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Influence of porosity on Young's modulus and Poisson's ratio in alumina ceramics

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Abstract

This work presents a study on the influence of the porosity level on Young's modulus and Poisson's ratio of sintered alumina. A non-destructive technique using ultrasonic waves was used to determine the different parameters. Longitudinal and transverse wave velocities are measured by reflection method respectively at two frequencies: 10 and 5 MHz. The elastic modulus and Poisson's ratio were calculated from the measured ultrasonic velocities. The porosity dependence of the Poisson's ratio constitutes an originality as this ratio is considered as constant in many papers. Moreover, the authors take into account the pore shape modification during the densification in Young's modulus and Poisson's ratio calculations.

Keywords: Al_2O_3; Elastic modulus; Mechanical properties; Poisson's ratio; Porosity

1. Introduction

Ceramic materials present interesting properties such as a good hardness, high wear and corrosion and temperature resistance which makes them candidates for thermomechanical applications. However, these advantages are counteracted by the brittleness of the ceramics. Consequently, the presence of small defects such as pores can lead to a dramatic decrease in strength value. Therefore, the use of ceramics as structural parts is conditioned by the development of non-destructive evaluation techniques such as ultrasonic methods which allow the porosity level and pore size determination through Young's modulus and Poisson's ratio measurements.

This paper reports on a study of the porosity influence in alumina parts on Young's modulus and Poisson's ratio by correlating the transverse and longitudinal ultrasonic wave velocities data with the measured density and porosity. The ultrasonic velocity method is non-destructive and very precise.[1,2] Moreover, the ultrasonic velocity through solids mainly depends on the intramolecular and intermolecular interaction potential. In the case of ceramics, this potential depends on the microstructural characteristics such as grain nature and size, porosity etc.

Several previous papers[2–4] were already devoted to the study of Young's modulus dependence versus the porosity taking into account the spherical or cylindrical shape of pores. By comparison between experimental and calculated data, these authors have chosen for all porosity values (from a few to 40%) the cylindrical shape. Moreover, in these works, the Poisson ratio is always assumed to be constant versus the porosity.

In this work, the pore shape modification with the densification level is taken into account in the Young's modulus and Poisson's ratio calculations.

2. Theoretical approach

The relationships between longitudinal (V_L) and transverse (V_T) wave velocities with bulk modulus (K) and shear modulus (G) are:

$$V_L = \left[\left(K + \frac{4}{3}G\right)/\rho\right]^{\frac{1}{2}} \tag{1}$$

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$$V_T = [G/\rho]^{\frac{1}{2}} \quad (2)$$

where ρ is the density of the material. For a porous medium, the density can be related to the porosity by the relation (3):

$$\rho = \rho_0(1-p) \quad (3)$$

where ρ_0 is the theoretical density and p is the porosity level

Eqs. (1) and (2) become:

$$V_L = \left[\frac{(K+4/3G)}{\rho_0(1-p)}\right]^{\frac{1}{2}} \quad (4)$$

$$V_T = [(G)/\rho_0(1-p)]^{\frac{1}{2}} \quad (5)$$

For isotropic materials, the bulk and shear moduli are linked to Young's modulus (E) and Poisson's ratio (σ) by:

$$K = \frac{E}{6(0.5-\sigma)} \quad G = \frac{E}{2(1-\sigma)} \quad (6)$$

In the hypothesis of pores presenting a spheroid symmetry, the Young's modulus-porosity dependence can be written following Boccaccini's[3] Eq. (7) derived from Ondracek's work.[5]

$$E(p) = E_0\left(1-p^{\frac{2}{3}}\right)^S \quad (7)$$

with

$$S = 1.21\left[\frac{z}{x}\right]^{\frac{1}{3}}\left[1+\left(\left(\frac{z}{x}\right)^{-2}-1\right)\cos^2(\phi)\right]^{\frac{1}{2}} \quad (8)$$

