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Global description of crack propagation in ceramics

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Abstract

The subject of crack propagation in brittle materials is sometimes confusing, since many terms or concepts are not always clearly defined, as toughness, sub-critical crack propagation, threshold, *R*-curve, etc. However, crack propagation can be simply and rigorously analysed by considering the equilibrium point in terms of energy release rate, as defined by the Griffith criteria, based on fundamentals of physics. The departure from this equilibrium leads to crack velocities related to dissipative mechanisms. Reinforcement mechanisms of any type can shift the equilibrium towards higher driving force values, giving an increase of crack resistance. The degradation of this reinforcement, as the case of cyclic fatigue, simply reduces the resistance to propagation. The total description of the behaviour is therefore included in the crack velocity—energy release rate ($v - G$) graph. However, the major point is the equilibrium, because in addition to its theoretical importance, it is of practical interest for brittle materials, since it corresponds to an energy value below which no propagation occurs.

Keywords: Fracture; Crack propagation; Fatigue

1. Introduction

The first description of brittle fracture and crack propagation was proposed by Griffith.[1,2] His idea was very simple and is still referred as the Griffith criterion. It is well known that this criterion is derived from an energy balance based on the first thermodynamics principle. It simply compares the magnitude of the strain energy released rate (so-called G, that represents the mechanical driving force) to that consumed by a crack increment (so-called R, that represents the resistance to cracking).[3] If the quantity $G - R$ is null the system is equilibrated, if it is positive or negative, the crack grows or heals, respectively.

A careful reading of the large bibliography on fracture of brittle solids published along this century would be somewhat perplex. From the earlier work of Griffith the description has been much refined and a new reader would discover the terms and concepts of toughness, catastrophic failure, sub critical crack growth, *R*-curve, fatigue crack propagation. If individually considered all those concepts can be easily explained. However, trying to assemble the pieces of the jigsaw is difficult and several aspects are not clear, at least in some works. For instance the Griffith criterion has often been described as a criterion for catastrophic failure, corresponding to toughness, which is according to Maugis[4] and to Lawn[5] a current misinterpretation or confusion. We can easily imagine this new reader asking several questions. What does the toughness exactly represent? Can the so-called sub-critical crack growth, often referred as propagation below the toughness, lie in the framework of the Griffith criterion? The presence of a threshold below which no propagation occurs is still in debates in the ceramic society; what is its meaning in the framework of the Griffith criteria? What do we measure on a system where a stable crack propagation occurs? What is the crack velocity influence on a *R*-curve measurement or the reciprocal?

Griffith applied his theory to the fracture of glass where the crack resistance is equal to the surface energy 2γ. He deduced his proposal from experiments conducted on an unstable crack geometry: tensile test. Maugis[6] thinks this is the reason why a confusion is often made by considering the criterion as describing unstability. For instance, the early experiments of Obreimoff[7] on cleavage of mica and of Roesler[8] on silica glass clearly were conducted on stable system. They clearly show that the Griffith analysis sets the equilibrium of the crack, but not the stability of this equilibrium. It is therefore not a rupture criteria.

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The analysis of fracture mechanics was completed by Irwin[9] who proposed the concept of stress intensity factor, K, as a measure of the local stress at the crack tip. A fracture criterion K_c is deduced and related to the resistance R from the well-known relationship $R = K_c^2/E$ for plane stress. As pointed out by Lawn[5] the advantage of the analysis based on stress intensity factor is the possibility of calculation for many crack configurations and their additivity. However, the analysis is still in the framework of Griffith theory.

A probable cause of difficulty with fracture mechanics is the time effect. It was first mentioned by Grenet[10] and Orowan[11] from the observation of strength degradation of glass with time and delayed fracture. Orowan showed that the so-called fatigue effect could be explained in the framework of the Griffith theory and he postulated that this implies the existence of a safe stress. Later Wiederhorn[12,13] showed that finite crack velocity could be measured, and that the velocity could be related to G by an univocal law, leading to the now well known $v - K$ or $v - G$ curves. An impressive collection of experimental data measured during the sixties was reviewed by Wiederhorn and Johnson.[14] Since the velocity can be low, it has been referred as slow crack growth and many times as sub-critical crack propagation. The phenomenon is highly promoted by water molecules, probably by a stress corrosion mechanism.[15–17]

The term sub-critical might be confusing for a reader out of the field. It should imply two different regimes 'critical' and 'sub'. The former would be defined by the Griffith criteria and the later corresponding therefore to a propagation with a negative balance energy. Under this misinterpretation, the existence of a threshold below which no propagation occurs is often questioned. The correct interpretation simply shows that the threshold is the point of the Griffith criteria and when the driving force increases, the velocity rises continuously from 0 to sonic values.[5]

A more complete thermodynamic analysis was proposed by Rice.[18,19] as an extent of the Griffith criterion in order to take into account kinetic analysis. He reformulated the analysis by considering an entropy contribution in agreement with the second thermodynamic law. Null crack velocity, leads to the Griffith criterion $G = R$ which is therefore included in the analysis and corresponds to a null entropy term.

Outside the equilibrium, application of the second thermodynamic law shows that the crack driving force $(G - R)$ and v are of the same sign, meaning that crack propagates for $G - R > 0$, and heals for $G - R < 0$. It even states that when $G - R$ increases, v increases following a $v - (G - R)$ relationship that will depend on the dissipation mechanism. However, the analysis is made on macroscopic scale thermodynamic and the possibility of healing will depend on the mechanism (reversibility of chemical reaction, crack closure impediment by debris or grains, etc.). Maugis[4] and Lawn[5] have applied this general concept to a more physical description, the entropy contribution being related to internal friction or chemical process at the crack tip, respectively.

The purpose of this paper is to show that the crack behaviour in ceramics, including the so-called sub-critical crack propagation, R-curve and cyclic fatigue can be simply described by the extended Griffith criterion. Within this framework, the description is only made with rigorous general terms that can be understood by the non-specialist of fracture mechanics.

