics and cementitious materials, and has been successfully used to explain the non-continuous crack phenomenon observed in fracture mechanics testing of such materials.[1–13] In the idealized form, however, the bridged crack may be seen as a series of collinear cracks lying ahead of the main crack tip. As a result, two ways may be used to model or analyse the stress intensities of the crack system. One considers the interaction of the microcracks with the main crack where the singularities at each crack tip are retained and the other assumes finite stresses acting over the bridges. These are referred to as the interaction model and the bridging model, respectively. As will be shown later two completely different conclusions result from these two models.

Stress Intensity Factor Analyses

Figure 1(a) shows the interaction which consists of a semi-infinite crack along the negative x axis and a finite crack located in $a<x<b$. The stress fields in the vicinity of the main crack tip can be significantly intensified because of the presence of the finite crack. In the bridging model, Fig. 1(b), the ligament between the main crack and the finite crack is considered as a bridge and its effects on crack growth can be modelled by the actual stress distribution $\sigma_b(x)$ on the bridge.

Systematic solutions for the interaction model have been obtained by using the complex potential technique.[14–17] The stress intensity factors K_t^O, K_t^a and K_t^b at the crack tips O, a and b are normalised by the applied stress intensity factor at the main crack tip K_a^O and are given by[14,16]

$$\begin{aligned} K_t^O/K_a^O &= C/k \\ K_t^a/K_a^O &= (C-k^2)/kk' \\ K_t^b/K_a^O &= (1-C)/k' \end{aligned} \tag{1}$$

where

$$\begin{aligned} C &= E(k')/K(k') \\ k^2 &= a/b \\ k' &= (1-k^2)^{1/2} \end{aligned} \tag{2}$$

E and K denote the complete elliptic integrals of the second and first kinds, respectively. The normalised true stress intensity factors are plotted in Fig. 2 where it is found that the true stress intensity factor at the main crack tip K_t^O is always the largest. Therefore, for homogeneous brittle materials, crack growth will always start from the main crack tip, and if K_a^O is constant, the link-up of the main crack with the collinear microcrack will proceed unstably.[16]

Further study by Rubinstein[14] found that the above analysis holds also for collinear multi-

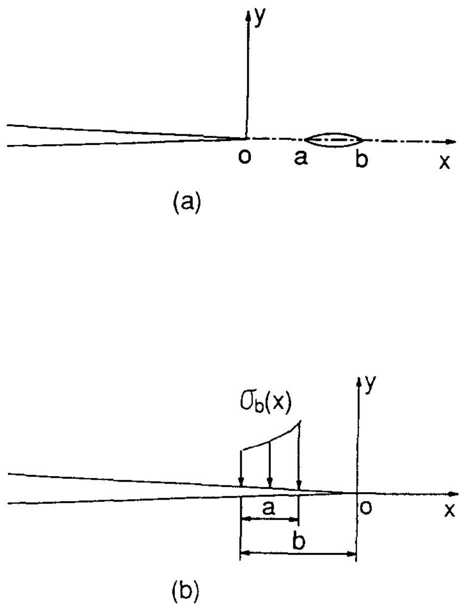

Fig. 1. Two models to study the collinear crack problem: (a) crack interaction model between a semi-infinite crack and a microcrack and (b) crack-wake bridging model.

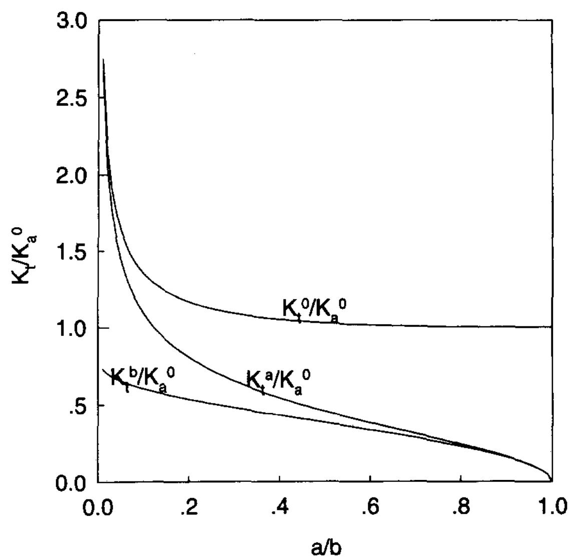

Fig. 2. True stress intensity factors normalized by the applied stress intensity factor.

microcracks. However, the stress intensity factor at the main crack tip is enlarged even further when the number of microcracks increases.