$\frac{z}{x}$ is the mean axial ratio (shape factor) of the pores (for a spherical shape pore: $\frac{z}{x}=1$, $\cos^2(\phi)$ is the factor of pore orientation (for an isotropic medium $\cos^2(\phi)=\frac{1}{3}$) this allows to deduce:

$$V_L = \left[\frac{E_0}{2\rho_0}\frac{(1-\sigma)}{(0.5-\sigma)(1+\sigma)}\frac{\left(1-p^{\frac{2}{3}}\right)^S}{(1-p)}\right]^{\frac{1}{2}} \quad (9)$$

$$V_T = \left[\frac{E_0}{2\rho_0(1+\sigma)}\frac{\left(1-p^{\frac{2}{3}}\right)^S}{(1-p)}\right]^{\frac{1}{2}} \quad (10)$$

The dynamic elasticity allows to write longitudinal and transverse wave velocities for isotropic solids:

$$V_L = \sqrt{\frac{\lambda+2\mu}{\rho}} \quad \text{and} \quad V_T = \sqrt{\frac{\mu}{\rho}} \quad (11)$$

Moreover, according to the static elasticity, the Young's modulus *(E)* and the Poisson's ratio (σ) are given by the following relations:

$$E = \mu.\frac{3\lambda+2\mu}{\lambda+\mu} \quad \text{and} \quad \sigma = \frac{\lambda}{2(\lambda+\mu)} \quad (12)$$

By combining the relations (11) and (12), we obtain the relations linking the elastic characteristics with the wave velocities:

$$E = \rho V_T^2\frac{4V_T^2-3V_L^2}{V_T^2-V_L^2} \quad \sigma = \frac{2V_T^2-3V_L^2}{2V_T^2-2V_L^2} \quad (13)$$

3. Experimental procedure

3.1. Elaboration of ceramics

Alumina pellets (diameter: 2.5 cm, thickness: 1 cm) were shaped by uniaxial pressing from Alcoa A16SG powder and then sintered at different temperatures (in the range 1300–1700°C) for 2 h in order to produce a wide range of porous samples (from 25% down to 2% of porosity). The porosity level was measured by Archimedes' method. The porosity of various alumina ceramics are presented in Table 1, P_o corresponds to the open porosity (fully interconnected pores of complex shapes), P_c, to the close porosity (fully isolated pores of nearly spherical shape) and P_T to the total porosity. We notice that the open porosity P_o is predominant in the materials sintered up to 1430°C, then beyond this sintering temperature, the closed porosity P_c becomes more important.

3.2. Experimental device

We have chosen to use the reflection method (pulse echo method) that needs the use of only a single probe. A coupling medium between the ceramic pellet and the ultrasonic transducer is necessary. The ultrasonic pulses are provided by a 5072PR SOFRANEL generator. An electrical impulse with high amplitude and short duration excites the piezoelectric transducer vibrating on the fundamental mode through the sample, and after reflections on the opposite face returns to the transducer. After propagation in the material, the

Table 1
Influence of the sintering temperature on the porosity

T (°C)	1300	1400	1410	1420	1430	1470	1500	1600	1700
P_o (%)	24.3	18	11.2	8.9	6.3	0.5	0.2	0.1	0.1
P_c (%)	0.8	0.6	0.8	1.1	1.9	4.5	3.7	2	1.7
P_f (%)	25.1	18.6	12	10	8	5	3.9	2	1.8

output signal is displayed on the screen of a numerical oscilloscope.

The longitudinal or transverse wave velocities are obtained from the measurement of the round-trip time between two echoes corresponding to the propagation in the sample, and the thickness of the sample.

The knowledge of the transit time through the thickness of the sample allows the determination of the longitudinal or transverse wave velocities according to the mode of mechanical vibration (longitudinal or transverse mode).

The relative precision on the velocity measurement by this method is estimated to 10^{-2}%.