1.1. $v - G$ relation: general pattern

The Griffith criterion is based on an energy conservation principle for a system that contains a crack of length c in equilibrium, submitted to traction force. The energy is composed on the one hand on mechanical energy (strain energy and work induced by loading) and on the other hand on the energy dissipated by the creation of new surfaces. The equilibrium of a crack is obtained when the derivative of the mechanical energy with respect to the crack length (so-called: strain energy released rate, G) is equal to the surface energy per unit length, i.e. the crack resistance. Since it was initially applied to glass or single crystals, i.e. materials with very small-scale microstructure or without microstructure, the crack resistance is simply equal to the surface energy γ:

$$G = 2\gamma \tag{1}$$

The criteria also states that crack progresses when the left member becomes higher or heals when it becomes lower than the right member. There is no mention of velocity, although of importance when conducting experiments. In the pioneer works of those who tried to confirm this relation and calculate γ, a delay to reach the equilibrium is often reported.[7,8] This has been later rationalised by measuring the crack velocity as a function of G. A ground breaking series of examples was conducted on mica by Wan et al.[20] Mica is very interesting since it presents a weak cleavage plane, that leads to a very planar crack, without mechanical crack closure impediment, thus, facilitating the crack retreat. They pointed out that the way to definitely determine the equilibrium is to record a series of forward and backward velocities around it. When only crack extension is performed, which is the case of most experiments, the equilibrium is more difficult to observe since it corresponds to a threshold with an apparent null crack velocity. It is clearly seen in mica (Fig. 1) and it is shown to depend on the humidity content of the environment. The real surface energy is only obtained for a virgin specimen broken in vacuum and in presence of a reactive or adsorbed atmosphere, the surface energy is reduced. As a generalisation, it is better to consider the right hand term of Eq. (1) as the Dupré work of adhesion, giving:

$$G = R = 2\gamma \quad \text{under vacuum} \tag{2a}$$

$$G = R_e = 2\gamma_e \quad \text{in a given environment} \tag{2b}$$

with $\gamma_e < \gamma$. Thus the equilibrium of a given crack may be changed either by variation of the loading or by the environmental conditions.

The initial Griffith criteria corresponds therefore to this point of equilibrium. When we move above or below this equilibrium point, the crack velocity increases (backward or forward). This is described by the generalised thermodynamic description of Rice, when the entropy term exists:

$$(G - R)v = \dot{S} \geq 0 \tag{3}$$

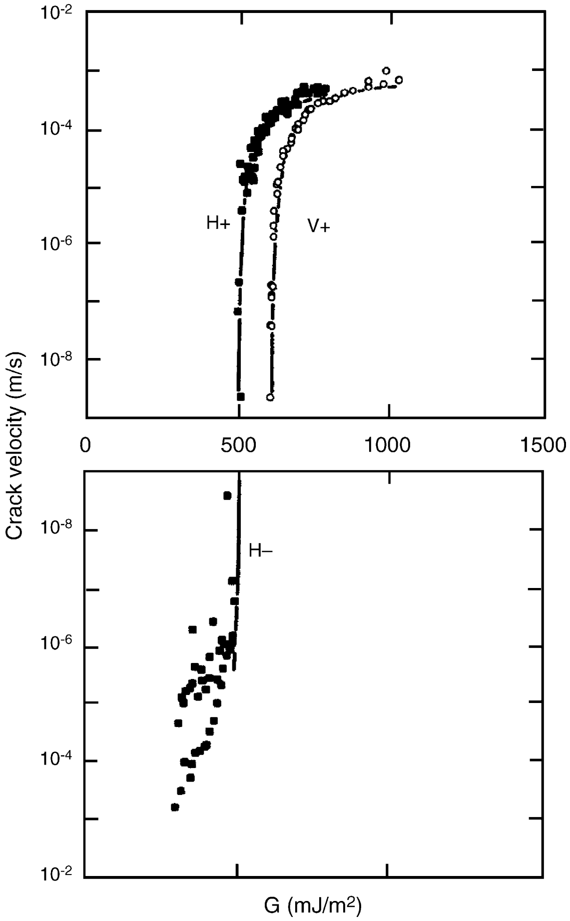

Fig. 1. Crack velocity vs. mechanical energy release rate measured on mica around the equilibrium. Increasing the load (i.e. G) leads to crack growth with increasing velocity, represented by V+ for virgin specimen. When the load is decreased below the threshold it moves backward (healing, represented by H−). However, note that owing to atom mismatch and probably atmosphere the crack re-propagate for a lower threshold, represented by H+ (reprint from Wan[20] copyright with permission from Journal of Materials Research and the author).

where $\dot{S}$ is the entropy creation rate. Rice description therefore shows that the velocity is related to irreversible (energy dissipative) mechanisms. Wan et al.[21] have proposed a general $v(G)$ law based on activation theory, i.e. probability of transition. The kinetics is attributed to discrete thermal fluctuations over energy barriers. This analysis is therefore phenomenological and is able to describe different mechanisms. They obtain a classical expression where the velocity is a hyperbolic sine function of G. These assumptions can be precisely illustrated if very low velocity can be measured (Fig. 2).

Maugis[4] considers that the velocity is given by a function that depends on the dissipative mechanism:

$$(G - R)v = R\chi(v) \tag{4}$$

where $\chi(v)$ is a function related to internal friction at the crack tip. This general approach has been satisfactory applied to polymers and problems of adhesion[4,6] but no echo appeared in the ceramic community, because viscoelastic losses are generally weak in ceramic materials. The mechanism rather proposed is a stress corrosion of the ceramic by water molecules at the crack tip. Fig. 3 shows typical examples of $v - G$ curve obtained in glass under different humidity contents (Wan et al.,[21] from data of Wiederhorn[22]). These curves are generally separated into three kinetics branches and a threshold that represents the equilibrium. As shown in Fig. 3 this equilibrium depends on the water content, in agreement with Eq. (2b).

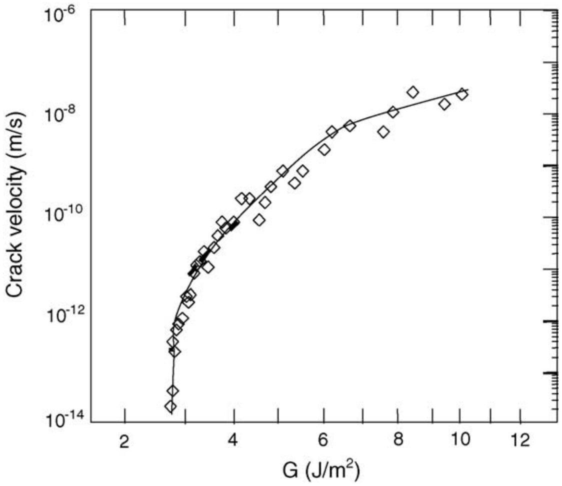

Fig. 2. Crack velocity vs. mechanical energy release rate measured on glass fibres in air. The graph is deduced from lifetime under constant load (so-called static fatigue) applied to a mesh of about 1000 filaments, after statistical treatment. Such a configuration allows very low velocity to be obtained and also a good definition of the curve since each point correspond to one rupture time in the experiments (after Pauchard[39]).

