Available online at www.sciencedirect.com
ScienceDirect
ELSEVIER
Journal of the European Ceramic Society 27 (2007) 3023–3026
www.elsevier.com/locate/jeurceramsoc

Exciting traveling waves in high Q structures using microstrip

A.C. Fowler [a], M.E. Tobar [a]*, C.R. Locke [a], E.N. Ivanov [a], J.G. Hartnett [a], J.D. Anstie [a], D. Cros [b]

[a] *Frequency Standards and Metrology Group, School of Physics, University of Western Australia, 35 Stirling Hwy, Crawley 6009, Australia*
[b] *XLIM-UMR CNRS, n 6172-123, Avenue Albert-Thomas, 87060 Limoges Cedex, France*

Available online 15 December 2006

Abstract

Exciting traveling waves in high-Q resonant structures can simplify low noise oscillator designs and also has applications for newly proposed Lorentz invariance tests. In this work we use microstrip probes to excite traveling waves in a sapphire dielectric resonator. Previous work has indicated that matching microstrip probes to a sapphire resonator can be a difficult requirement. A model has been developed, which takes into account leakage of the microstrip line in the reverse direction to which we excite the traveling wave. From such a model we can define the standing wave ratio (SWR). By comparing the model with experiment we find that we can excite a traveling wave with an SWR of 0.35 and an unloaded Q-factor of 1.5×10^5 in a $WGH_{12,0,2}$ mode at 9.997 GHz, in a sapphire cylinder of 5 cm diameter and 3 cm height. From the model, we also propose new oscillator designs for low noise applications.

Keywords: Dielectric properties; Al_2O_3; Microwave applications

1. Introduction

Whispering gallery modes (WGMs) in low-loss dielectrics such as sapphire have been well utilised in the development of ultrastable low noise microwave oscillators and precise tests of fundamental physics.[1–4] In this paper, WGMs refer to electromagnetic resonance modes in a cylindrical dielectric with large azimuthal mode number. It is informative to describe these modes as $WGE_{m,n,p}$ (quasi-TE) or $WGH_{m,n,p}$ (quasi-TM), where m, n and p are the azimuthal, radial and axial mode number, respectively. The mode energy exists around the edge of the cylinder, and can be thought of as a linear combination of two traveling waves of clockwise and counter-clockwise propagation. A standard magnetic or electric field probe excites both directions in equal amounts to form a standing wave resonance. For most applications, a standing wave regime is ideal. However, a traveling wave regime can simplify low noise oscillator designs[1–3] and is required in newly proposed Lorentz invariance tests.[4]

Excitation of traveling waves in a whispering gallery mode resonator has been described previously,[2] where parallel microstrip lines were used to create a "unidirectional" excitation method. However, the model assumed coupling into and out of the resonator parallel to the initial direction of travel in the microstrip only. These ideal conditions were not found to occur in our experimental investigation and in another reference.[5] In order to interpret these results, the model must include coupling in the antiparallel direction. This results in both directions of traveling waves coexisting in the resonator, with the same resonance frequency. In Section 2, we present new theory and simulations of such a model. In Sections 3 and 4, we present experimental results and possible applications.

2. Model allowing bidirectional coupling

The previous model assumed a single transmission coefficient κ_1 into and out of the resonator parallel to the direction of the traveling wave, as shown in Fig. 1a.[2] On resonance, the output at B looks like the reflection port of a standing wave resonator. The model predicts no signal at the input port A. To interpret our initial experimental investigations, which showed a strong response at the input with well matched microstrips, we describe a new model which includes an additional coupling coefficient κ_2 in the antiparallel direction. Four coupling configurations are considered, as shown in Fig. 1. We assume the microstrip is perfectly impedance matched at its joints, so line reflections are not a significant contribution. The complex response functions

* Corresponding author.
E-mail address: mike@physics.uwa.edu.au (M.E. Tobar).