For the bridging model the ligament between the two cracks is considered as a bridge, Fig. 1(b). The true stress intensity factor K_t^O at the crack tip can be expressed as the superposition of the applied stress intensity factor K_a^O and stress intensity factor increment ΔK induced by the bridging stress on the ligament, i.e.

$$K_t^O = K_a^O + \Delta K \tag{3}$$

ΔK is usually negative and is referred to as the fracture resistance increment denoted by ΔK_R. Previous studies showed that ΔK_R due to the bridging stress $\sigma_b(x)$ can be written as[18]

$$\Delta K_R = \sqrt{\frac{2}{\pi}} \int_{b-a}^{b} \frac{\sigma_b(x)}{\sqrt{x}}\, dx \tag{4}$$

For quasi-brittle materials such as non-transformable ceramics and cements, the bridging stress $\sigma_b(x)$ usually decreases with crack face separation $w(x)$, i.e. it obeys a strain-softening law which can be approximated by[2]

$$\sigma_b(x) = \sigma_m\, [1 - w(x)/w_c]^n \tag{5}$$

where σ_m is the maximum bridging stress, n, softening exponent and w_c is the critical crack face separation corresponding to the saturated bridging zone length X, at which the bridging stress is zero. Further assumption that the crack faces remain straight during crack propagation leads to a simpler relationship between the bridging stress $\sigma_b(x)$ and the distance x to the crack tip, [6]

$$\sigma_b(x) = \sigma_m(1 - x/X)^n \tag{6}$$

The stress intensity factor increment due to the bridging stress can be estimated by substituting eqn (6) into eqn (4) and completing the integration. For the purposes of discussion ΔK_R for two bridging stress distributions: linear strain-softening and constant bridging stress, i.e. $n = 1{\cdot}0$ and 0 in eqn (6), respectively are calculated. Substitute eqn (6) with $n = 1{\cdot}0$ into eqn (4) and complete the integration, ΔK_R due to the linear strain-softening bridging stress can be obtained as

$$\Delta K_R = \sqrt{\frac{8}{\pi}}\, \sigma_m \sqrt{b}\, \{1 - k' - \frac{b}{3X}[1-(k')^3]\} \tag{7}$$

For the constant bridging stress σ_m, i.e. $n = 0$, we have

$$\Delta K_R = \sqrt{\frac{8}{\pi}}\, \sigma_m \sqrt{b}\,(1 - k') \tag{8}$$

As expected, the bridging stress reduces the stress intensity factor at the crack tip. The amount of crack shielding due to the bridging stress is related to the material parameters n and σ_m. This is different to the interacting collinear microcrack model where the stress intensity factors only depend on the dimensions of the microcracks and their spatial distribution. Corresponding to the multi-microcrack problem, the incremental resistances ΔK_R due to N ligaments with these two bridging stress functions are

$$\Delta K_R = \sqrt{\frac{8}{\pi}}\, \sigma_m \sqrt{b}\sum_{i=1}^{N} \{(\sqrt{i} - \sqrt{i - a/b} - \frac{b}{3X}[i^{3/2} - (i - a/b)^{3/2}]\} \tag{9}$$

for $n = 1{\cdot}0$ and

$$\Delta K_R = \sqrt{\frac{8}{\pi}}\, \sigma_m \sqrt{b}\sum_{i=1}^{N} \{(\sqrt{i} - \sqrt{i - a/b})\} \tag{10}$$

for $n = 0$.

Instability of Crack System

The instability of materials containing cracks is dependent on the resistance to crack propagation. For ideally brittle materials, unstable crack propagation would proceed if the net stress intensity factor at a crack tip is equal to or above the critical stress intensity factor K_{Ic}. In the interaction model, the true stress intensity factor K_t^0 at $x = 0$ is always the largest. Therefore, the instability of the system is always preceded by unstable propagation of the main crack. This implies that the interaction situation can not be observed unless there are pre-existing microcracks ahead of the main crack. Based on eqn (1), it is also seen that the fracture resistance reduces because of the presence of the collinear microcrack.