A small perturbation from the equilibrium gives a steady state crack velocity for a given G. This velocity may be limited by a reaction rate between the corrosive species and the ceramic bound, leading to the so-called first stage. For a given water activity, the velocity can be limited by the arrival of the water molecule to the crack tip, leading to a second stage. For even higher velocity crack extension operates without the need of water, under quasi vacuum conditions. Such a mechanism has

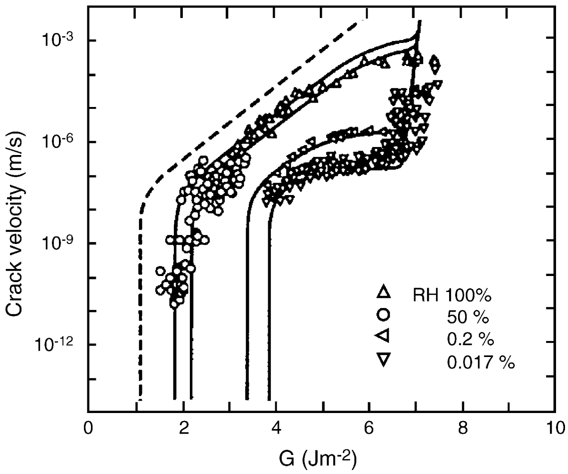

Fig. 3. Crack velocity vs. mechanical energy release rate of soda-lime glass in air under different humidity content (reprint from Wan[21], copyright, with permission from Elsevier Science).

been modelled by thermally activated processes and gaseous diffusion. In the first stage, the crack rate is then related to G by:

$$v_{\mathrm{I}} = 2\nu_{\mathrm{o}}a_{\mathrm{o}} \exp\left(\frac{-\Delta F}{kT}\right) \sinh\left(\alpha\frac{G-\gamma_{\mathrm{e}}}{kT}\right) \quad (5)$$

where a_{o} is the atomic spacing; ν_{o}, fundamental lattice vibration frequency; ΔF and α physical parameters of the activation mechanism. Note that other mechanisms with two sets of velocity limitation could give such type of curves, but still in the framework of Griffith equilibrium and Rice description of velocities.

In the second stage water diffusion controls the crack rate which is then given by

$$v_{\mathrm{II}} = \frac{64G}{3\pi Ea_{\mathrm{o}}\ \ln(a/a_{\mathrm{o}})}\frac{a_{\mathrm{o}}^{3}p}{\sqrt{2\pi mkT}} \quad (6)$$

where E is the Young modulus; p, the partial pressure of the atmosphere and m the molecular mass of the fluid species.

In fact the two processes acting in series, the overall velocity is given by:

$$v = \left(\frac{1}{v_{\mathrm{I}}} + \frac{1}{v_{\mathrm{II}}}\right)^{-1} \quad (7)$$

For higher velocities the propagation occurs under vacuum condition. A formalism similar to Eq. (5) has been proposed by Wan et al.,[21] but where the crack velocity is null for $G=\gamma$. However, in this stage, crack velocity can also be related to internal friction, close to crack tip.[4,6] The phenomenological relation $v = AK_{\mathrm{I}}^{n}$ (with n showing high values), frequently used to described the velocity, is compatible with the above analysis.

It is more difficult to make precise measurements on polycrystalline ceramics, especially of coarse microstructure. Yttria stabilised zirconia polycrystal (Y-TZP) is interesting owing to its fine grain size (lower than 1 μm). Fig. 4 presents a set of results measured in different environments over a large range of crack velocities. The measurements conducted in air are the most precise since the easiest to conduct. Clearly the behaviour is similar to that observed on glass and single crystals, with the three different propagation stages mentioned above. At low crack driving force, crack velocity tends to a vertical asymptote, showing again the existence of an equilibrium value, related to the crack resistance in air defined by the Griffith Criteria. Results are also presented for vacuum condition. Although more difficult to obtain, these results suggest the presence of an equilibrium. It is surprising that a threshold under vacuum condition has not been the subject of consideration in the literature, although representing a major meaning in terms of fracture. As for glass, stages I and II can be fitted to Eqs. (5)–(7).

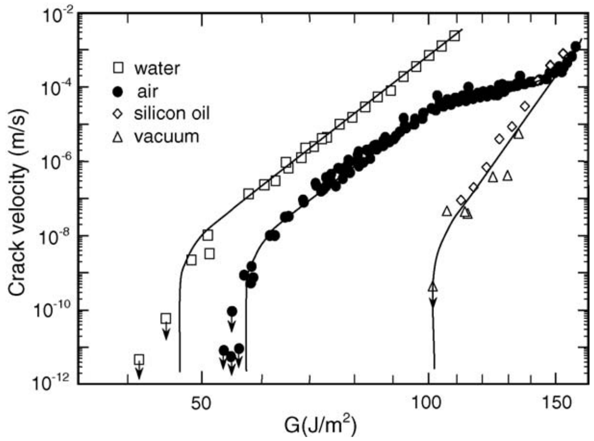

Fig. 4. Crack velocity vs. mechanical energy release rate of a 3% yttria stabilised zirconia in different environments. The data were obtained by double torsion by constant loading and constant displacement. This fine grain material allows a good reproducibility of the measurements (data from Ref. 34).