0955-2219/$ – see front matter © 2006 Published by Elsevier Ltd.
doi:10.1016/j.jeurceramsoc.2006.11.040

at A and B are found by summing all possible traveling wave revolutions as geometric series, giving

$$\Gamma_\mathrm{A} = \frac{\kappa_1\kappa_2\,\mathrm{e}^{-(\alpha+\mathrm{i}\phi)}}{1-\sqrt{1-\kappa_1^2-\kappa_2^2}\,\mathrm{e}^{-(\alpha+\mathrm{i}\phi)}} + \frac{\kappa_1\kappa_2\,\mathrm{e}^{-(\alpha-\mathrm{i}\phi)}}{1-\sqrt{1-\kappa_1^2-\kappa_2^2}\,\mathrm{e}^{-(\alpha-\mathrm{i}\phi)}} \quad (1)$$

and

$$\Gamma_\mathrm{B} = \frac{\kappa_1^2\,\mathrm{e}^{-(\alpha+\mathrm{i}\phi)}}{1-\sqrt{1-\kappa_1^2-\kappa_2^2}\,\mathrm{e}^{-(\alpha+\mathrm{i}\phi)}} + \frac{\kappa_2^2\,\mathrm{e}^{-(\alpha-\mathrm{i}\phi)}}{1-\sqrt{1-\kappa_1^2-\kappa_2^2}\,\mathrm{e}^{-(\alpha-\mathrm{i}\phi)}} - \sqrt{1-\kappa_1^2-\kappa_2^2} \quad (2)$$

where α is the attenuation and ϕ is the phase shift in one revolution around the resonator. As in the previous model,[2] we can consider $\beta = (\kappa_1^2+\kappa_2^2)/2\alpha$ as the equivalent coupling into the resonator. The derivation assumes the reverse coupling is below a critical value, given by $\kappa_2^2/2\alpha < 1$, which is valid experimentally for the traveling wave regime. Resonance occurs when $\phi = 2\pi m$, where m is the azimuthal mode number. Also, the complex amplitude of the wave circulating in the resonator is given by

$$T_\mathrm{c} = \frac{\kappa_1\,\mathrm{e}^{-(\alpha+\mathrm{i}\phi)}}{1-\sqrt{1-\kappa_1^2-\kappa_2^2}\,\mathrm{e}^{-(\alpha+\mathrm{i}\phi)}} + \frac{\kappa_2\,\mathrm{e}^{-(\alpha-\mathrm{i}\phi)}}{1-\sqrt{1-\kappa_2^2-\kappa_2^2}\,\mathrm{e}^{-(\alpha-\mathrm{i}\phi)}} \quad (3)$$

Thus κ_1 and κ_2 characterise the amount of wave in the resonator in the counterclockwise and clockwise directions, respectively. In this case, one can define a standing wave ratio, $\mathrm{SWR} = \kappa_2/\kappa_1$. If $\mathrm{SWR} = 0$ there is a pure traveling wave. Fig. 2 shows the response at ports A and B, respectively, for $\alpha = 10^{-5}$ and $\alpha = 10^{-6}$, where $2\beta\alpha$ was fixed at 1.4×10^{-3}. The dotted, solid and dashed lines are for SWR = 1, 0.4 and 0, respectively. The input $|\Gamma_\mathrm{A}|^2$ and output $|\Gamma_\mathrm{B}|^2$, respectively, look like the transmission and reflection ports of a standing wave resonator. The size of the transmission peak is dependent on the SWR, the ratio between κ_1 and κ_2. The largest response is when SWR = 1. In the limit of unidirectional coupling, SWR = 0 and the signal is zero. The Q-factor of the transmission response at port A increased with decreasing α. For $\alpha = 10^{-6}$ in Fig. 2, the Q-factor of the response at port B can be seen to decrease with increasing SWR. This broadening effect is caused by doublet formation, which is clearly visible when α is decreased to 10^{-7}, as shown by the thicker line. The doublet splitting arises from a coupling between the two traveling waves, only visible when the bandwidth becomes less than the doublet splitting. As $\alpha = 10^{-7}$ corresponds to Q-factors of around 10^8, doublet splitting will not be visible at room temperature for low coupling. To further investigate this splitting regime, the model must be extended to allow $\kappa_2^2/2\alpha \gtrsim 1$. Doublet effects are often seen in standing wave whispering gallery mode resonators at cryogenic temperatures or when a mode is overcoupled, and can be explained in terms of localised perturbations.[6]