4. Results and discussion

4.1. Wave velocities values

The measured transverse and longitudinal velocities plotted versus total porosity (P_T) are shown in Fig. 1. It can be observed that the slope is approximately twice bigger in the case of longitudinal waves which would suggest that compression waves are more sensitive to the porosity than shear waves.

4.2. Young's modulus calculation

From the values of velocities for a pore-free sample deduced by extrapolation (no porosity) $V_{L0} = 10904$ m/s and $V_{T0} = 6399$ m/s, a Young's modulus value of 401 GPa is calculated; this value is very close to literature data[6,7].

Fig. 2 shows the variation of Young's modulus calculated from measured velocities and Eq. (13) versus the pore volume fraction. It can be noted that the Young's modulus presents a relatively linear decrease according to the empirical relation: $E(p) = E_0$ (1–2.12p). On the same figure, the Young's modulus values calculated from Boccaccini' s relation by using $\frac{z}{x} = 1.5$ which corresponds to cylindrical shape and $\frac{z}{x} = 1$ corresponding to spherical shape are presented. It can be noted that the data obtained with $\frac{z}{x} = 1.5$ is in agreement with experimental data for high porosity samples (>12%) which confirms the cylindrical shape for open pores.

For the lower porosity level, a close porosity appears which certainly induces theoretically a shape modification of the pores towards a spherical shape. The experimental data are close to the theoretical values obtained with the z/x ratio equal to unity.

4.3. Poisson's ratio calculation

The Poisson's ratio variation with porosity calculated from velocity values [relation (13)] is presented in Fig. 3. It can be noted that the porosity influence is not so negligible, contrary to the often-used hypothesis.[2–4] Moreover the relations (9) and (10) show that if the porosity level does not influence the Poisson's ratio, the V_L and V_T dependence versus the porosity would be similar, which is not experimentally observed. Indeed, Fig. 1 presenting the porosity level dependence of transverse and longitudinal velocities shows a slope twice as big for the longitudinal wave. This means that Eqs. (9) and (10) deduced from Boccaccini's model do not describe correctly the wave velocity/porosity dependence. So, we propose to take into account the different behaviours of transerve and longitudinal waves versus porosity by adapting another model, the Phani's one.[8]

The Phani's[8,9] relation, reported by Kathrina and Rawlings[10] expresses the wave velocity versus porosity by $V = V_0(1-p)^m$ where m is a constant, V_0 the velocity in the pore-free material. By considering different behaviors for longitudinal (compression strain) and transverse (shear strain) waves, so we propose the following relations:

$$V_L = V_{L0}(1-p)^m \quad \text{and} \quad V_T = V_{T0}(1-p)^n \tag{14}$$

with $m \neq n$ and V_{L0}, V_{T0} are transverse and longitudinal velocities for the pore-free material.

By fitting these relations with experimental data, we obtain 1.17 and 1.03 for m and n coefficients respectively.

These results show that the polarization of the wave is very sensitive to the pore shape (in the case of a spherical symmetry, m and n should be equal). This analysis is comparable to G. Ondracek's one[5], that has shown the influence of the grain orientation and shape as well as of the porosity on the electrical field direction in dielectric alumina based composites.

Relationships (13) and (14) allow by undertaking a second order limit development versus the porosity, to determine a new expression of the Poisson's ratio:

$$\sigma = \sigma_0\left[1 + pF(m,n) - p^2G(m,n)\right] \tag{15}$$

with:

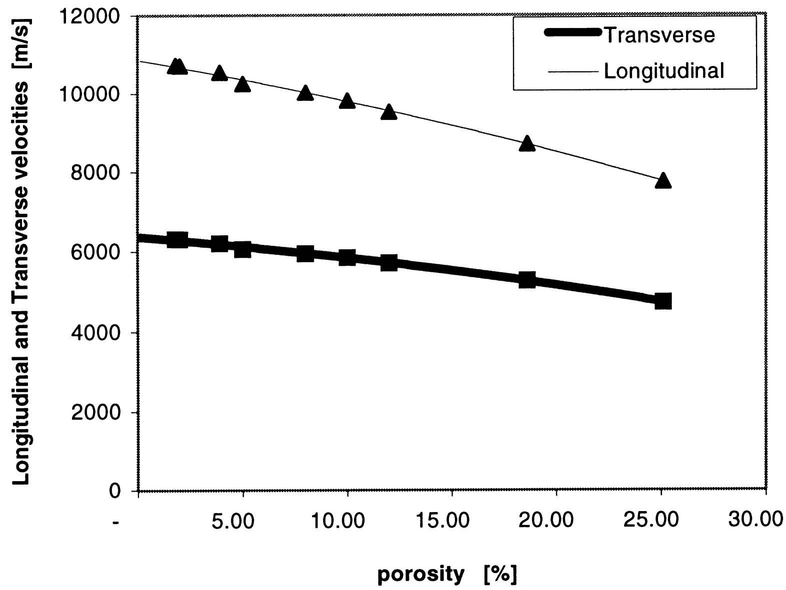

Fig. 1. Porosity influence on longitudinal and transverse velocities.

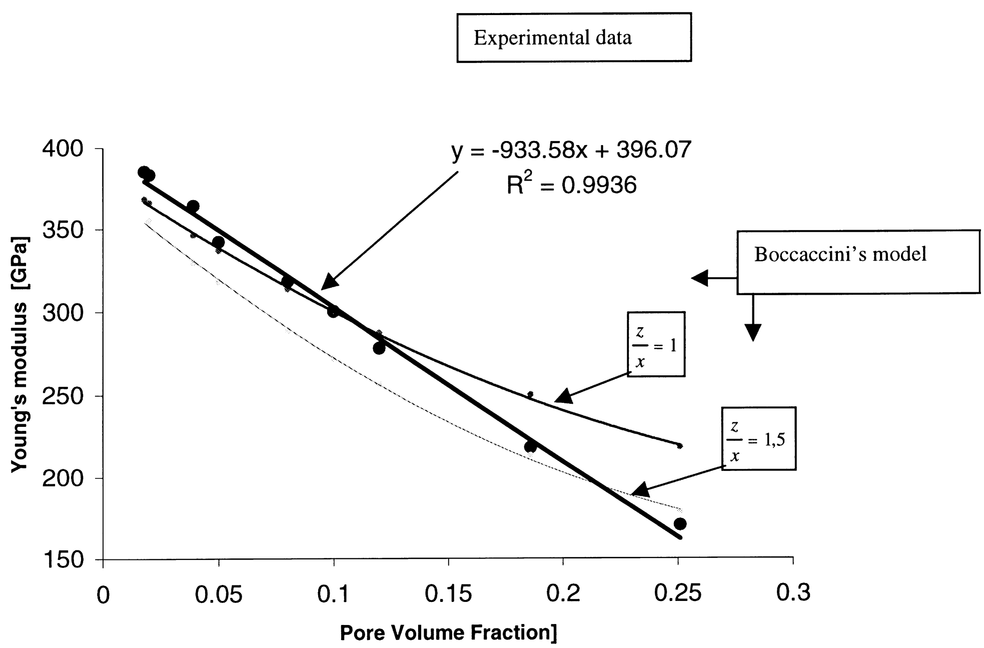

Fig. 2. Porosity influence on Young's modulus.

$$F(m,n) = 2(m-n)\frac{V_{L0}^2 V_{T0}^2}{\left(3V_{L0}^2 - 2V_{T0}^2\right)\left(V_{L0}^2 - V_{T0}^2\right)} \quad (16)$$

$$G(m,n) = 4\frac{\left(3mV_{L0}^2 - 2nV_{T0}^2\right)\left(mV_{L0}^2 - nV_{T0}^2\right)}{\left(3V_{L0}^2 - 2V_{T0}^2\right)\left(V_{L0}^2 - V_{T0}^2\right)} \quad (17)$$

It can be noted that in the case of a spherical symmetry *(m=n)* the relations (16) and (17) become:

$F(m,n) = 0$ and $G(m,n) = 4$

Therefore, it is stated that only the term of the second order in "p^2" contributes to the Poisson's ratio.