1.2. Reinforcement and $v-G$ relations

For Y-TZP, threshold in air and vacuum give a crack resistance of 30 and of 50 J/m^2, respectively (cf. Fig. 4). In contrast to the case of glass and single crystals,[a] these values are far above the surface energy that can be measured by the heat of vaporisation in ceramics (on the order of 1 J/m^2 [5]). This means that the crack resistance R (or R_{e}) is not only related to the surface energy but also to additional energy dissipation. The two main mechanisms contributing to energy dissipation in polycrystalline ceramics are phase transformation toughening and crack bridging. The Eq. (3) must be transformed as:

$$R = 2\gamma + R_{\phi} \quad \text{under vacuum} \quad (8a)$$

$$R_{\mathrm{e}} = 2\gamma_{\mathrm{e}} + R_{\phi} \quad \text{in a given environment} \quad (8b)$$

where R_{ϕ} is the contribution of microstructural related reinforcement to the crack resistance. This contribution in the case of Y-TZP is about ten times that of the surface energy. The same order of magnitude is observed for most polycristalline ceramics and the difference is even much higher in some polymers or metals. Since the crack propagation in polycrystalline ceramics is always at least partly intergranular, bridging will (more or less) operate. It is known that[23,24] its contribution increases with crack extension (the longer the crack the higher the number of bridging ligaments). Thus, R_{ϕ} is function of the crack length, a. A typical example is shown in Fig. 5a) for three alumina ceramics of grain sizes ranging between 1.9 and 12.7 μm. The shape is classical of that observed in ceramics: a rising part followed by a plateau caused by the extension and the saturation of the bridging. This reinforcement depends on the microstructure and is higher for coarse materials (a transition from inter to transgranular propagation is however observed for coarser grain size, leading to a drop of reinforcement). This reinforcement leads to a shift of the $v-G$ curve towards high G values which depends on the microstructure (Fig. 5b). The subtraction of the reinforcement, $R_{\phi}(a)$ leads to a single curve, therefore independent of the microstructure (Fig. 5c). In the end since the reinforcement depends both on the crack length and on the microstructure, the $v-G$ relation is not an intrinsic property of a material. It may

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[a] Even in the case of single crystal or glass there is a slight discrepancy between the surface energy measured by vaporisation and the crack resistance, owing to the dissipation unavoidably associated to mechanical fracture.

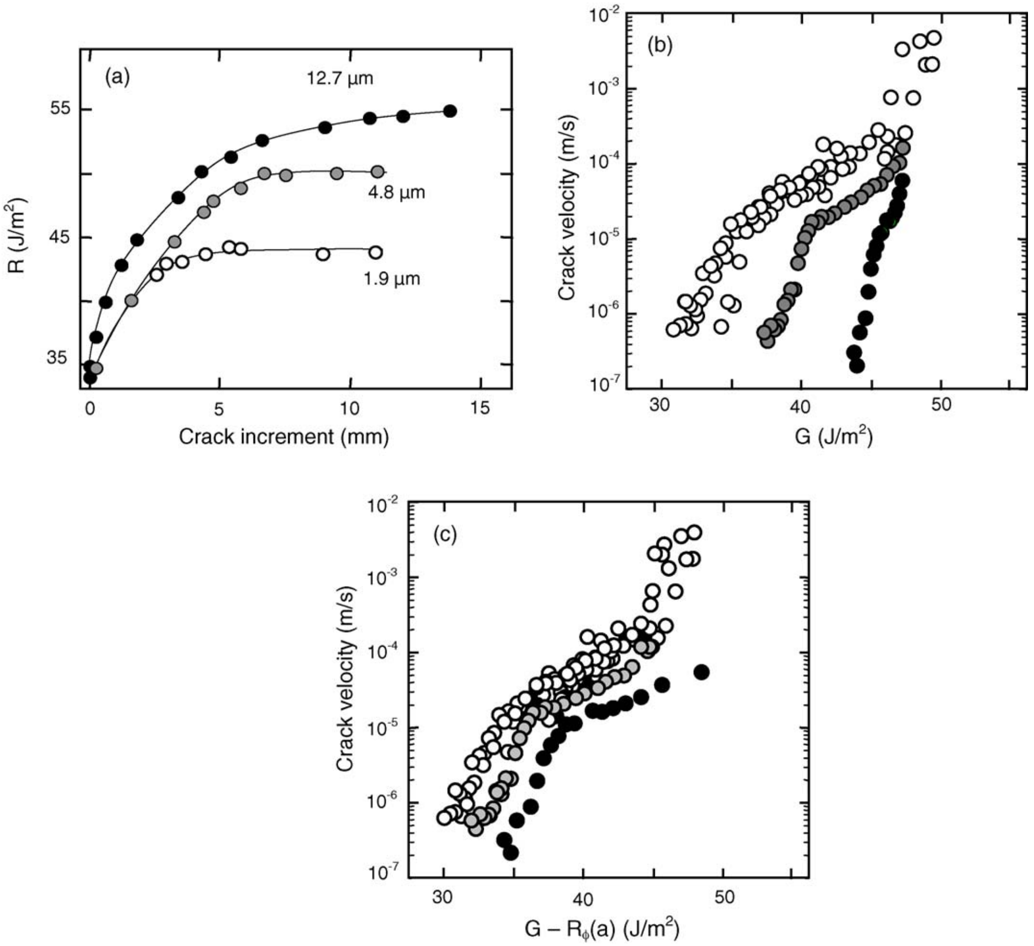

Fig. 5. Example of crack behaviour in alumina of different grain sizes (data from Ref. 25. (a) Reinforcement due to dissipative mechanisms, R_Φ (a) vs. crack length. In this range an increase of grain size leads to an increase of the reinforcement, mostly by bridging. (b) Crack velocity vs. mechanical energy release rate. The data are obtained by double torsion under constant displacement (load relaxation). In this case the initial (recorded) velocities are the highest and decreases when crack length increases, leading to an augmentation of the reinforcement. (c) Crack velocities as a function of $G - R_\Phi$ (a). By suppressing the reinforcement, the different materials define a unique plot.

depend on the method used for the measurement.[26,27] Moreover, even for a given method, it should depend on the loading configuration (for instance constant loading where crack velocity, da/dt, increases with crack length, a or load relaxation where da/dt decreases with crack length). In contrast, an intrinsic property is a $v - G^*$ relation where:

$$G^* = G - R_\Phi(a) \tag{9}$$

G^* represents the net crack tip driving force.