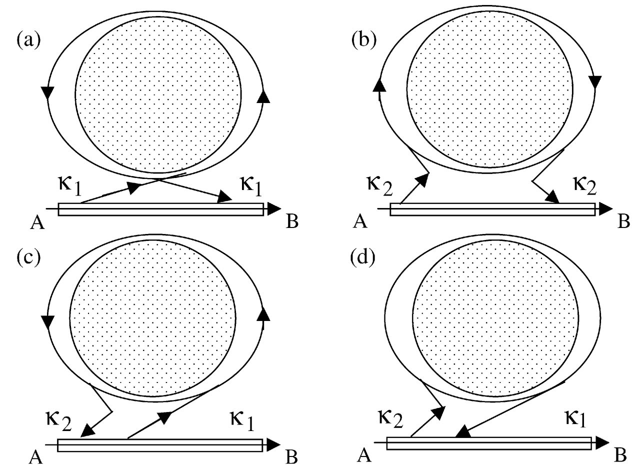

Fig. 1. Travelling wave resonator excited by adjacent microstrip line. Transmission into and out of the resonator is characterized by coefficients κ_1 and κ_2 parallel and antiparallel to the microstrip, respectively.

3. Experiment

WGMs in a cylindrical sapphire resonator of height 30.05 mm and diameter 49.95 mm were excited using a microstrip launcher provided by XLIM and a network analyzer. The launcher was well matched at the joints between the central conductor and the SMA microwave cable. The resonance frequencies of the fundamental modes had already been predicted using method of lines and measured using standing wave probes. These were verified in this experiment using the microstrip launchers as an alternative method of excitation. Resonant modes showed a reflection-like response at the output port, as predicted by Eq. (2). In addition, a transmission-like response was observed at the input port, as predicted by Eq. (1) for a non-zero κ_2. The modes chosen for further study had a symmetric shape and were identified as the $\mathrm{WGE}_{9,2,0}$ and $\mathrm{WGH}_{12,0,2}$ modes.

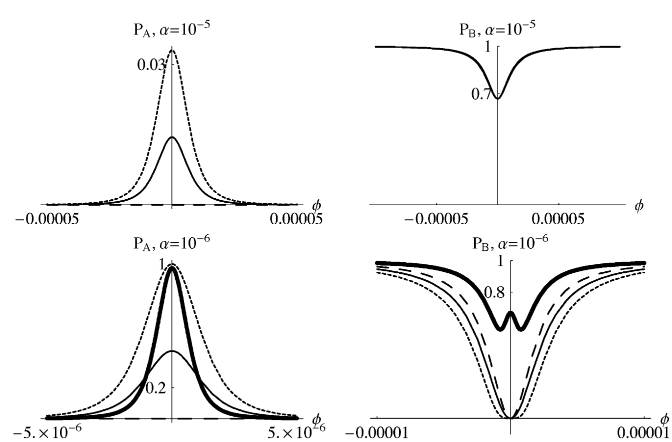

Fig. 2. $|\Gamma_\mathrm{A}|^2$ and $|\Gamma_\mathrm{B}|^2$ for $\alpha = 10^{-5}, 10^{-6}$, respectively. The dotted, solid and dashed lines are for SWR = 1, 0.4 and 0, respectively. The thick line shows SWR = 0.4, $\alpha = 10^{-7}$.