This analysis was applied to the experimental data obtained for alumina samples with:

$V_{L0} = 10904\ m/s$; $V_{T0} = 6399$ m/s;
$m = 1.17$ and $n = 1.03$

and has allowed to deduce: $F(m,n)=0.07$ and $G(m,n)=6.16$. The calculated values for the Poisson's ratio according to our model are fitted in Fig. 3 for

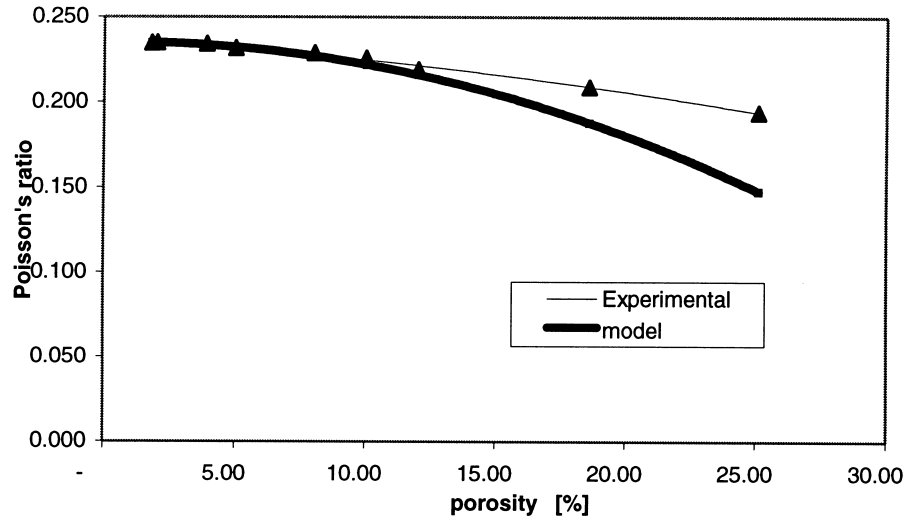

Fig. 3. Porosity influence on Poissons ratio.

comparison with experimental values. An excellent correlation between the model and the experiment is obtained up to 12% of pore fraction value. According to Table 1 data, it is clear that above this pore fraction value the whole porosity is open and presents interconnected structure.[11] We think that the divergence of the model is probably due to the interconnection between complex shape pores (threshold of percolation of the porosity is estimated approximately to 15 vol.%), what induces a diffusion phenomenon of ultrasonic waves. In this case, the material behaves as a two-phase medium and therefore very dispersive. In this effect, when the size of the obstacles become close to the wavelength of the propagating wave, the dispersion phenomena appear showing the non-homogeneity of the propagating medium (solid, emulsion,. . .).[12–15] Moreover, the change of the shape and orientation of the obstacles with respect to the propagation direction induce inevitably a modification of dispersion curves. In this way the correlation between the velocity variation and the geometry of pores could be a very interesting tool to characterize the shape of the pores in porous media like ceramic materials.

5. Conclusion

A very precise non-destructive technique on ultrasonic waves was used to determine the influence of the porosity on the compression and shear wave velocities through alumina samples. From those velocity values, Young's modulus and Poisson's ratio were determined for alumina parts containing different pore fractions. A linear decrease of Young's modulus versus the porosity level was observed. Boccaccini's model describing the Young's modulus dependence on the porosity can be applied on condition that the porosity shape change is taken into account. More sensitivity of the longitudinal velocity to the porosity was observed which suggests a non negligible influence of the porosity on the Poisson's ratio.

An adaptation of Phani's model allowed the deduction of the Poisson's ratio-porosity dependence. This model fits very well with the experimental data up to the pore percolation limit. For a higher porosity value, the pore interconnection induces dispersion phenomena of ultrasonic waves which necessitates a more complex interpretation of acoustic results.

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