The case of alumina is relatively simple since at least as first approximation R_Φ depends only on the crack length. Other cases are more complex, and R_Φ can also depend on the local stress at the crack tip, so on G. This is the case of the mechanisms acting at the crack front, such as microcracking and phase transformation toughening. A good example of the last case is the reinforcement in zirconia ceramics. It is based on the tetragonal to monoclinic transformation of particles at the crack tip due to the local high stresses. The associated volume expansion leads to stress shielding. It can be shown that transformation zone size is related to local stresses at the crack tip, so to G^*. From the model of McMeeking and Evans[28] the zone width, h is proportional to G^*:

$$h = \alpha\, G^* \tag{10}$$

The reinforcement is also proportional to the zone width (the larger the zone width, the larger the reinforcement):

$$R_\Phi = \alpha'\, h \tag{11}$$

Thus, R_Φ is proportional to G^*:

$$R_\Phi = CG \tag{12}$$

This means that the reinforcement increases with the external load, leading therefore to lower slope of the $v - G$ graph, as shown schematically in Fig. 6. A comparison of $v - G$ curves in single crystal (without transformation toughening) and of one Y-TZP polycrystal is shown Fig. 7. There is an important shift between the two sets of curves that increases with G in agreement with Eq. (12).

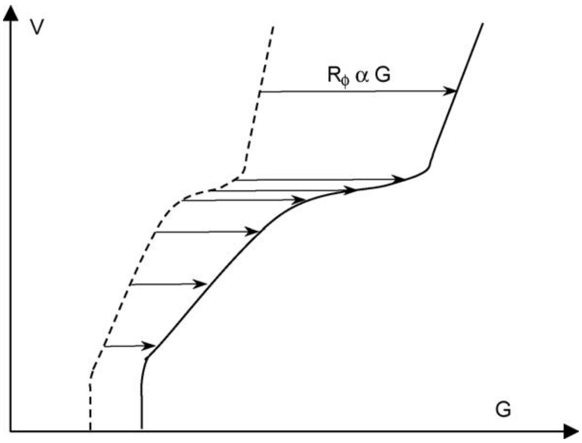

Fig. 6. Schematic drawing of the effect of a reinforcement proportional to the load on the shape of the crack velocity vs. mechanical energy release rate graph. The graph of a material without reinforcement is shifted, leading to a decrease of the slope.

A generalisation of Eq. (9) can therefore be expressed as:

$$G^* = G - R_\phi(a, G^*) \tag{13}$$

1.3. $v - G$ relations and cyclic fatigue

For many years the crack propagation mechanisms of ceramics was solely attributed to stress corrosion, even under varying stresses. For instance, Evans and Fuller[29] showed that cyclic fatigue of glass or china clay materials could be predicted from integration of double torsion results. The first evidence of cyclic effect in ceramics was reported by Dauskardt et al.[30] on a Mg-PSZ at room temperature. They pointed out an acceleration of the crack propagation under cyclic loading. They also showed that crack propagation under cyclic loading could occur at stresses lower than sub-critical crack growth threshold under static fatigue. As an analogy to metals, they analysed their results in terms of a Paris law: $v = B(\Delta K)^m$, but two major differences were observed. On the one hand, the stress exponent m was equal to 24 so higher than that generally found for metals. On the second hand, they noted that the parameter $R = K_{\min}/K_{\max}$ (where $K_{\min}$ and $K_{\max}$ represent the minimum and the maximum stress intensity factors during one cycle, respectively) played a significant role which was not described by the Paris law. Following those results an extensive amount of literature was published during the 90s. In the early studies[30–32] the analysis and the modeling of fatigue were only based on pure mechanical cycling effects, without any consideration of stress corrosion (so-called static fatigue). It is now well recognised[33–34] that the intrinsic response to static or cyclic loading is the same, but there is a degradation of the reinforcement under alternate loading. It has been shown[35] that frictional and crack bridging degradations occur, leading to a decrease of R_ϕ. In this respect, the relation (13) can be modified to take into account cyclic effect:

$$G^* = G - (R_\phi(a,\ G^*) - \Delta R_{\phi(\text{cyclic})}) \tag{14}$$

where $\Delta R_{\phi(\text{cyclic})}$ corresponds to the mechanical degradation of R_ϕ. An example of such degradation in alumina can be shown in Fig. 8. It was obtained by first propagating a crack under constant loading, leading to an increase of R_ϕ. The sample was further submitted to cyclic loading for a given number of cycles taking care not to propagate the crack. The sample was subsequently reloaded monotonically in order to re-propagate the crack. The degradation for a constant crack length can therefore be quantified by the drop of R_ϕ. The degradation is a function of the number of cycles N.[36] The actual behaviour is more complex since crack length is not constant but generally propagates during cyclic loading. The net crack driving force is the resultant of two phenomena: increase of R_ϕ associated to crack extension and decrease of R_ϕ by damage accumulation. In this respect, Jacobs and Chen[33] have described the crack velocity as a balance between those two effects. During crack propagation under

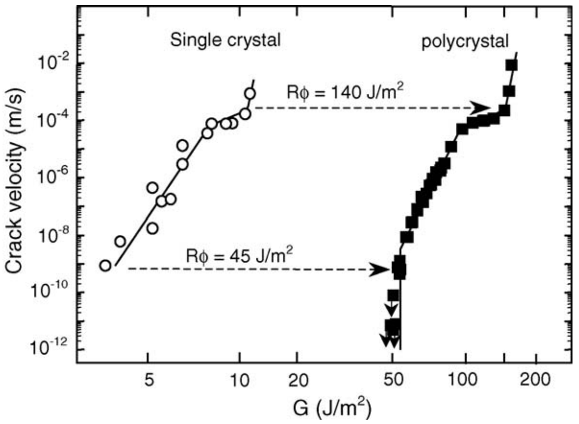

Fig. 7. Crack velocity vs. mechanical energy release rate (log–log scale) measured on zirconia single crystals and polycrystalline 3% yttria stabilised zirconia. The shift that corresponds to the reinforcement increases with load ($R_\phi = 45$ and 140 for $G \sim 50$ and 110, respectively).