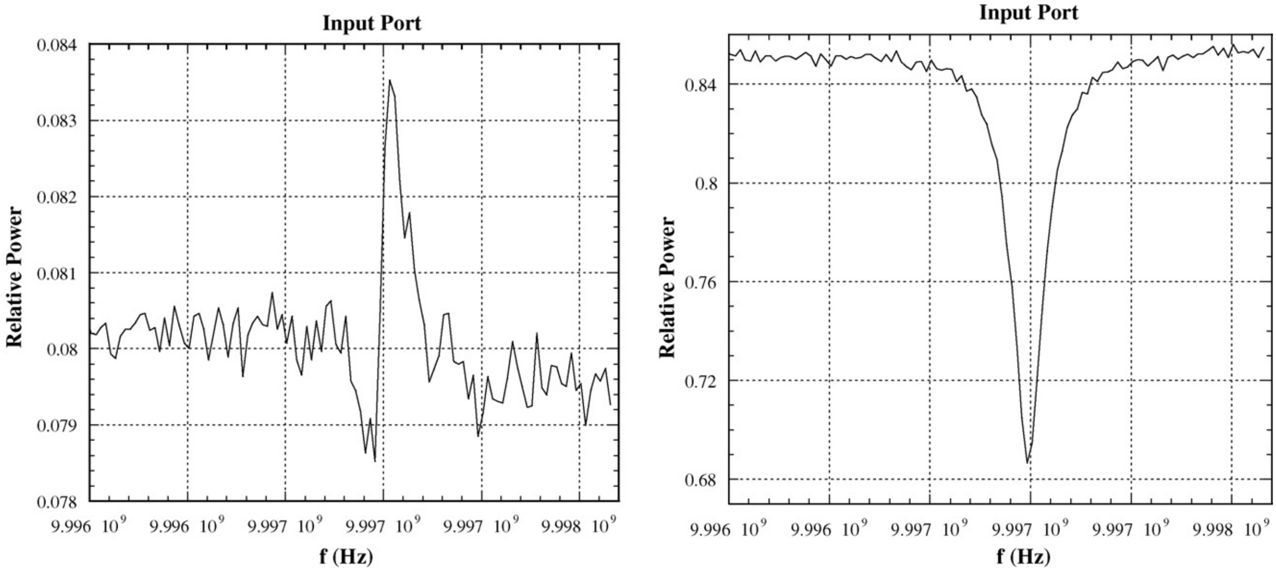

Fig. 3. Transmission-like response at input port and reflection-like response at output port for $WGH_{12,0,2}$ mode.

The Q-factor at room temperature was measured as a function of the distance between the microstrip probe and the crystal, reproducing the exponential dependence observed previously.[2] The loaded Q factor of the $WGE_{9,2,0}$ mode at 9.222 GHz, decreased from 8.3×10^4 with coupling 0.01 at 5.08 mm, to 4.6×10^3 with a coupling of 0.02 at 0.08 mm from the crystal, indicating that the prime source of loss is launcher itself. The dielectric and metal in the launcher add loss to the system, so that the effective unloaded Q-factor also varied exponentially with distance.

The responses at both ports for the $WGH_{12,0,2}$ mode at 9.997 GHz are shown in Fig. 3. From measurement of the Q factor and coupling coefficient, β, α was estimated to be 3.8×10^{-4}, with $Q_0 = 1.5 \times 10^5$. By then fitting the experimentally measured transfer functions to the expressions in Eqs. (1) and (2), κ_1 and κ_2 were estimated to be 0.0060 and 0.0021 in either order, and within the regime of assumptions made in the derivation. The model allows this degeneracy, but it is more likely that $\kappa_1 > \kappa_2$, as the TEM mode in the microstrip line propagates mostly in the forward direction, if well matched. Thus we can estimate that SWR ≈ 0.35 and we have excited a mode which has both a standing and traveling wave component.

4. Applications

It has been suggested that using microstrip to excite traveling modes in high Q structures could simplify low noise oscillator design and even further decrease the noise by removing the need for a circulator to separate the input and output wave from the resonator.[2] The proposed technique required two parallel microstrip lines. For example, Fig. 4 shows a traveling wave resonator with two ports incorporated into an interferometric frequency discriminator. The loop oscillator is driven by positive feedback from the transmission-like response at port 2 to the input port 1. The reflection-like response at port 3 is combined with the transmission-like repsonse in a hybrid coupler to form an interferometer. The dark port (DP) of the interferometer is amplified in a low-noise amplifier (LNA) and fed to the RF port of the mixer to create a low noise phase detector which provides a correction signal. The intrinsic noise in circulators is avoided as no circulator is required to separate the two ports. Our model shows that this technique is still valid even with a non-zero standing wave ratio.