For the bridging model with $n = 1{\cdot}0$, the fracture resistance increment ΔK_R can be expressed as

$$\Delta K_R \Big/ \sqrt{\frac{8}{\pi}}\,\sigma_m \sqrt{X} = \sqrt{\frac{b}{X}}\,\{1-k' - \frac{b}{3X}\,[1-(k')^3]\} \qquad (11)$$

and for $n = 0$, one has

$$\Delta K_R \Big/ \sqrt{\frac{8}{\pi}}\,\sigma_m \sqrt{X} = \sqrt{\frac{b}{X}}\,(1-k') \qquad (12)$$

Shown in Fig. 3 is the change of ΔK_R, due to a bridge of fixed width a with increasing distance from the crack tip. It can be seen that ΔK_R decreases as the crack advances. This indicates that the single bridging ligament can not result in stable crack propagation when subjected to steady-state loading even if it enhances the effective fracture toughness. However, if discrete bridges are developed continuously as the crack advances, an increasing ΔK_R can be obtained because the bridging stress effects from the bridges near the crack tip are much larger than those far away from it. As a result, stable crack growth can be observed when ΔK_R is larger than the crack driving stress intensity factor increment, ΔK_a.

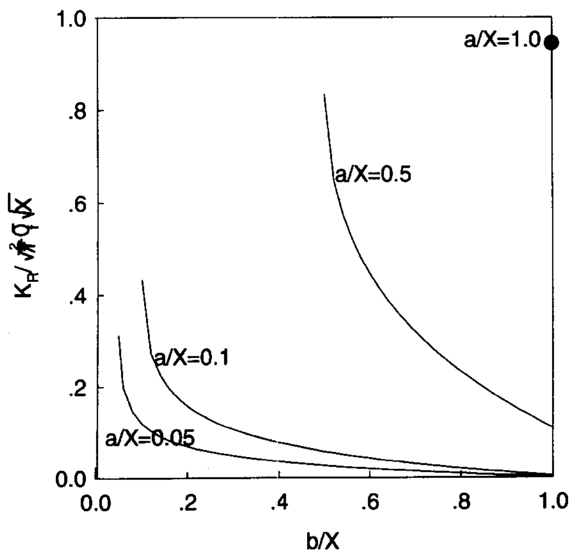

Fig. 3. Fracture resistance curves due to linear strain-softening bridging stress.

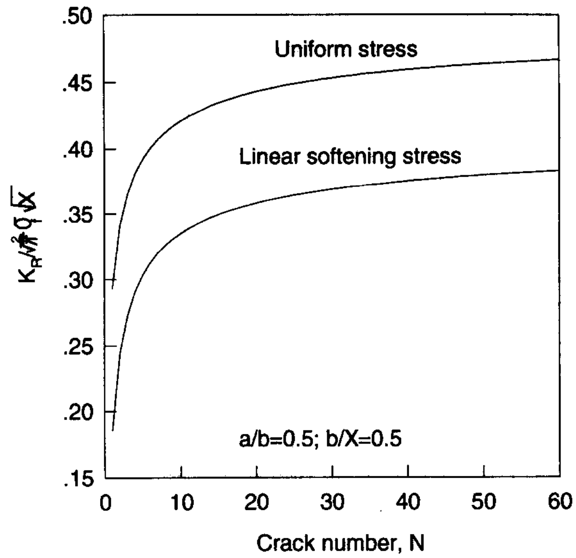

Fig. 4. Change of fracture resistance increment ΔK_R with microcrack number N.

The effect of crack number N on ΔK_R is shown in Fig. 4 where the distance between main crack tip and the most remote microcrack tip is assumed to be a constant b ($b/X = 0{\cdot}5$). For a given a/b ($0{\cdot}5$), the crack shielding effects due to bridging stresses of the ligaments increase with increasing crack number N and tend to a plateau value dependent on a/b.

Conclusions

The collinear crack problem can be simplified as two idealised mechanics models: crack interaction and crack bridging. But the two simplifications lead to two completely different conclusions. Crack propagation based on the interaction model is unstable. However, stable crack propagation can be achieved based on the crack bridging model if discrete bridges are continuously developed with the advancing crack. The increased fracture resistance from the bridging model are dependent on both geometry and material properties. Unless pre-existing microcracks are observed the collinear crack system found in the fracture of quasi-brittle materials is usually caused by crack bridging.

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