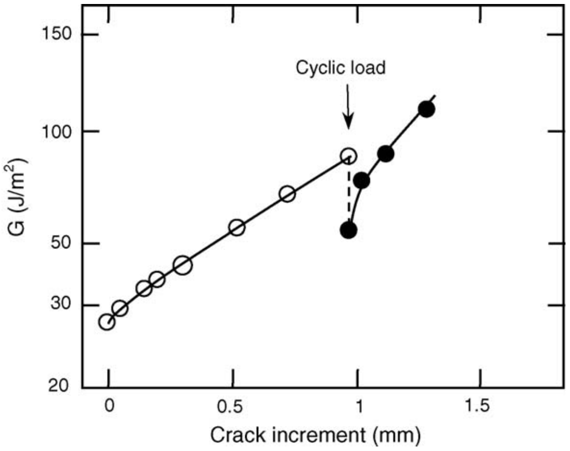

Fig. 8. Degradation of the reinforcement by cyclic load on a alumina (data from Ref. 36). The crack is propagated by applying a constant load, which leads to an increase of the reinforcement, up to point (A). A cycling load is subsequently applied such that no propagation occurs. When a subsequent constant load is applied the load required to propagate the crack, point (B) is lower, i.e. the reinforcement has been reduced.

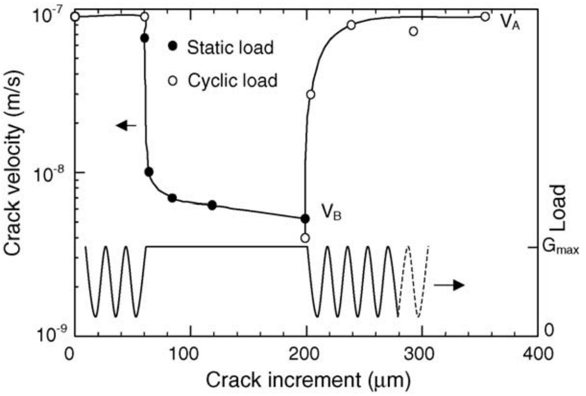

Fig. 9. Degradation of the reinforcement by cyclic load on a Y-TZP (data from Ref. 34). A cyclic load is applied between 0 and the $G_{\max}$ (the present configuration is such that the mechanical energy release rate is proportional to the load and independent of the crack length), leading to a velocity V_A. A constant load equal to $G_{\max}$ is further applied. This leads to a decrease of velocity, up to reach again a constant velocity V_B, owing to the increase of the reinforcement R_ϕ. Again a cyclic load is applied, and the crack velocity return backs to its initial value V_A. It is interesting to note that immediately after the application of the cyclic load, there is a transient velocity lower than V_B. This appears when there is not yet degradation, and because the average cyclic load is lower that the constant load.

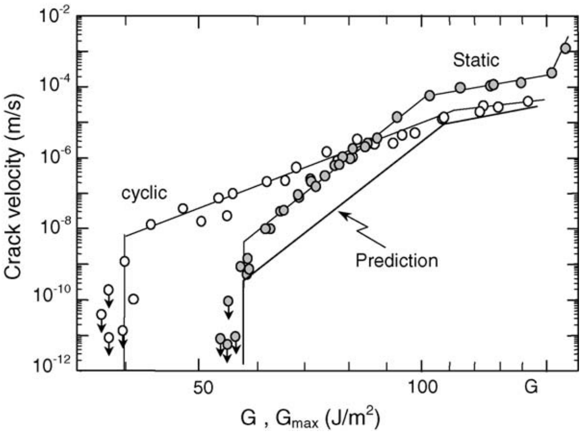

Fig. 10. Comparison of crack velocities under static loading (function of G) and cyclic loading (function of $G_{\max}$) for a Y-TZP ceramic, as measured in air at room temperature (data from Ref. 34). The solid line represents prediction of cyclic fatigue from static data under alternate loading, i.e. with the hypothesis of no cyclic effect, which underestimates cyclic fatigue.

cyclic loading a steady state can be reached and a constant crack velocity can be obtained. This is illustrated in Fig. 9 for again a Y-TZP. The specimen was first loaded under static loading under a constant value, so called G_{static}, and the crack velocity is recorded as a function of the crack extension. The specimen was then loaded under alternate loading ($G(t) = G_{\text{static}} \sin(\omega t)$), so that the maximum value was the same as that applied statically. The evolution of the crack velocity depends on two phenomena: on the one hand the average G value is lower than G_{static} and on the other hand there a progressive degradation of R_ϕ. At the early stage, the former is prominent since degradation is still close to zero. Then a progressive increase of the crack rate is related to a progressive degradation of R_ϕ and a plateau is reached. It corresponds to the steady state where damage accumulation and reinforcement just balance. Another transient regime followed by a plateau is again observed when further loading under G_{static}. It is interesting to note that in this example of fine microstructure (average grain size of 0.5 μm) the equilibrium velocity is reached after crack extension of about 20–30 μm, revealing that R_ϕ and/or ΔR_ϕ are steep function of crack length. In coarser microstructure materials this equilibrium would be shifted to larger crack extent[37] or even not reached if the specimen was too small.

The degradation of R_ϕ is obviously directly related to the number of cycles. However, since this process operates around a crack that propagates at a given velocity, the degradation is a function of time and hence frequency.

The behaviour under alternate loading can be characterised in terms of crack velocity versus $G_{\max}$ (representing the maximum value of G during one cycle). The example of the Y-TZP, where steady state velocity is easily reached is shown in Fig. 10 and compared to static crack propagation. This curve that was obtained at a frequency of 1 Hz and ratio $G_{\max}/G_{\min} = 0.1$ would be different for other solicitation parameters.[34] The shift between the two curves is more important for lower crack velocities (especially near the threshold). Indeed, at low velocity the number of cycles during a given crack extension is important and the effect of fatigue is more pronounced. On the contrary at high velocity, the number of cycles for the same crack increment becomes low and damage accumulation negligible. In this case (for velocity higher than 10^{-5} m/s for the present material) the crack velocity under cyclic loading is just given by the integration of the $v - G$ curve during one cycle.

Again the threshold represents the Griffith equilibrium given by Eq. (13) with R_ϕ lower than under static conditions. The shift of this equilibrium value should depend on the $G_{\max}/G_{\min}$ ratio but not on time (null crack velocity) and therefore not on the frequency.

1.4. General discussion

The general cracking behaviour depends on a series of mechanisms acting at different scales. At the atomic scale the crack rate is linked to the bound fracture rate and more precisely to the atom reorganisation in the near crack tip field. In highly brittle single crystal ceramics the behaviour is quasi elastic even at crack tip[5] and the sole dissipation mechanism is the atomic re-organisation.