In this work we have shown that exciting a whispering gallery mode traveling wave resonator using microstrip can produce a significant standing wave ratio, giving a transmission-like response at the input port and a reflection-like response at the output. These ports may both be utilized in oscillator stabilization designs, in which two parallel microstrip lines may be replaced with a single microstrip line. The transmission-like response at the input port can be used to drive the loop, while the reflection-like response at the output port may be used to produce an error signal for feedback. For example, the second microstrip line in Fig. 4 could be removed, since port 1 provides a transmission-like response for a non-zero standing wave ratio. Fig. 5 shows a modified Galani oscillator stabilisation technique, incorporating a traveling wave resonator with a standing wave ratio. A

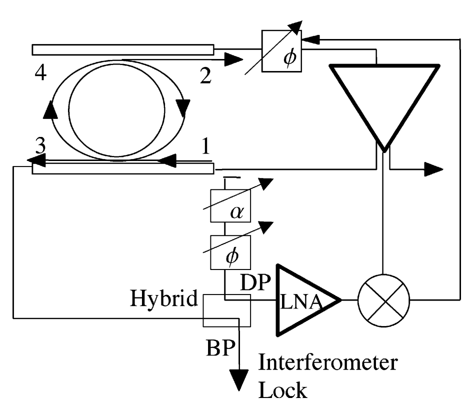

Fig. 4. Traveling wave resonator with two ports incorporated into interferometric frequency discriminator for oscillator stabilisation.

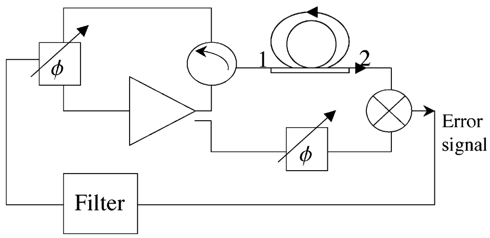

Fig. 5. Modified Galani oscillator stabilisation technique utilising travelling wave resonator with standing wave ratio.

circulator or hybrid power splitter is required to separate the input and the reflection responses. However, the circulator would be in series with the amplifier, the larger noise of which would dominate. A modified Pound lock system would also be possible. These modifications make use of the responses predicted by our bidirectional coupling model to simplify low noise oscillator design.

5. Conclusion

The subject of the excitation of traveling waves in high-Q dielectric structures has been a long-standing question in the field. In this work we have developed a more complete model to describe the coupling of microstrip lines to high-Q cylindrical dielectric resonators. The model takes into account leakage of the microstrip line in the reverse direction to which we excite the traveling wave. From such a model we can define the standing wave ratio (SWR) from the amount of forward and reverse traveling wave in the resonator. In our experiment, we were able to excite a traveling wave in a sapphire cylinder with a Q-factor of 1.5×10^5 and an SWR of 0.35. New oscillator designs for low noise applications can be further simplified by this non-zero standing wave ratio. Interferometric and Galani configurations were proposed.

References

1. Tobar, M.E., Ivanov, E.N., Blondy, P., Cros, D. and Guillon, P., Whispering gallery travelling wave resonator for oscillator frequency stabilization. In Proceedings of the 1999 IEEE International Frequency Contribution Symposium/13th European Frequency and Time Forum, 1999, pp. 589–592.
2. Tobar, M.E., Ivanov, E.N., Blondy, P., Cros, D. and Guillon, P., High-Q sapphire whispering gallery travelling wave resonators for oscillator frequency stabilization. *IEEE Trans. UFFC*, 2000, **47**(2), 421–426.
3. Tsarapkin, D.P. and Shtin, N.A., Performance limits of microwave oscillators with combined stabilization. In Proceedings of the 16th European Frequency and Time Forum, 2002, pp. C028-C031.
4. Tobar, M.E., Wolf, P., Fowler, A. and Hartnett, J.G., New methods of testing Lorentz violation in electrodynamics. *Phys. Rev. D*, 2005, **71**(2), 025004.
5. Tsarapkin, D.P. and Shtin, N.A. Whispering gallery traveling wave interferometer for low phase noise applications. In Proceedings of the 2004 IEEE IFCS, 2004, pp. 762–765.
6. Bourgeois, P.-Y. and Giordano, V., Simple model for the mode splitting effect in whispering gallery mode resonators. *IEEE Trans. MTT*, 2005, **53**(10), 3185–3190.