At a higher microscopic level, plastic or pseudo-plastic mechanisms (phase transformation, bridging ...) can operate and decrease the net crack tip driving force. Even in glass, acknowledged as brittle, the size of the plastic zone is about 100 nm,[38] which obviously drives the $v - G$ curve. However, these microscopic mechanisms do not modify the intrinsic $v - G^*$ response.

The role of the environment on the cracking behaviour can be viewed as decreasing the surface energy but also as another dissipation mechanism which modifies the shape of the $v - G^*$ relation (e.g. stages I and II in air). The cracking behaviour of a given material thus depends on its affinity to water. It can be easily inferred that materials presenting a polar surface (e.g. some

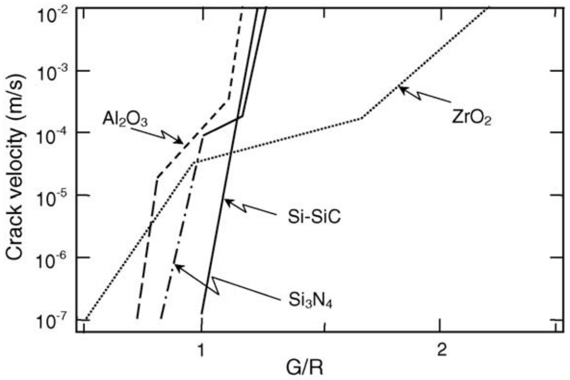

Fig. 11. Crack velocity vs. mechanical energy release rate rationalised by the crack resistance measured in vacuum (*G*/*R*), measured on different ceramic materials. The sensitivity to humidity (small slope that starts below *R*) appears as strongly linked to the ionic fraction of the bond The most covalent being a Si–SiC, without glassy phase, and on the contrary the most ionic being zirconia.

planes of materials with mainly ionic bonds) will be more sensitive to water than covalent bond materials. This is illustrated in Fig. 11 where the crack velocity in air is represented as function of G rationalised by the crack resistance in vacuum (G/R). On the one hand, the covalent Si–SiC material shows an abrupt $v-G$ relation and crack occurs only for $G>R$. On the other hand, the materials exhibiting high ionic fraction (for example, ZrO_2 with an ionic fraction of 0.6) show propagation well below R.

This general description allows to give simple answers to the questions arisen in the introduction. The most important being the definition and the measurement of the toughness. Toughness measurements are very often conducted at high speed. Despite the technical interest for such method, it is clear that such “toughness” only corresponds to one point of the $v-G$ curve, corresponding to relatively high velocity. More intrinsic parameters should be the resistance R and R_e, which must be measured at very low velocity. Such values are also interesting on a practical point of view since they control the long-term integrity of ceramic components. In this respect, slow crack growth, often referred as propagation below the toughness (subcritical crack growth) must be viewed as propagation above the threshold (which actually corresponds to G_c). The R-curve generally described as an increase of toughness with crack length simply reveals an increase of crack resistance R_ϕ. As a general pattern, R_ϕ can depend on crack length but also on loading conditions. For a given $v-G^*$, this will affect the measured $v-G$ relation and make extrapolation difficult for life-time prediction from the velocity function. However, again for practical use, the threshold appears as the fundamental parameter.

Fig. 12 represents the general description of crack propagation in brittle materials. The intrinsic response of a crystal structure is identical on a $v-G^*$ or $v-G$ graph, since there is no reinforcement. This response depends on the environment, especially on the sensitivity of atomic bounds to water. For more real polycrystalline ceramics, this response is shifted towards higher energy values, as a consequence of reinforcement. On the opposite, the partial degradation of reinforcement, for example by cyclic fatigue, shifts partially back the $v-G$ diagram to lower energies.

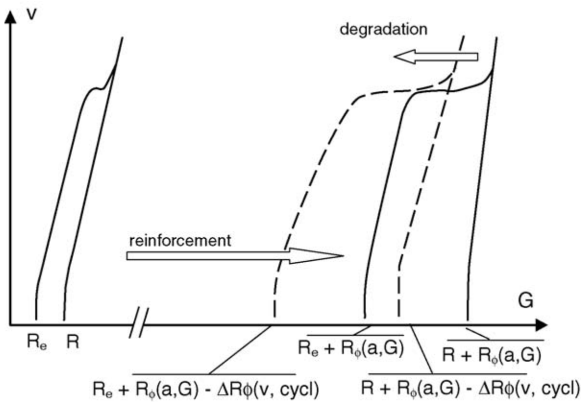

Fig. 12. Schematic drawing representing a synthesis of the cracking behaviour in ceramics. Near purely brittle materials exhibit a résistance R and R_e, that is shifted with reinforcement under static load. The presence of a cyclic loading leads to partial reinforcement degradation and hence lower resistance.

2. Conclusion

Although sometimes confusing because treated is separated items in the literature, the crack propagation in brittle materials can be simply described. The central aspect is the equilibrium of the crack, in terms of energy release rate, stated by the Griffith criteria. This equilibrium is funded by the physics and of practical importance since it corresponds to an energy value below which no propagation occurs.

References

1. Griffith, A. A., The phenomena of rupture and flow in solids. *Phil. Trans. R. Soc. A*, 1920, **221**, 163–198.
2. Griffith, A. A., The theory of rupture. In: *Proceedings of the First International Congress of Applied Mechanics*. Delft, 1920, pp. 55–63.
3. Irwin, G. R. and Kies, J. A., Fracturing and fracture dynamics. *Welding J.*, 1952, **31**, 95–100.
4. Maugis, D., Subcritical crack growth, surface energy, fracture toughness, stick-slip and embrittlement. *J. Mater. Sci.*, 1985, **20**, 3041–3073 (Review).
5. Lawn, B., Fracture of brittle solids. In *Cambrige Solid State Science Series*, ed. W. A. Davis and I. M. Ward. 2nd ed. Cambrige University Press, 1993.
6. Maugis, D., Adherence solids mechanical aspects adhesion adherence. *Rev. Métall.*, 1997, **94**(5), 690.
7. Obreimov, I. B., The splitting strength of mica. *Proc. R. Soc. A*, 1930, **127**, 290–297.
8. Roesler, F. C., Brittle fracture near equilibrium. *Proc. Phys. Soc. B*, 1956, **69**, 981–992.
9. Irwin, G. R., *Encyclopedia of Physics*, *Vol. 6*. Flügge, Springer-Verlag, Berlin, 1958, pp. 551–590.
10. Grenet, L., Recherches sur la résistance mécanique des verres. *Bull. Soc. Encour. Ind. Nat.*, 1899, **4**, 838–848.
11. Orowan, E., The fatigue of glass under stress. *Nature*, 1944, **154**, 341–349.
12. Wiederhorn, S. M., In *Environment Sensitive Mechanical Behaviour*, ed. A. R. C. Westwod and N. S. Stoloff. Gordon and Breach, New York, 1966, 293 pp.
13. Wiederhorn, S. M., Moisture assisted crack growth in ceramics. *Int. J. Fracture Mech.*, 1968, **4**(2), 171–177.

14. Wiederhorn, S. M. and Johnson, H., Effect of electrolyte pH on crack propagation in glass. *J. Am. Ceram. Soc.*, 1973, **56**, 192–197.
15. Michalske, T. A. and Freiman, S. W., A molecular interpretation of stress corrosion in silica. *Nature*, 1982, **295**(5849), 511–512.
16. Michalske, T. A. and Freiman, S. W., A molecular mechanism for stress corrosion in vitreous silica. *J. Am. Ceram. Soc.*, 1983, **66**(4), 284–288.
17. Michalske, T. A. and Fuller Jr., E. R., Closure and repropagation of heald crack in silica glass. *J. Am. Ceram. Soc.*, 1985, **68**(11), 586–590.
18. Rice, J. R. and Thomson, R. M., Ductile vs brittle behaviour of crystals. *Phil. Mag.*, 1974, **31**, 647.
19. Rice, J. R., Thermodynamics of the quasi-static growth of Griffith cracks. *J. Mech. Phys. Solids*, 1978, **26**, 61–78.
20. Wan, K. T., Aimard, N., Lathabai, S., Horn, R. G. and Lawn, B. R., Interfacial energy states of moisture-exposed cracks in mica. *J. Mater. Res.*, 1990, **5**(1), 172–182.
21. Wan, K. T., Lathabai, S. and Lawn, B. R., Crack velocity functions and thresholds in brittle solids. *J. Eur. Ceram. Soc.*, 1990, **6**, 259–268.
22. Wiederhorn, S. M., In *Mechanical and Thermal Properties of Ceramics*, ed. J. B. Wachtman. National Bureau of Standards, Gaithersburg, MD, 1969, p. 217 (Special Technical publication 303 NBS).
23. Knehans, R. and Steinbrech, R. W., Memory effect of crack resistance during slow crack growth in notched Al_2O_3 bend specimens. *J. Mater. Sci. Lett.*, 1982, **1**, 327–329.
24. Reichl, A. and Steinbrech, R. W., Determination of crack bridging forces in alumina. *J. Am. Ceram. Soc.*, 1988, **71**(6), C299–C301.
25. Ebrahimi, M. E., Chevalier, J. and Fantozzi, G., Slow crack-growth behavior of alumina ceramics. *J. Mater. Res.*, 2000, **15**(1), 142–147.
26. Fett, T. and Munz, D., Evaluation of *R*-curve effects in ceramics. *J. Mater. Sci.*, 1993, **28**, 742–752.
27. Fett, T., Munz, D., Geraghty, R. D. and White, K. W., Influence of specimen geometry and relative crack size on the *R*-curve. *Eng. Fracture Mech.*, 2000, **66**, 375–386.
28. McMeeking, R. M. and Evans, A. G., Mechanics of transformation toughening in brittle materials. *J. Am. Ceram. Soc.*, 1982, **65**(6), 242–246.
29. Evans, E. G. and Fuller, E. R., Crack propagation in ceramic materials under cyclic loading conditions. *Metall. Trans.*, 1974, **5**, 27–32.
30. Dauskardt, R., Yu, W. K. and Ritchie, R. O., Fatigue crack propagation in transformation toughened zirconia ceramics. *J. Am. Ceram. Soc.*, 1987, **70**(10), 248–252.
31. Liu, S. and Chen, I. W., Fatigue of yttria-stabilized zirconia: I. Fatigue damage, fracture origins, and lifetime prediction. *J. Am. Ceram. Soc.*, 1991, **74**(6), 1197–1205.
32. Liu, S. and Chen, I. W., Fatigue of yttria-stabilized zirconia: II. Crack propagation, fatigue striations, and short-crack behavior. *J. Am. Ceram. Soc.*, 1991, **74**(6), 1206–1216.
33. Jacobs, D. S. and Chen, I. W., Cyclic fatigue in ceramics: a balance between crack shielding accumulation and degradation. *J. Am. Ceram. Soc*, 1995, **78**(3), 513–520.
34. Chevalier, J., Olagnon, C. and Fantozzi, G., Subcritical crack propagation in 3Y-TZP ceramics: static and cyclic fatigue. *J. Am. Ceram. Soc.*, 1999, **82**(11), 3129–3138.
35. Lathabai, S., Rödle, J. and Lawn, B., Cyclic fatigue from frictionnal degradation at bridging grains in alumina. *J. Am. Ceram. Soc.*, 1991, **74**(6), 1340–1348.
36. El Attaoui, H., Saadaoui, M., Chevalier, J. and Fantozzi, G., Crack shielding degradation in alumina during cyclic fatigue. In: *Proceedings of the 7th Conference and Exhibition of The European Ceramic Society*. *Key Eng. Mater.*, 2002, 1137–1140.
37. Dauskart, R. H., Marshall, D. B. and Ritchie, R. O., Cyclic fatigue crack propagation in Mg-PSZ ceramics. *J. Am. Ceram. Soc.*, 1990, **73**(4), 893–903.
38. Henaux, S. and Creuzet, F., Kinetic fracture of glass at the nanometer scale. *J. Mater. Sci. Lett.*, 1997, **16**(12), 1008–1011.
39. Pauchard, V., *Etude des mécanismes de rupture des fibres dans les composites UD verre/époxy sous sollicitation de fatigue longitudinale en milieu humide*. PhD thesis, Ecole Centrale de Lyon, 2001, 159 pp.